Code: A-01/C-01/T-01 **Subject: MATHEMATICS-I**

Time: 3 Hours

DECEMBER 2006 **Max. Marks: 100**

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q. 1. must be written in the space provided for it in the answer book supplied and nowhere else.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or best alternative in the following: (2x10)

a. The value of limit $\lim_{(x,y)\to(0,1)} \tan^{-1}\left(\frac{y}{x}\right)$

(A) 0 **(B)** $\pi/2$

(C) $-\pi/2$ **(D)** does not exist

b. Let a function f(x, y) be continuous and possess first and second order partial derivatives at a point (a, b). If $P(a, b)$ is a critical point and $r = f_{xx}(a, b)$, $s = f_{xy}(a, b)$, $t = f_{yy}(a, b)$ then the point P is a point of relative maximum if

(A) $rt - s^2 > 0, r > 0$ **(B)** $rt - s^2 > 0$ and $r < 0$

(C) $rt - s^2 < 0, r > 0$ **(D)** $rt - s^2 > 0$ and $r = 0$

c. The triple integral $\iiint_T dx\, dy\, dz$ gives

(A) volume of region T **(B)** surface area of region T
(C) area of region T **(D)** density of region T

d. If $A^2 = A$ then matrix A is called

(A) Idempotent Matrix **(B)** Null Matrix
(C) Transpose Matrix **(D)** Identity Matrix

e. Let λ be an eigenvalue of matrix A then A^T, the transpose of A, has an eigenvalue as

(A) $\frac{1}{\lambda}$ (B) $1+\lambda$

(C) λ (D) $1-\lambda$

f. The system of equations is said to be inconsistent, if it has

(A) unique solution (B) infinitely many solutions

(C) no solution (D) identity solution

g. The differential equation $M(x, y)dx + N(x, y)dy = 0$ is an exact differential equation if

(A) $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$ (B) $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

(C) $\frac{\partial M}{\partial x} = \frac{\partial N}{\partial y}$ (D) $\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = 1$

h. The integrating factor of the differential equation $x(1+y^2)dy + y(1+x^2)dx = 0$ is

(A) $\frac{1}{x}$ (B) $\frac{1}{y}$

(C) xy (D) $\frac{1}{xy}$

i. The functions x, x^2, x^3 defined on an interval I, are always

(A) linearly dependent (B) homogeneous

(C) identically zero or one (D) linearly independent

j. The value of $J_1''(x)$, the second derivative of Bessel function in terms of $J_2(x)$ and $J_1(x)$ is

(A) $xJ_2(x) + J_1(x)$ (B) $\frac{1}{x}J_2(x) + J_1(x)$

(C) $\frac{1}{x}J_2(x) - J_1(x)$ (D) $J_2(x) - \frac{1}{x}J_1(x)$

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. Show that the function

$$f(x,y) = \begin{cases} \frac{x^2+y^2}{|x|+|y|}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

is continuous at (0, 0) but its partial derivatives f_x and f_y do not exist at (0, 0). **(8)**

b. Find the linear and the quadratic Taylor series polynomial approximation to the function $f(x,y) = 2x^3 + 3y^3 - 4x^2y$ about the point (1, 2). Obtain the maximum absolute error in the region $|x-1| < 0.01$ and $|y-2| < 0.1$ for the two approximations. **(8)**

Q.3 a. Find the shortest distance between the line $y = 10 - 2x$ and the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$. **(8)**

b. Evaluate the double integral $\iint_R xy\ dxdy$, where R is the region bounded by the x-axis, the line y = 2x and the parabola $x^2 = 4ay$. **(8)**

Q.4 a. Evaluate the integral $\iint_R (x-y)^2 \cos^2(x+y) dxdy$, where R is the parallelogram with successive vertices at $(\pi, 0)$, $(2\pi, \pi)$, $(\pi, 2\pi)$ and $(0, \pi)$. **(8)**

b. Show that $J_0^2 + 2(J_1^2 + J_2^2 + \dots\dots\dots) = 1$, where $J_n(x)$ is the Bessel function of n^{th} order. **(8)**

Q.5 a. Show that

$$\int_{-1}^{1} (1-x^2) P_m'(x)\, P_n'(x) dx = \begin{cases} 0 & \text{if } m \neq n \\ \frac{2n(n+1)}{2n+1}, & \text{if } m = n \end{cases}.$$ **(6)**

where $P_k(x)$ are the Legendre polynomials.

b. Find the power series solution about x =2, of the initial value problem

$$4y'' - 4y' + y = 0,\ y(2) = 0,\ y'(2) = \frac{1}{e}.$$

Express the solution in closed form. **(10)**

Q.6 a. Solve the initial value problem $y''' - 6y'' + 11y' - 6y = 0$ y(0) = 0, $y'(0) = 1$, $y''(0) = -1$. **(8)**

b. Solve $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = \frac{x^3}{1+x^2}$. **(8)**

Q.7 a. Show that set of functions $\left\{x, \frac{1}{x}\right\}$ forms a basis of the differential equation $x^2y'' + xy' - y = 0$. Obtain a particular solution when $y(1) = 1, y'(1) = 2$. **(6)**

b. Solve the following differential equations:

(i) $(2xy + x^2)y' = 3y^2 + 2xy$

(ii) $(6x - 4y + 1)dy - (3x - 2y + 1)dx = 0$ **(2×5 = 10)**

Q.8 a. Let $T : R^3 \rightarrow R^2$ be a linear transformation defined by $T\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{pmatrix} y+z \\ y-z \end{pmatrix}$. Taking $\left\{\begin{pmatrix}1\\1\\0\end{pmatrix}, \begin{pmatrix}0\\1\\1\end{pmatrix}, \begin{pmatrix}1\\0\\1\end{pmatrix}\right\}$ as a basis in R^3, determine the matrix of linear transformation. **(8)**

b. If $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ then show that $A^n = A^{n-2} + A^2 - I$, for $n \geq 3$. Hence find A^{50}. **(8)**

Q.9 a. Examine whether matrix A is similar to matrix B, where $A = \begin{bmatrix} 5 & 5 \\ -2 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ -3 & 4 \end{bmatrix}$. **(8)**

b. Discuss the consistency of the following system of equations for various values of λ:

$$2x_1 - 3x_2 + 6x_3 - 5x_4 = 3$$
$$x_2 - 4x_3 + x_4 = 1$$
$$4x_1 - 5x_2 + 8x_3 - 9x_4 = \lambda$$

and if consistent, solve it.

(8)