Code: AE01/AC01/AT01
Time: 3 Hours

JUNE 2011

Subject: MATHEMATICS-I
Max. Marks: 100

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 Minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: (2 × 10)

a. The value of $\lim\limits_{(x,y)\to(0,0)} \frac{x+y}{x-y}$ is

(A) 0 **(B)** 1
(C) −1 **(D)** limit does not exist

b. If $z = \log\left(x^2 + xy + y^2\right)$, then the value of $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}$ is

(A) 0 **(B)** 1
(C) 2 **(D)** 4

c. If u=f(x-y, y-z, z-x), then $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$ is

(A) x **(B)** y
(C) z **(D)** 0

d. The value of integral $\int_0^1 \int_0^{\sqrt{1-y^2}} x^3 y\,dx\,dy$ is

(A) $\frac{1}{24}$ **(B)** $\frac{1}{12}$
(C) $\frac{1}{6}$ **(D)** $\frac{1}{4}$

e. The solution of the differential equation $y\frac{dy}{dx} + x = 0$ is

(A) $x^2 - y^2 = c^2$ **(B)** $x^2 + y^2 = c^2$
(C) $x^2 y^2 = c^2$. **(D)** None of these.

f. The solution of the differential equation $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = e^{3x}$ is

(A) $y = (c_1 + c_2 x)e^{3x} + xe^{3x}$

(B) $y = c_1 e^{x} + c_2 e^{-3x} + xe^{3x}$

(C) $y = c_1 e^{x} + c_2 e^{3x} + \frac{xe^{3x}}{2}$

(D) $y = c_1 e^{-x} + c_2 e^{3x} + \frac{x}{2}e^{3x}$

g. The rank of the matrix $\begin{bmatrix} 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \\ 5 & 6 & 7 & 8 \\ 10 & 11 & 12 & 13 \end{bmatrix}$ is

(A) 4
(B) 3
(C) 2
(D) 1

h. If two eigen values of $\begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$ are 2 and 3, then third eigen value is

(A) 1
(B) -1
(C) 4
(D) 6

i. The value of $\frac{d}{dx}(J_0(x))$ is

(A) $J_1(x)$
(B) $-J_1(x)$
(C) $xJ_1(x)$
(D) $-xJ_1(x)$

j. The value of $\int_{-1}^{1} P_2(x)P_3(x)dx$ is

(A) 0
(B) 1
(C) -1
(D) none of these

Answer any FIVE Questions out of EIGHT Questions.
Each Question carries 16 marks.

Q.2 a. If z is a homogeneous function of x,y of degree n, prove using Euler's theorem that $x^2\frac{\partial^2 z}{\partial x^2} + 2xy\frac{\partial^2 z}{\partial x \partial y} + y^2\frac{\partial^2 z}{\partial y^2} = n(n-1)z$. **(8)**

b. If $x^x y^y z^z = c$, show that at x=y=z, $\frac{\partial^2 z}{\partial x \partial y} = -(x\log(ex))^{-1}$ **(8)**

Q.3 a. Expand sin(xy) in power of (x-1) and $\left(y-\dfrac{\pi}{2}\right)$, upto the second degree terms **(8)**

b. Change the order of integration and evaluate the integral

$$\int_0^3 \int_1^{\sqrt{4-y}} (x+y)\,dx\,dy$$ **(8)**

Q.4 a. Discuss the maximum and minimum values of $\sin x \sin y \sin(x+y)$ **(8)**

b. Solve the differential equation $(1+x+y+xy)^2 \dfrac{dy}{dx} = 1$ **(8)**

Q.5 a. Solve the differential equation $\dfrac{d^2y}{dx^2} + 4\dfrac{dy}{dx} + 3y = \sin x + xe^{3x}$ **(8)**

b. Use method of variation of parameters to solve $\dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} + y = e^x \log x$. **(8)**

Q.6 a. Solve the differential equation $x^2\dfrac{d^2y}{dx^2} - 2x\dfrac{dy}{dx} - 4y = x^2 + 2\log x$. **(8)**

b. Use elementary row transformations to find the inverse of $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$ **(8)**

Q.7 a. For what values of k, the equation x+y+z=1, 2x+y+4z=k and $4x+y+10z=k^2$ have a solution and solve them completely in each case. **(8)**

b. Find the eigen values and eigen vectors of $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ **(8)**

Q.8 a. Define a unitary matrix and show that $\begin{bmatrix} \alpha + i\gamma & -\beta + i\delta \\ \beta + i\delta & \alpha - i\gamma \end{bmatrix}$ is unitary matrix if $\alpha^2 + \beta^2 + \gamma^2 + \delta^2 = 1$. **(8)**

b. Solve in series the equation $\left(1-x^2\right)\dfrac{d^2y}{dx^2} + 2x\dfrac{dy}{dx} + y = 0$. **(8)**

Q.9 a. Show that $J_n(x) = \frac{x}{2n}\left[J_{n-1}(x) + J_{n+1}(x)\right]$ **(8)**

b. Show that $\int_{-1}^{+1} xP_n(x)P_{n-1}(x)dx = \frac{2n}{4n^2 - 1}$ **(8)**