FIITJEE Talent Reward Exam

for student presently in

Class 11

A

PAPER–1

Time: 3 Hours | **CODE 11A** | ***Maximum Marks: 214***

Instructions:

Caution: Question Paper CODE as given above MUST be correctly marked in the answer OMR sheet before attempting the paper. Wrong CODE or no CODE will give wrong results.

1. This Question Paper Consists of 7 Comprehension Passages based on Physics, Chemistry and Mathematics which has total **29 objective type questions.**
2. All the Questions are Multiple Choice Questions having only **one correct answer**. Each question from **Q. 1 to 9** carries **+6 marks** for correct answer and **–2 marks** for wrong answer. Each question from **Q. 10 to 29** carries **+8 marks** for correct answer and **–3 marks** for wrong answer.
3. Answers have to be marked on the OMR sheet.
4. The Question Paper contains blank spaces for your rough work. No additional sheets will be provided for rough work.
5. Blank papers, clip boards, log tables, slide rule, calculator, cellular phones, pagers and electronic devices, in any form, are not allowed.
6. **Before attempting paper write your Name, Registration number** and **Test Centre** in the space provided at the bottom of this sheet.

Note:

Check all the sheets of this question paper. Please ensure the same SET is marked on header of all the sheets inside as indicated above 'Maximum Marks' of this page. In case SET marked is not the same on all pages, immediately inform the invigilator and CHANGE the Questions paper.

Registration Number : ☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐

Name of the Candidate : ______________________________

Test Centre : ______________________________

Comprehension

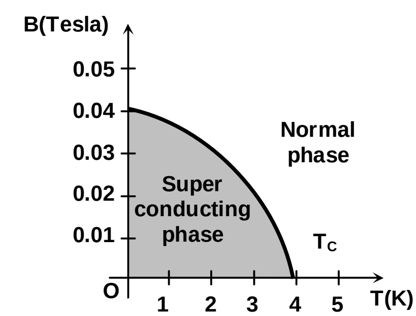
SECTION –I

COMPREHENSION – 1 (For question No. 1-3)

If temperature of some metals including its alloys and oxides is decreased, its resistance decreases. Below a certain temperature called critical temperature $\mathbf{T_C}$ (which varies from material to material) resistance becomes zero. In this state, the material is called a superconductor.

If such a material is initially placed in an external magnetic field and the temperature of the material is lowered, the transition to the superconducting state takes place at a lower critical temperature, i.e., $\mathbf{T_C}$ is lower than the value what it was in the absence of the magnetic field. Thus, the critical temperature $\mathbf{T_C}$ of a material depends on the strength of external magnetic field B in which it is placed. It is observed that if strength of external magnetic field is increased, the value of $\mathbf{T_C}$ decreases.

In the adjacent graph, B versus $\mathbf{T_C}$ is plotted for mercury, which is nearly a parabola. The graph says that beyond the value of B = 0.04T (T = tesla), $\mathbf{T_C}$ for mercury does not exist, i.e., transition to the superconducting state is not possible. The mathematical relation between magnetic field strength B (in tesla) and absolute temperature T (in Kelvin) is given by, $B = a - bT^2$, where a and b are positive constants.



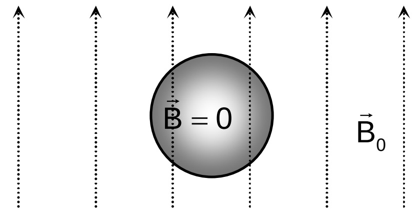
The temperature of the material also decides existence of magnetic field inside it. If its temperature is above $\mathbf{T_C}$, magnetic field inside it is nearly equal to the value of external field in which it is placed. If its temperature is decreased below $\mathbf{T_C}$, magnetic field inside the material vanishes, i.e., inside the super conductor magnetic field does not exist and external magnetic field lines get distorted.

Read above passage carefully and answer the following questions.

