This Question Paper contains 8 Printed Pages.

050(E)

(JULY, 2008)

Time : 3.00 Hours] ***[Maximum Marks : 100***

Instructions :

1. Answer **all** the questions.
2. Write your answers according to the instructions given.
3. Begin each question from a new page.

SECTION - A

Given below are **15** *multiple choice questions, each carrying* **ONE** *mark. Write the serial number [(A) or (B) or (C) or (D)] which you feel is the correct answer of the questions.* **15**

1. In Δ ABC, if A(1, –6), B(–5, 2) and the centroid is G(–2, 1), then Co-ordinates of vertex C are ?

(A) (– 2, 1) (B) (–2, 6)

(C) (3, 2) (D) (–2, 7)

2. $d\{(a, 0), (0, b)\} = ?$

(A) a (B) b

(C) $|a-b|$ (D) $\sqrt{a^2+b^2}$

3. The t point of Parabola $y^2 = 20x$ is ? ($t \in$ R)

(A) $(5t, 4t^2)$ (B) $(5t^2, 4t)$

(C) $(5t^2, 10t)$ (D) $(t, 2t)$

4. If $y = 2x + c$ touches a parabola $y^2 = 16x$, then value of c is ...

(A) 2 (B) – 2

(C) 8 (D) $\sqrt{2}$

5. The equation of director circle of ellipse $\frac{x^2}{9}+\frac{y^2}{16}=1$ is ...

(A) $x^2+y^2=9$ (B) $x^2+y^2=16$

(C) $x^2+y^2=25$ (D) $x^2+y^2=7$

6. The eccentricity of hyperbola $x^2-y^2=144$ is ...

(A) $\sqrt{21}$ (B) $\sqrt{2}$

(C) $\sqrt{7}$ (D) $\sqrt{3}$

7. For non-null vectors $\bar{a}, \bar{b}, \bar{c}, \bar{d} \in R^3$ are distinct vectors, then $(\bar{a}\times\bar{b})\cdot(\bar{c}\times\bar{d})$ is ...

(A) $\begin{vmatrix} \bar{a}\cdot\bar{c} & \bar{a}\cdot\bar{d} \\ \bar{b}\cdot\bar{c} & \bar{b}\cdot\bar{d} \end{vmatrix}$ (B) $\begin{vmatrix} \bar{b}\cdot\bar{c} & \bar{b}\cdot\bar{d} \\ \bar{a}\cdot\bar{c} & \bar{a}\cdot\bar{d} \end{vmatrix}$

(C) $\begin{vmatrix} \bar{a}\cdot\bar{d} & \bar{a}\cdot\bar{c} \\ \bar{b}\cdot\bar{d} & \bar{b}\cdot\bar{c} \end{vmatrix}$ (D) $\begin{vmatrix} \bar{b}\cdot\bar{d} & \bar{b}\cdot\bar{c} \\ \bar{a}\cdot\bar{d} & \bar{a}\cdot\bar{c} \end{vmatrix}$

8. The projection of $\bar{a}=(1,1,1)$ on $\bar{b}=(2,2,1)$ is ...

(A) $\frac{5}{9}(2,2,1)$ (B) (1, 3, 2)

(C) (0, 0, 1) (D) $\frac{1}{9}(1,3,2)$

9. The direction of a line passing through points (3, 2, 1) and (5, 6, 7) is ...

(A) (8, 8, 8) (B) (2, 4, 3)

(C) (4, 3, 2) (D) (2, 4, 6)

10. The perpendicular distance between $6x-3y+2z=1$ and $12x-6y+4z=21$ is ...

(A) $\frac{63}{17}$ (B) $\frac{6}{31}$

(C) $\frac{12}{7}$ (D) $\frac{19}{14}$

11. The centre of sphere $|\bar{r}|^2-\bar{r}\cdot(2,4,6)+5=0$ is ...

(A) (2, 4, 6) (B) (1, 2, 3)

(C) (2, 1, 3) (D) (2, 3, 5)

12. $N(a, \delta)$ form of the set $\{x / |x+1| < 3,\ x \in R\}$ is ...

