S.E. sem 4 EXTC Signals and Systems. 19/12/08

Con. 5700-08. **RC-6162.**

(3 Hours) **[Total Marks : 100**

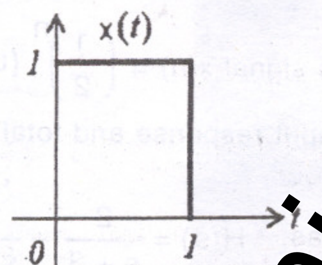
N.B.: (1) Question No. **1** is **compulsory**.
(2) Attempt any **four** questions out of remaining **six** questions.
(3) Assume **suitable data if required.**

1. (a) A LTI system is stable if, $\int_{-\infty}^{\infty} |h(t)|\, dt < \infty$. Justify. **20**

(b) Determine which of the following signals are periodic or nonperiodic. If the sequence is periodic, determine its fundamental period.

(i) $x(n) = \cos(3\pi n)$ (ii) $x(n) = \cos\left(\frac{n}{8}\right)\cos\left(\frac{\pi n}{8}\right)$

(c) Find out the even and odd components of the signal shown in **figure**.



(d) Determine whether the following discrete time signals are linear or nonlinear.
(i) $y(n) = x(n^2)$ (ii) $y(n) = x^2(n)$

(e) Determine whether the following continuous time signals are causal or noncausal.
(i) $y(t) = x(t) \cos(t + 1)$ (ii) $y(t) = x(2t)$

2. (a) Determine magnitude and phase coefficients of the Fourier coefficients of the signal **10**

$$x(t) = 1 + \sin \omega_0 t + 2 \cos \omega_0 t + \cos\left(2\omega_0 t + \frac{\pi}{4}\right)$$

(b) What is orthogonal function space or signal space ? Explain with sketches. Assuming that an arbitrary function f(t) is approximated by a orthogonal set of functions $g_r(t)$, r = 0, 1, 2, 3, ... **10**

$$f(t) \approx \sum_{r=0}^{n} C_r\, g_r(t)$$

Derive an expression for the general coefficient C_r.

3. (a) Derive an expression for convolution sum formula for a continuous time system **8**

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

State properties of continuous time convolution.

(b) Compute the output y(t) for a continuous time LTI system whose impulse response h(t) and the input x(t) are given by **8**

$$h(t) = e^{-at} u(t) \qquad x(t) = e^{\alpha t} u(-t)$$

(c) Find linear convolution of two sequences **4**

$x_1(n) = \{2, 1, 1, 2\}$ and $x_2(n) = \{0, 1, 2, 3\}$

4. (a) Find and sketch the Fourier transform $X(\omega)$ of the rectangular pulse signal $x(t)$ defined by 8

$$x(t) = \begin{cases} 1 & |t| < a \\ 0 & |t| > a \end{cases}$$

(b) Explain and prove Time Shifting and Frequency Shifting property of Fourier Transform . 8

(c) Explain Gibb's phenomenon. 4

5. (a) A continuous time function $x(t)$ is sampled by a periodic impulse train $\sum_{n=0}^{\infty} \delta(t - nT)$ with period 'T'. 10

The sampled function $x_s(t)$ is given by $x_s(t) = \sum_{n=0}^{\infty} x(nT)\,\delta(t - nT)$. Show that the z-transform of $x(nT)$ equals the Laplace Transform of $x_s(t)$ with $z = e^{sT}$.

(b) Determine z-transform including ROC for the signal $x(n) = \left(\frac{1}{2}\right)^n \{u(n) - u(n-10)\}$. 6

(c) Explain what is zero state response, zero input response and total response. 4

6. (a) The transfer function of the system is given as, $H(s) = \frac{2}{s+3} + \frac{1}{s-2}$. 8

Determine the impulse response if the system is,
(i) Stable (ii) Causal
Whether this system will be stable and causal simultaneously ?

(b) State and prove initial and final value theorem in z-transform. 6

(c) Determine Fourier transform of. 6
(i) Continuous time signal $x(t) = \cos \omega_0 t$
(ii) Discrete time signal $y(n) = \cos\omega_0 n$
(iii) Comment on the results in parts (i) and (ii).

7. (a) Develop the block diagram and state variable model of the system described by the differential equation 8

$$\frac{d^2 y(t)}{dt^2} + \frac{3\,dy(t)}{dt} + 2y(t) = u(t)$$

where $y(t)$ is the output, and $u(t)$ is any input.

(b) Obtain the state transition matrix for the system matrix given by— 8

$$A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$$

(c) State properties of state transition matrix. 4