Code: AE-01/AC-01/AT-01 **Subject: MATHEMATICS-I**

JUNE 2007

Time: 3 Hours **Max. Marks: 100**

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q. 1. must be written in the space provided for it in the answer book supplied and nowhere else.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or best alternative in the following: **(2x10)**

a. The value of limit $\lim_{(x,y)\mapsto(0,0)} \frac{xy}{\sqrt{(x^2+y^2)}}$ is

(A) 0 **(B)** 1
(C) limit does not exist **(D)** -1

b. If $u = x^y$ then the value of $\frac{\partial u}{\partial x}$ is equal to

(A) 0 **(B)** yx^{y-1}
(C) xy^{x-1} **(D)** $x^y \log(x)$

c. If $z = \sin^{-1} \frac{x^2+y^2}{x+y}$, then the value of $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}$ is

(A) z **(B)** 2z
(C) tan(z) **(D)** sin(z)

d. The value of integral $\int_0^1 \int_{x^2}^{2-x} xy\, dx\, dy$ is equal to

(A) $\frac{3}{4}$ **(B)** $\frac{3}{8}$
(C) $\frac{3}{5}$ **(D)** $\frac{3}{7}$

e. The differential equation of a family of circles having the radius r and the centre on the x-axis is given by

(A) $y^2\left(1+(\frac{dy}{dx})^2\right)=r^2$

(B) $x^2\left(1+(\frac{dy}{dx})^2\right)=r^2$

(C) $r^2\left(1+(\frac{dy}{dx})^2\right)=x^2$

(D) $(x^2+y^2)\left(1+(\frac{dy}{dx})^2\right)=r^2$

f. The solution of the differential equation $\frac{d^2y}{dx^2}+y=0$ satisfying the initial conditions y(0) = 1, y(π/2) = 2 is

(A) y = 2 cos(x) + sin(x) **(B)** y = cos(x) + 2 sin(x)
(C) y = cos(x) + sin(x) **(D)** y = 2 cos(x) + 2 sin(x)

g. If the matrix $A=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix},\ B=\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix},\ C=\begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}$ then

(A) C=Acos(θ) – Bsin(θ) **(B)** C=Asin(θ) + Bcos(θ)
(C) C=Asin(θ) – Bcos(θ) **(D)** C=Acos(θ) + Bsin(θ)

h. The three vectors (1,1,-1,1), (1,-1,2,-1) and (3,1,0,1) are

(A) linearly independent **(B)** linearly dependent
(C) null vectors **(D)** none of these.

i. The value of $\int_{-1}^{1} P_3(x)\,P_4(x)dx$ is equal to

(A) 1 **(B)** 0
(C) $\frac{2}{9}$ **(D)** $\frac{2}{7}$

j. The value of the integral $\int \frac{1}{x} J_2(x)dx$ is

(A) $\frac{1}{x} J_1(x)+c$ **(B)** $\frac{1}{x} J_{-1}(x)+c$
(C) $-\frac{1}{x} J_1(x)+c$ **(D)** $J_1(x)+c$

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. For the function $f(x,y)=\begin{cases}\frac{xy(2x^2-3y^2)}{x^2+y^2}, & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0)\end{cases}$ show that $f_{xy}(0,0)\neq f_{yx}(0,0)$. **(8)**

b. Find the absolute maximum and minimum values of the function $f(x,y)=4x^2+9y^2-8x-12y+4$ over the rectangle in the first quadrant bounded by the lines x = 2, y = 3 and the coordinate axes. **(8)**

Q.3 a. If $f(x,y)=\tan^{-1}(xy)$, find an approximate value of f(1.1,0.8) using the Taylor's series quadratic approximation. **(8)**

b. Evaluate the integral $\iint_R \sqrt{(x^2+y^2)}\,dxdy$ by changing to polar coordinates, where R is the region in the x-y plane bounded by the circles $x^2+y^2=4$ and x^2+y^2 =9. **(8)**

Q.4 a. Find the solution of the differential equation (y-x+1)dy – (y+x+2) dx = 0. **(6)**

b. Solve the differential equation

$$\cot 3x\frac{dy}{dx}-3y=\cos 3x+\sin 3x,\ 0<x<\pi/2$$

(6)

c. Show that the functions 1, sin x, cos x are linearly independent. **(4)**

Q.5 a. Using method of undetermined coefficients, find the general solution of the equation $y''-4y'+13y=12e^{2x}\sin 3x$. **(8)**

b. Solve $x^2\frac{d^2y}{dx^2}-3x\frac{dy}{dx}+y=\log x\frac{\sin(\log x)+1}{x}$. **(8)**

Q.6 a. In an L-C-R circuit, the charge q on a plate of a condenser is given by $L\frac{d^2q}{dt^2}+R\frac{dq}{dt}+\frac{q}{C}=E\sin pt$. The circuit is tuned to resonance so that $p^2=1/LC$. If initially the current I and the charge q be zero, show that, for small values of R/L, the current in the circuit at time t is given by (Et/2L)sinpt. **(8)**

b. Find a linear transformation T from $\mathbb{R}^3$ into $\mathbb{R}^3$ such that

$$T\begin{pmatrix}1\\1\\1\end{pmatrix}=\begin{pmatrix}6\\2\\4\end{pmatrix},\ T\begin{pmatrix}1\\-1\\1\end{pmatrix}=\begin{pmatrix}2\\-4\\2\end{pmatrix},\ T\begin{pmatrix}1\\-2\\3\end{pmatrix}=\begin{pmatrix}6\\6\\5\end{pmatrix}.$$

(8)

Q.7 a. Examine, whether the matrix A is diagonalizable. $A=\begin{pmatrix}-2 & 2 & -3\\ 2 & 1 & -6\\ -1 & -2 & 0\end{pmatrix}$. If, so, obtain the matrix P such that $P^{-1}AP$ is a diagonal matrix. **(8)**

b. Investigate the values of μ and λ so that the equations $2x+3y+5z=9,\ 7x+3y-2z=8,\ 2x+3y+\lambda z=\mu$, has (i) no solutions (ii) a unique solution and (iii) an infinite number of solutions. **(8)**

Q.8 a. Find the power series solution about the point $x_0 = 2$ of the equation $y''+(x-1)y'+y=0$. **(11)**

b. Express f(x)= $x^4+2x^3-6x^2+5x-3$ in terms of Legendre Polynomial. **(5)**

Q.9 a. Express $J_5(x)$ in terms of $J_0(x)$ and $J_1(x)$. **(8)**

b. If $f(x)=\begin{cases}0, & -1<x\le 0\\ x, & 0<x<1\end{cases}$ show that

$$f(x)=\frac{1}{4}P_0(x)+\frac{1}{2}P_1(x)+\frac{5}{16}P_2(x)-\frac{3}{32}P_4(x)+--$$. **(8)**