Space for Rough Work

1. Choose the correct graph for super conducting material (mercury) in presence of external field $\vec{B}_0$ at temperature T < $\mathbf{T_C}$ (critical temperature).

(A)



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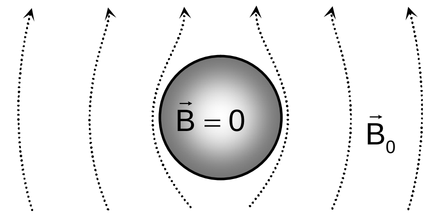

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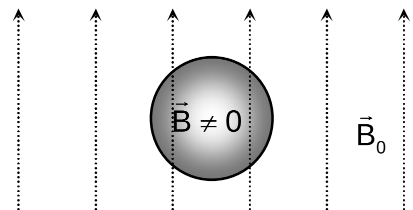

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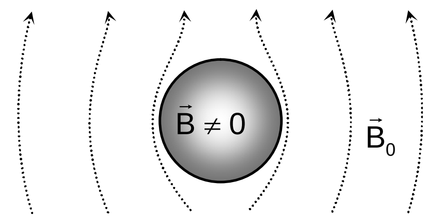


2. With the help of graph, find the value of (a × b).
(A) 0.0004 $(\text{Tesla/K})^2$
(B) 0.0003 $(\text{Tesla/K})^2$
(C) 0.0002 $(\text{Tesla/K})^2$
(D) 0.0001 $(\text{Tesla/K})^2$

3 Find the value of temperature (approximately) if $B_C(T) = 0.01$ Tesla.
(A) 1.5 K
(B) 2.5 K
(C) 3.5 K
(D) 4.0 K

Space for Rough Work

COMPREHENSION – 2 (For question No. 4-6)

In inorganic qualitative analysis, various basic radicals or cations are classified into different groups based on the solubility product principle. According to this principle, generally, a salt with lowest K_{sp} value precipitates first and the other with highest K_{sp} value precipitates in the last. Precipitation of a salt starts when its ionic product just exceeds the K_{sp} value of the salt.
Also, if some non–volatile solute (whose vapour presence is nearly zero) is added into a solvent, its freezing point decreases. The decrease in freezing point (ΔT_f) is directly proportional to the number of solute particles in the solution and is called a colligative property.
ΔT_f is calculated by using the following expression,

$$\Delta T_f = i \times K_f \times m \ldots\ldots\ldots..(i)$$

where K_f = molal depression constant of solvent and it is characteristic of solvent.
m = molal conc. of solution
and i = Van't Hoff factor which is calculated by

$$i = 1 + (y-1)\alpha \ldots\ldots\ldots\ldots.(2)$$

where y = no. of moles of ions or particles obtained by the dissociation or association of one mole of a solute and α is the degree of dissociation or association of solute.
For example for CH_3COOH, y = 2, and solution with higher ΔT_f has lower freezing point.

Four readily soluble salts A_2SO_4, BPO_4, CCl and DSO_4(all are strong electrolytes) and each having same molar concentration of 0.1M are present in 1 litre solution in a container. Now, the other readily soluble salt NaX (strong electrolyte) is added dropwise into this solution. Assume that Na_2SO_4. Na_3PO_4 and NaCl are readily soluble salts and do not precipitate under these conditions.

K_{sp} of $AX = 2 \times 10^{-6} M^2$; $K_{sp}(BX_3) = 1 \times 10^{-10} M^4$; $K_{sp}(CX) = 1 \times 10^{-8} M^2$ & $K_{sp}(DX_2) = 1 \times 10^{-13} M^3$.