(A) N (1, 3) (B) N(2, 3)

(C) N(3, 1) (D) N (–1, 3)

13. For $\sqrt{x} - \sqrt{y} = \sqrt{a}$, $a > 0$, $\frac{dy}{dx} = ?$

(A) $\sqrt{x}$ (B) $\sqrt{y}$

(C) $\sqrt{\frac{y}{x}}$ (D) $\sqrt{\frac{x}{y}}$

14. $\int \frac{1}{x^2 + 4x + 5} dx = ?$

(A) $\tan^{-1}(x + 5) + c$ (B) $\tan^{-1}(x + 4) + c$

(C) $\tan^{-1}(x + 2) + c$ (D) $\tan^{-1}(5x + 4) + c$

15. $\int_1^4 \left(\frac{x^2+1}{x}\right)^{-1} \cdot dx = ?$

(A) $\log\left|\frac{17}{2}\right|$ (B) $\frac{1}{2}\log\left|\frac{17}{2}\right|$

(C) $2\log|17|$ (D) None of these

SECTION - B

Instruction : *In the following* ***16*** *to* ***30*** *questions each carries 1-1 mark. Answer your questions as requirement.* **15**

16. If a line $(a + 3)x + (a^2 - 9)y + (a - 3) = 0$ passes through origin, then find the value of a.

OR

Find K ; if the following lines

$2x - 5y + 3 = 0$

$5x - 9y + K = 0$

and $x - 2y + 1 = 0$ are concurrent.

17. Find the equation of parabola whose focus is $S(4, 0)$ and equation of its directrix is $x + 4 = 0$.

18. Find the tangents to the parabola $y^2 = 8x$ that is perpendicular to the line $x + 2y + 5 = 0$.

19. Prove that $(\bar{x} - \bar{y}) \times (\bar{x} + \bar{y}) = 2\,(\bar{x} \times \bar{y})$.

20. Obtain the cosine formula for a triangle by using vectors.

21. If the equation $|\bar{r}|^2 - \bar{r} \cdot (2, 1, 1) + 3 = 0$ represents a sphere, then find its radius.

22. Obtain equation of a sphere having extremities of its diameter are (1, 1, 1) and (2, 2, 1).

23. Find K if $f(x) = \begin{cases} kx - 1, & x < 2 \\ x & x \geq 2 \end{cases}$

is continuous at $x = 2$.

OR

Obtain $\lim\limits_{x \to 0} \dfrac{(2006)^x + (2005)^x - 2}{x}$.

24. Prove $f(x) = e^{\frac{1}{x}}$ is decreasing function for $x \neq 0$.

25. Find the approximate value of $\sqrt{28}$.

26. Verify Rolle's theorem for $f(x) = x^2$, $x \in [-2, 2]$.

27. Evaluate $\int \dfrac{\log x}{x}\, dx$.

OR

Evaluate : $\int [\sin^2 x + \sin 2x] e^x \, dx$.

28. Show that $\int_0^{\pi} x\, f(\sin x)dx = \frac{\pi}{2} \int_0^{\pi} f(\sin x)dx$.

29. Solve the differential equation $x\frac{dy}{dx} = y+2$.

30. Write down the order of the differential equation $\frac{d^2y}{dx^2} + 3y = 0$.

SECTION - C

Instruction : *In the following questions* ***31*** *to* ***40****, each question carries 2 marks.* **20**

31. Let A be $(3,-1)$ and $B(0,4)$. If $P(x,y) \in \overline{AB}$, obtain the maximum and minimum values of $3y - x$.

OR

Find the equations of lines containing the diagonals of the rectangle formed by the lines $x = 2$, $x = -1$, $y = 6$ and $y = -2$.

32. Find the maximum and minimum distances of points on the circle $x^2 + y^2 - 4x - 2y - 20 = 0$ from the point (10, 7).

OR

Prove that for every value of K, the circle $2x^2 + 2y^2 - 12x + Ky + 18 = 0$ touches the X axis.

