JUNE 2008

Code: AE01/AC01/AT01 **Subject: MATHEMATICS-I**
Time: 3 Hours **Max. Marks: 100**

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q. 1. must be written in the space provided for it in the answer book supplied and nowhere else.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or best alternative in the following: (2 x 10)

a. The value of $\lim\limits_{(x,y)\to(2,-2)} \dfrac{x^2+xy+x+y}{x+y}$ is

(A) 3 **(B)** –3
(C) limit does not exist **(D)** –1

b. If $u = f(y/x)$, then

(A) $x\dfrac{\partial u}{\partial x} - y\dfrac{\partial u}{\partial y} = 0$ **(B)** $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = 0$
(C) $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = 2u$ **(D)** $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = 1$

c. If $x = r\cos\theta,\ y = r\sin\theta$, then the value of $\dfrac{\partial(x,y)}{\partial(r,\theta)}$ is

(A) 1 **(B)** r
(C) 1/r **(D)** 0

d. The value of integral $\int_0^2\int_0^x (x+y)\,dx\,dy$ is equal to

(A) – 4 **(B)** 3
(C) 4 **(D)** –3

e. The solution of the differential equation $\dfrac{dy}{dx} + \dfrac{y}{x} = x^2$ under the condition y(1)=1 is given by

(A) $4xy = x^3 + 3$ **(B)** $4xy = x^4 + 3$

(C) $4xy = y^4 + 3$ **(D)** $4xy = y^3 + 3$

f. The particular integral of the differential equation $\frac{d^2y}{dx^2} + a^2y = \sin ax$ is

(A) $-\frac{x}{2a}\cos ax$ **(B)** $\frac{x}{2a}\cos ax$

(C) $-\frac{ax}{2}\cos ax$ **(D)** $\frac{ax}{2}\cos ax$

g. The sum of the eigen values of $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 3 & -1 \\ 0 & -1 & 3 \end{pmatrix}$ is equal to

(A) 6 **(B)** -8

(C) 7 **(D)** -6

h. If $A\begin{bmatrix} 0 & 1 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix}$. Then, the matrix A is equal to

(A) $\begin{bmatrix} 2 & 1 \\ 0 & 0 \end{bmatrix}$ **(B)** $\begin{bmatrix} 0 & 1 \\ 2 & -1 \end{bmatrix}$

(C) $\begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix}$ **(D)** $\begin{bmatrix} 2 & 1 \\ -1/2 & -1/2 \end{bmatrix}$

i. The value of $\int_{-1}^{1} x^m P_n(x)dx$ (m being an integer $< n$) is equal to

(A) 1 **(B)** –1

(C) 2 **(D)** 0

j. The value of the $J_{-1/2}(x)$ is

(A) $\sqrt{(2/\pi x)}\cos x$ **(B)** $\sqrt{(2/\pi x)}\sin x$

(C) $\sqrt{(1/\pi x)}\cos x$ **(D)** $\sqrt{(2/\pi)}\cos x$

Answer any FIVE Questions out of EIGHT Questions.
Each Question carries 16 marks.

Q.2 a. Compute $f_{xy}(0,0),\ f_{yx}(0,0)$ for the function

$$f(x,y)=\begin{cases}\dfrac{xy^3}{x+y^2}, & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0)\end{cases}$$

Also discuss the continuity of $f_{xy},\ f_{yx}$ at (0,0). **(8)**

b. Find the minimum values of $x^2+y^2+z^2$ subject to the condition $xyz=a^3$. **(8)**

Q.3 a. The function $f(x,y)=x^2-xy+y^2$ is approximated by a first degree Taylor's polynomial about the point (2,3). Find a square $|x-2|<\delta, |y-3|<\delta$ with centre at (2,3) such that the error of approximation is less than or equal to 0.1 in magnitude for all points within the square. **(8)**

b. Find the Volume of the ellipsoid $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}+\dfrac{z^2}{c^2}=1$ **(8)**

Q.4 a. Solve the differential equation
$\left(3x^2y^3e^y+y^3+y^2\right)dx+\left(x^3y^3e^y-xy\right)dy=0$ **(8)**

b. Using the method of variation of parameters, solve the differential equation
$y''+3y'+2y=2e^x$.
(8)

Q.5 a. Find the general solution of the equation $y''+4y'+3y=x\sin 2x$. **(8)**

b. The eigenvectors of a 3 x 3 matrix A corresponding to the eigen values1, 1, 3 are $[1,0,-1]^T, [0,1,-1]^T, [1,1,0]^T$ respectively. Find the matrix A. **(8)**

Q.6 a. Test for consistency and solve the system of equations
$5x+3y+7z=4,\ 3x+26y+2z=9,\ 7x+2y+10z=5$ **(8)**

b. Given that $A=\begin{bmatrix}0 & 1+2i\\ -1+2i & 0\end{bmatrix}$ show that $(I-A)(I+A)^{-1}$ is a unitary matrix.
(8)

Q.7 a. Show that the transformation
$y_1 = x_1 - x_2 + x_3,\ y_2 = 3x_1 - x_2 + 2x_3,\ y_3 = 2x_1 - 2x_2 + 3x_3$ is non-singular. Find the inverse transformation. **(8)**

b. If $u = f(x, y), x = r\cos\theta, y = r\sin\theta,$ then show that
$$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = \left(\frac{\partial u}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial u}{\partial \theta}\right)^2.$$ **(8)**

Q.8 a. Find the power series solution about the origin of the equation
$x^2y'' + 6xy' + (6 + x^2)y = 0$. **(11)**

b. Find the value of $P_3(2.1)$. **(5)**

Q.9 a. Prove the orthogonal property of Legendre Polynomials. **(8)**

b. Show that $J_0^2 + 2J_1^2 + 2J_2^2 + \ldots\ldots = 1$ **(8)**