Now answer the following questions:

4. The salt that precipitates first is
(A) DX_2
(B) CX
(C) AX
(D) BX_3

5. The salt that precipitates in the last is
(A) CX
(B) DX_2
(C) BX_3
(D) AX

6. If these four electrolytes A_2SO_4, BPO_4, CCl and DSO_4 each having 0.1M are present in separate containers, then increasing order of their freezing point will be (assume molality = molarity and each solute to be strong electrolyte)
(A) $A_2SO_4 < BPO_4 = CCl = DSO_4$
(B) $A_2SO_4 < BPO_4 < CCl = DSO_4$
(C) $CCl < A_2SO_4 < DSO_4 < BPO_4$
(D) $BPO_4 = CCl = DSO_4 < A_2SO_4$

Space for Rough Work

COMPREHENSION – 3 (For question No. 7-9)

Let P be a point (not the vertex) on a parabola;
Q be the point where the tangent to the parabola at P, meets the tangent at vertex of the parabola;
S be the focus of parabola.
Now according to a property : The tangent at vertex of the parabola is also the tangent (with point of contact as Q) to the circle described on the segment SP as diameter.
Let P, Q, S are taken at points (7, 13), (2, – 2) and (–1, –1) respectively and equation of parabola is f(x, y) = 0, then

7. The equation of tangent at vertex of the parabola is
(A) $x - 8y + 14 = 0$
(B) $8x - y + 14 = 0$
(C) $x + 8y + 14 = 0$
(D) $8x + y + 14 = 0$

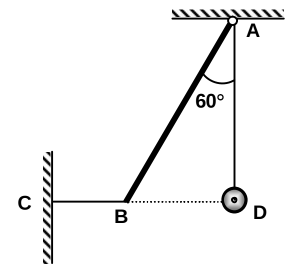
8. The equation of directrix of the parabola is
(A) $x - 8y + 19 = 0$
(B) $8x + y + 19 = 0$
(C) $8x - y + 19 = 0$
(D) $x + 8y + 19 = 0$

9. If the coefficient of y^2 in f (x, y) is 1, then the value of f (0, 0) is
(A) 321
(B) – 321
(C) 231
(D) – 231

Space for Rough Work

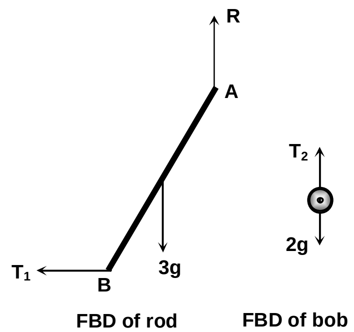
COMPREHENSION – 4 (For question No. 10-14)

In the adjacent figure, a uniform rod AB of mass 3 kg and length 60 cm is hinged at the end A to rotate in a vertical plane. A small sphere of mass 2 kg is suspended from the hinge (A) by a light and inextensible string AD of length 30 cm. The rod is taken aside, such that it makes an angle of 60° with the vertical, and to keep it in this position, its lower end B is connected to a fixed wall by a light and inextensible horizontal string BC. The rod as well as the bob is in equilibrium. Read above passage carefully and answer the following questions. (given g = 10 m/s^2)



10. Choose the correct free body diagram of the rod AB and bob (diagrams are not drawn in actual scale).

(A)



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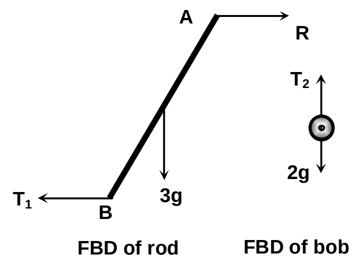

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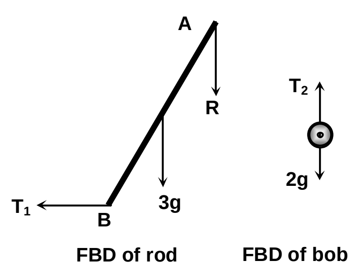
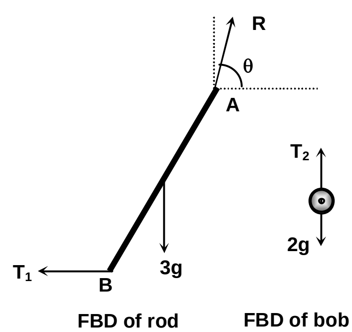


(D)



Space for Rough Work

11. If the tension in the string BC is T_1 and tension in string AD is T_2, the ratio of T_1/T_2 is
(A) 3 : 2 (B) 2 : 3
(C) $3\sqrt{3} : 4$ (D) $4 : 3\sqrt{3}$

Now the string BC is cut and rod is allowed to rotate. Now answer the following questions.