33. Find the equation of Ellipse passing through the points (1, 4) and (– 6, 1).

34. Find the measure of angle between the asymptotes of hyperbola $3x^2 - 2y^2 = 1$.

35. Find a unit vector orthogonal to (2, 1, 1) and (1, 2, 3) .

36. Find the area of a parallelogram if its diagonals are $2\bar{i} + \bar{k}$ and $\bar{i} + \bar{j} + \bar{k}$.

37. Obtain : $\lim_{x \to \pi} \frac{\sqrt{10 + \cos x} - 3}{(\pi - x)^2}$

OR

Obtain : $\lim_{x \to 1} (1 - x) \tan\left(\frac{\pi x}{2}\right)$

38. Find : $\lim_{n \to \infty} \sum_{r=1}^{n} \left(\frac{1}{4r^2 - 1}\right)$

39. Find : $\int \frac{\sin 2x \, dx}{m^2 \sin^2 x - n^2 \cos^2 x}$.

40. Evaluate : $\int_0^1 x \sqrt{\frac{1-x^2}{1+x^2}} \, dx$.

OR

Show that : $\int_0^{\pi/2} \frac{dx}{2 + \cos x} = \frac{\pi}{3\sqrt{3}}$

SECTION - D

Instructions : *Given below are **41** to **50** questions.*
Each question carries 3 marks. Write your answer carefully. **30**

41. If G and I are respectively the centroid and incentre of the triangle whose vertices are A(–2, –1), B(1, –1) and C(1, 3), find IG.

42. If circle $x^2 + y^2 + 2x + fy + K = 0$ touches both the axes, then find f and K.

43. If $\bar{x} + \bar{y} + \bar{z} = \bar{0}$, then prove that $\bar{x} \times \bar{y} = \bar{y} \times \bar{z} = \bar{z} \times \bar{x}$.

OR

If the vectors $(a, 1, 1)$, $(1, b, 1)$ and $(1, 1, c)$ are coplaner vectors, then show that $\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 1$.

44. Find the shortest distance between the lines

$x = y = z$ and $\frac{x+1}{1} = \frac{y}{2} = \frac{z}{3}$.

45. Find the vector and cartesian equation of plane and distance from origin to the plane which passes through points A(1, 1, 0), B(0, 1, 1) and C(1, 0, 1).

46. Obtain : $\lim_{x \to 0} \frac{(1+mx)^n - (1+nx)^m}{x^2}$; $m, n \in \mathrm{N}$.

47. If $y = a \cos(\log x) + b \sin(\log x)$, then prove that $x^2 y_2 + x y_1 + y = 0$.

48. Using the mean value theorem, prove that

$$\frac{1}{1+x^2} < \frac{\tan^{-1} x - \tan^{-1} y}{x - y} < \frac{1}{1+y^2} \quad (x > y > 0).$$

OR

Show that curves $y = ax^3$ and $x^2 + 3y^2 = b^2$ are orthogonal curves. $(a \neq 0, \ b \neq 0)$.

49. Solve the differential equation :

$$x\frac{dy}{dx} - y + x \sin\left(\frac{y}{x}\right) = 0.$$

50. If the time is taken for horizontal range R is T, prove that angle of projection has measure $\tan^{-1}\left(\frac{g\mathrm{T}^2}{2\mathrm{R}}\right)$.

OR

Velocity of a projectile at the maximum height is $\sqrt{\frac{2}{5}}$ times its velocity at half the maximum height. Prove that angle of projection has measure $\frac{\pi}{3}$.

SECTION - E

***Instructions :** Each question carries 5 marks of the following **51** to **54** questions.*

Answer the following questions. **20**

51. In ΔABC, C is $(4, -1)$. The line containing the altitude from A is $3x + y + 11 = 0$ and the line containing the median $\overline{AD}$ through A is $x + 2y + 7 = 0$. Find the equations of lines containing the three sides of the triangle.

OR

Find the equation of the line that passes through the point of intersection of $3x - 4y + 1 = 0$ and $5x + y - 1 = 0$ and that cuts off intercepts of equal magnitude on the two axes.

52. $f(x) = \begin{cases} e^x & ; \ x \geq 0 \\ \log \ (x + e) & ; \ x < 0 \end{cases}$

If f continuous at $x = 0$? It is differentiable at $x = 0$? Why ?

53. Obtain : $\int \frac{dx}{\sin x + \sec x}$.

54. Obtain : $\int_1^4 x^3 \, dx$ as the limit of a sum.

OR

Prove that $\int_0^{\pi/2} \frac{x.\sec x}{1 + \tan x} dx = \frac{\pi}{2\sqrt{2}} \log \left(\sqrt{2} + 1\right)$.