12. The speed of approach of the rod for the bob just before collision is?
(A) 1.5 m/s (B) 2 m/s
(C) 2.5 m/s (D) 3 m/s

13. Angular speed of the rod just after collision with the bob is (assuming collision is perfectly elastic)
(A) 4/3 rad/sec (B) 5/3 rad/sec
(C) 2/3 rad/sec (D) 8/3 rad/sec

14. The maximum height attained by the bob after impact (assuming collision is perfectly elastic) with rod will be?
(A) 10 cm (B) 15 cm
(C) 20 cm (D) 25 cm

Space for Rough Work

COMPREHENSION – 5 (For question No. 15-19)

According to the theory of hybridization, in polyatomic molecules the atomic orbitals of almost same or exactly same energy on the central atom redistribute their energy and form new atomic orbitals, exactly same in number and identical in energy and shape. These orbitals, also known as hybrid orbitals, overlap axially with the atomic orbitals of surrounding atoms. If the central atom in the molecule belongs to third row or below in the periodic table, then the lone pair will occupy a stereochemically inactive s–orbital and bonding will be through almost pure p–orbitals. In the presence of anionic field, all the five degenerate d-orbitals of the central atom split into two sets of orbitals namely $\left(d_{xy}, d_{yz}, d_{zx}\right)$ and $\left(d_{x^2-y^2}, d_{z^2}\right)$. All the orbitals in each of these sets are of almost same energy. Further a molecule can have some symmetry elements e.g. plane of symmetry, centre of symmetry and axis of symmetry.
A plane of symmetry divides the molecule into two identical halves which are the mirror images of each other. Further this plane can pass through any chemical bond or atom. Now answer the following questions:

15. In which of the following options, the covalent bond with maximum percentage of s-characters is
(A) P–H bond in phosphine (PH_3)
(B) As – H bond in Arsine (AsH_3)
(C) N–H bond in ammonia (NH_3)
(D) All bonds have equal % of s-character

16. Hybridization of nitrogen atom in cationic and anionic moiety of ammonium nitrate, respectively, is
(A) sp^3, sp^3
(B) sp^2, sp^3
(C) sp^3, sp^2
(D) sp^2, sp^2

17. Sulphur in SF_6 undergoes sp^3d^2 hybridization and attains octahedral geometry. The atomic orbitals used by sulphur atom in the formation of S–F bonds, in this molecule, are
(A) 4s, $4p_x$, $4p_y$, $4p_z$, $4d_{xy}$, $4d_{z^2}$
(B) 4s, $4p_x$, $4p_y$, $4p_z$, $4d_{x^2-y^2}$, $4d_{z^2}$
(C) 4s, $4p_x$, $4p_y$, $4p_z$, $4d_{xy}$, $4d_{x^2-y^2}$
(D) 4s, $4p_x$, $4p_y$, $4p_z$, $4d_{zx}$, $4d_{z^2}$

18. An alkaline earth metal 'M' which imparts brick–red colour to the flame, combines directly with the gas 'X' which is the predominant gas present in air, to produce a compound (A). Compound (A) reacts with water to produce a Gas (B) which dissolves in conc. H_2SO_4 to produce another compound (C). So, the number of planes of symmetry, in the cationic moiety of compound (C) are
(A) 02
(B) 03
(C) 05
(D) 06

19. Number of planes of symmetry in the molecule PF_3Cl_2 is/are
(A) 01
(B) 02
(C) 03
(D) 04

Space for Rough Work

COMPREHENSION – 6 (For question No. 20-24)

If we consider two variable positive numbers a and b whose sum and product are respectively S and P, then we know that:
If P is fixed, then $S_{min} = 2\sqrt{P}$, whereas
If S is fixed, then $P_{max} = \frac{S^2}{4}$.
These min/max occur when a = b. Now,

20. The minimum value of $y = \frac{4\pi}{\alpha} - \frac{9\beta}{\pi}$, where α and β are opposite angles (in radians) of a cyclic quadrilateral, is
(A) 0
(B) 1
(C) 2
(D) 3

21. If in the previous question $y = y_{min}$, then the value of tan α is
(A) 1
(B) $\sqrt{3}$
(C) 0
(D) $-\sqrt{3}$

22. The maximum value of $y = f(x)\, f\left(\frac{1}{x}\right)$, where $f(x) = \frac{1}{1+x}$, $x > 0$, is
(A) 2
(B) 1/4
(C) 1/2
(D) 1

23. The exhaustive set of values of $y = \log_e\left(\log_e\left(\frac{x}{4e} + \frac{e^3}{x}\right)\right)$ is
(A) $[0, \infty)$
(B) $[1, \infty)$
(C) $(-\infty, \infty)$
(D) $[-1, \infty)$

24. The minimum value of $y = \text{cosec}^2\theta - \cos^2\theta$ is
(A) 0
(B) 1/2
(C) 1
(D) 3/2

Space for Rough Work

COMPREHENSION – 7 (For question No. 25-29)

Let us define : For n, m $\in$ N

F_n = The sum of all the terms of a G.P. with first term $(1 + x)^n$ ($|x| < 1$); common ratio to be $\left(\frac{1+x}{2}\right)$ and the number of terms to be (n + 1).

S_m = $1 + x + x^2 + \ldots + x^m$; ($|x| < 1$). Now

25. The value of F_n is

(A) $\frac{2^{n+1}(1+x)^{n+1}-(1+x)^{2n+1}}{2^n(1-x)}$

(B) $\frac{2^n(1+x)^{n+1}-(1+x)^{2n}}{2^{n-1}(1-x)}$

(C) $\frac{2^{n+1}(1+x)^{n}-(1+x)^{2n+1}}{2^n(1-x)}$

(D) $\frac{2^n(1+x)^{n}-(1+x)^{2n}}{2^{n-1}(1-x)}$

26. The value of $2^n \cdot F_n$ is

(A) $\left[2^{n+1}(1+x)^n-(1+x)^{2n+1}\right]\cdot \lim\limits_{m\to\infty} S_m$

(B) $\left[2^{n+1}(1+x)^n-(1+x)^{2n}\right]\cdot \lim\limits_{m\to\infty} S_m$

(C) $\left[2^{n}(1+x)^n-(1+x)^{2n+1}\right]\cdot \lim\limits_{m\to\infty} S_m$

(D) $\left[2^{n}(1+x)^n-(1+x)^{2n}\right]\cdot \lim\limits_{m\to\infty} S_m$

27. Let P (x) be a polynomial of degree n < m, then the coefficient of x^n in $(P(x) \cdot S_m)$ is

(A) P (0)
(B) P′ (0)
(C) P (1)
(D) P′ (1)

28. The coefficient of x^n in F_n is

(A) 2^n
(B) 2^{n+1}
(C) 2^{2n}
(D) 2^{2n+1}

29. The value of $\sum_{r=0}^{n} {}^{n+r}C_n \left(\frac{1}{2}\right)^r$, is

(A) 2^n
(B) 2^{n+1}
(C) 2^{2n}
(D) 2^{2n+1}

Space for Rough Work

FIITJEE TALENT REWARD EXAM

(FTRE-2013)

CLASS XI
HINTS (SET-A)
PAPER-1

1. If its temperature is decreased below $\mathbf{T_C}$, magnetic field inside the material vanishes, i.e., inside the super conductor magnetic field does not exist and external magnetic field lines get distorted.

2. $B = a - bT^2$

   when T = 0, B = 0.04 $\Rightarrow$ a = 0.04

   when T = 4k $\Rightarrow$ B = 0 $\Rightarrow$ b = $\frac{a}{T^2} = \frac{0.04}{16}$

   $$a \times b = \frac{0.04}{100} \times \frac{0.04}{16 \times 100} = 10^{-4} = 0.0001\,(\text{Tesla/K})^2$$

3. $$0.01 = 0.04 - \frac{0.04}{16}T^2$$

   $$\Rightarrow \quad \frac{0.2}{4}T = \sqrt{0.03} = \frac{\sqrt{3}}{10} = \frac{1.732}{10}$$

   $$T = \frac{1.732 \times 4 \times 10}{10 \times 2} = 3.5\text{K}$$

Sol.: 4 & 5

$$\left[X^-\right]_{AX} = \left(\frac{K_{sp}}{\left[A^+\right]}\right) = \frac{2 \times 10^{-6}}{2 \times 10^{-1}} = 10^{-5}\text{M}$$

$$\left[X^-\right]_{BX_3} = \left(\frac{K_{sp}}{\left[B^{3+}\right]}\right)^{1/3} = \left(\frac{1 \times 10^{-10}}{1 \times 10^{-1}}\right)^{1/3} = \left(10^{-9}\right)^{1/3} = 10^{-3}\text{M}$$

$$\left[X^-\right]_{CX} = \left(\frac{K_{sp}}{\left[C^+\right]}\right) = \frac{1 \times 10^{-8}}{1 \times 10^{-1}} = 10^{-7}\text{M}$$

$$\left[X^-\right]_{DX_2} = \left(\frac{K_{sp}}{\left[D^{2+}\right]}\right)^{1/2} = \left(\frac{1 \times 10^{-13}}{1 \times 10^{-1}}\right)^{1/2} = 10^{-6}\text{M}$$

So, precipitation of these salts occurs in the order.

$CX < DX_2 < AX < BX_3$

6. A

$\Delta T_f \ \alpha \ i \times \text{molality}(m)$

Since 'm' is same for each electrolyte, so

$\Delta T_f \alpha \ i$

Now, for $A_2SO_4 \longrightarrow 2A^{1+} + SO_4^{2-}$ y = 3

$i = 1 + (3-1) \times 1 = 3$

Similarly for $BPO_4\,(i=2)$, $CCl(i=2)$ & $DSO_4\,(i=2)$

So, (A) is the correct answer

7. The circle is (x + 1) (x – 7) + (y + 1) (y – 13) = 0
i.e., $x^2 + y^2 - 6x - 12y - 20 = 0$.
The tangent to it at Q (2, – 2) will be 2x – 2y – 3 (x + 2) – 6 (y – 2) – 20 = 0.
i.e., x + 8y + 14 = 0.

8. Let it be x + 8y + k = 0.
Its distance from focus must be twice of the distance of tangent at vertex from focus.

So, $\frac{-1-8+k}{\sqrt{65}} = 2\left(\frac{-1-8+14}{\sqrt{65}}\right)$

$\Rightarrow$ k – 9 = 10 $\Rightarrow$ k = 19.

9. The equation of parabola : $(x + 1)^2 + (y + 1)^2 = \frac{(x+8y+19)^2}{65}$

$\Rightarrow 65(x + 1)^2 + 65(y + 1)^2 - (x + 8y + 19)^2 = 0$
As coefficient of y^2 in it is 1,
$f(0, 0) = 65 + 65 - 19^2 = -231$.

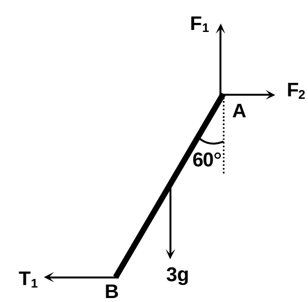
10. Self explanatory

11. According to translational equilibrium of Rod
$F_2 = T_1$ and $mg = F_1$
According to rotational equilibrium of rod, taking torque about A

$T_1 \times \ell\cos 60° \text{ (cw)} + Mg\frac{\ell}{2}\sin 60° \text{ (acw)} = 0$

$\frac{T_1}{2} = \frac{Mg\sqrt{3}}{4}$

$\Rightarrow \quad T_1 = \frac{Mg\sqrt{3}}{2} = 15\sqrt{3}\text{ N}$



FBD of rod

$F_2 = 15\sqrt{3}$ N, $F_1 = 30$ Kg

Hinge Reaction = $15\sqrt{(\sqrt{3})^2 + (2)^2}$ = $15\sqrt{7}$ N

$T_2 = 20$ N

$\frac{T_1}{T_2} = \frac{3\sqrt{3}}{4}$

12-14. Let ω_0 is the angular speed of the rod just before collision.
Using conservation of mechanical energy of the rod

$\frac{1}{2}I\omega_0^2 = Mg\frac{\ell}{2}(1-\cos\theta) \Rightarrow \omega_0 = 5$ rad/sec

Speed of approach of the rod just before collision = $\omega_0\frac{\ell}{2} = 1.5$ m/s.

Let after collision rod starts rotating with angular speed ω and bob starts moving horizontally with speed u.

Now using the conservation mechanical energy of rod and bob

$\frac{1}{2}I\omega_0^2 = \frac{1}{2}I\omega^2 + \frac{1}{2}mu^2 \Rightarrow 9 = 0.36\,\omega^2 + 2u^2$ …(i)

Using conservation of angular momentum about point A

$I\omega_0 = I\omega = mu\ell\cos\theta \Rightarrow \frac{1}{3}M\ell^2\omega_0 = \frac{1}{3}M\ell^2\omega + mu\ell\cos\theta \Rightarrow \omega = 5 - \frac{5}{3}u$ …(ii)

Putting the ω in equation (i), we get

So, $\omega = 5 - \frac{5}{3}\times 2 = 5 - \frac{10}{3} = \frac{5}{3}$ rad/sec

Using the conservation of mechanical energy of bob

$mg\left[\frac{\ell}{2} - \frac{\ell}{2}\cos\beta\right] = \frac{1}{2}mu^2$

$\cos\beta = \frac{1}{3}$

$\beta = \cos^{-1}\left(\frac{1}{3}\right)$ and height obtained by the bob

h = $h = \frac{\ell}{2} - \frac{\ell}{2}\cos\beta$ = 20 cm

Concept Involved:

1. FBD
2. Equilibrium (translation and rotational)
3. Definition of line of impact, speed of approach and speed of separation
4. Conservation of angular momentum and conservation of mechanical energy
5. conservation of mechanical energy

15. C

Since, Se and As both belong to 4^{th} period, so in these molecules bonding will be through pure p–orbitals. But nitrogen belongs to 2^{nd} period, so undergo sp^3 hybridisation in NH_3 and that's why N–H bond in NH_3 possesses maximum % of s–character.

16. C

$\underset{(sp^3)}{NH_4^+}\ \underset{(sp^2)}{NO_3^-}$

17. B

When hybridization involving d–orbitals are considered then all the five d–orbitals are not degenerate, rather $d_{x^2-y^2}, d_{z^2}$ and d_{xy}, d_{yz}, d_{zx} form two different sets of orbitals and since $d_{x^2-y^2}$ and d_{z^2} are of almost same energy, so these are involved in sp^3d^2 hybridization.

18. D

Metal 'M' is calcium (Ca)

$$3Ca + \underset{(X)}{N_2} \longrightarrow \underset{(A)}{Ca_3N_2}$$

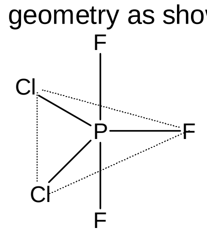
$$Ca_3N_2 + 6H_2O \longrightarrow 3Ca(OH)_2 + \underset{(B)}{2NH_3}(g)$$

$$2NH_3 + Conc.H_2SO_4 \longrightarrow \underset{(C)}{(NH_4)_2SO_4}$$

So, the cation present in compound (C) is NH_4^+ which is regular tetrahedron in shape and thus possess six planes of symmetry

19. B

PF_3Cl_2 possess trigonal bibyramidel geometry as shown below



Thus it possess only two planes of symmetry

20. $y = \frac{4\pi}{\alpha} - \frac{9\beta}{\pi} = \frac{4\pi}{\alpha} - \frac{9}{\pi}(\pi - \alpha) = \left(\frac{4\pi}{\alpha} + \frac{9\alpha}{\pi}\right) - 9 \geq 2\sqrt{36} - 9 = 3$.

21. $\frac{4\pi}{\alpha} = \frac{9\alpha}{\pi} \Rightarrow \alpha = \frac{2\pi}{3}$; so $\tan\alpha = -\sqrt{3}$.

22. $y = f(x)f\left(\frac{1}{x}\right) = \left(\frac{1}{1+x}\right)\left(\frac{x}{1+x}\right) \leq \frac{\left(\frac{1}{1+x} + \frac{x}{1+x}\right)^2}{4} = \frac{1}{4}$.

23. $\frac{x}{4e} + \frac{e^3}{x} \geq 2\sqrt{\frac{e^2}{4}} = e$;

So, $y = \log\left(\log\left(\frac{x}{4e} + \frac{e^3}{x}\right)\right) \geq \log(\log e) = 0$.

24. $y = \text{cosec}^2\theta - \cos^2\theta$

$= \frac{1}{\sin^2\theta} - (1 - \sin^2\theta) = \left(\sin^2\theta + \frac{1}{\sin^2\theta}\right) - 1 \geq 2 - 1 = 1$.

25. $F_n = \frac{(1+x)^n\left(1 - \left(\frac{1+x}{2}\right)^{n+1}\right)}{1 - \left(\frac{1+x}{2}\right)} = \frac{2^{n+1}(1+x)^n - (1+x)^{2n+1}}{2^n(1-x)}$.

26. $2^nF_n = \left(2^{n+1}(1+x)^n - (1+x)^{2n+1}\right) \cdot \frac{1}{(1-x)} = \left(2^{n+1}(1+x)^n - (1+x)^{2n+1}\right) \cdot \lim_{m\to\infty} S_m$.

27. Coefficient of x^n in $\left(\left(a_0 + a_1x + a_2x^2 + ... + a_nx^n\right)\left(1 + x + x^2 + ... + x^m\right)\right)$,
(where n < m), is $a_0 + a_1 + a_2 + \ldots + a_n = p(1)$.

28. Coefficient of x^n in 2^nF_n
= coefficient of x^n in $(2^{n+1}(1 + x)^n - (1 + x)^{2n+1})(1 + x + x^2 + x^3 + \ldots)$
= Sum of all the coefficients of terms from degree zero to degree n in $(2^{n+1}(1 + x)^n - (1 + x)^{2n+1})$
$= 2^{n+1}(1 + 1)^n - \frac{1}{2}\left(2^{2n+1}\right) = 2^{2n+1} - 2^{2n} = 2^{2n}$
So, coefficient of x^n in $F_n = 2^n$.

29. $\sum_{r=0}^{n} {}^{n+r}C_n \left(\frac{1}{2}\right)^r$ = coefficient of x^n in $\sum_{r=0}^{n}(1+x)^{n+r}\left(\frac{1}{2}\right)^r$
= Coefficient of x^n in $\sum_{r=0}^{n}(1+x)^n\left(\frac{1+x}{2}\right)^r$ = coefficient of x^n in $F_n = 2^n$.