SECTION - A

$10 \times 2 = 20$

VERY SHORT ANSWER TYPE QUESTIONS

Attempt ALL questions. Each question carries 2 marks.

1. If $x^2 - 6x + 5 = 0$, $x^2 - 12x + p = 0$ have a common root, find p.

2. Find the equation whose roots are squares of the roots of $x^3 - x^2 + 8x - 6 = 0$.

3. If $A = \begin{bmatrix} 2 & 3 & 1 \\ 6 & -1 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 & -1 \\ 0 & -1 & 3 \end{bmatrix}$, find the matrix X such that $A + B - X = O$.

4. If $\begin{vmatrix} 1 & 0 & 0 \\ 2 & 3 & 4 \\ 5 & -6 & x \end{vmatrix} = 45$, find x.

5. Find the number of 5 letter words that can be formed using the letters of the word NATURE that begin with N when repetition is allowed.

6. If $10 \cdot {}^nC_2 = 3 \cdot {}^{(n+1)}C_3$, find n.

7. If the coefficients of $(2r+4)$th and $(r-2)$nd terms in the expansion of $(1+x)^{18}$ are equal, find r.

8. Show that $\frac{1}{2} - \frac{1}{2} \cdot \frac{1}{2^2} + \frac{1}{3} \cdot \frac{1}{2^3} - \frac{1}{4} \cdot \frac{1}{2^4} + \ldots = \log_e\left(\frac{3}{2}\right)$.

9. If $P(A) = 0.5$, $P(B) = 0.4$, $P(A \cap B) = 0.3$, find the probability that neither A nor B occurs.

10. Find the least number of times a fair coin must be tossed so that the probability of getting atleast one head is atleast 0.8.

SECTION - B

$5 \times 4 = 20$

SHORT ANSWER TYPE QUESTIONS

Attempt any 5 questions. Each question carries 4 marks.

11. If x is real, find the minimum and maximum values of $\dfrac{x^2 + 14x + 9}{x^2 + 2x + 3}$.

12. Show that ${}^nP_r = {}^{(n-1)}P_r + r \cdot {}^{(n-1)}P_{r-1}$.

13. A double decker minibus has 8 seats in the lower and 10 seats in the upper deck. Find the number of ways of arranging 18 persons in the bus if 3 children want to go to the upper deck and 4 old people can not go to the upper deck.

14. Resolve $\dfrac{x^4}{(x-1)(x-2)}$ into partial fractions.

15. Show that $\dfrac{3\cdot5}{1!}x + \dfrac{4\cdot6}{2!}x^2 + \dfrac{5\cdot7}{3!}x^3 + \ldots = (x^2 + 7x + 8)\,e^x - 8$.

16. Show that every square matrix can be uniquely expressed as a sum of a symmetric matrix and a skew symmetric matrix.

17. A, B, C are three news papers published from a city. 20% of the population read A, 16% read B, 14% read C, 8% read both A and B, 5% read both A and C, 4% read both B and C and 2% read all the three. Find the percentage of the population who read at least one paper.

SECTION - C

$5 \times 7 = 35$

LONG ANSWER TYPE QUESTIONS

Attempt **any 5** questions. Each question carries 7 marks.

18. Solve the equation $x^4 - 9x^3 + 27x^2 - 29x + 6 = 0$, one root being $2 - \sqrt{3}$.

19. Show that $\dfrac{C_1}{C_0} + 2\dfrac{C_2}{C_1} + 3\dfrac{C_3}{C_2} + \ldots + n\dfrac{C_n}{C_{n-1}} = \dfrac{n(n+1)}{2}$.

20. If a_1, a_2, a_3, a_4 are the coefficients of 2nd, 3rd, 4th and 5th terms respectively in the expansion of $(1+x)^n$ where n is a positive integer then show that
$$\frac{a_1}{a_1 + a_2} + \frac{a_3}{a_3 + a_4} = \frac{2a_2}{a_2 + a_3}.$$

21. Solve the equations by Cramer's rule.
$2x - y + 3z = 8,\ -x + 2y + z = 4,\ 3x + y - 4z = 0.$

22. Show that $\begin{vmatrix} a+b & b+c & c+a \\ b+c & c+a & a+b \\ c+a & a+b & b+c \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$.

23. Three boxes numbered I, II, III contain 1 white, 2 black and 3 red balls; 2 white, 1 black and 1 red ball; 4 white, 5 black and 3 red balls respectively. One box is randomly selected and a ball is drawn from it. If the ball is red then find the probability that it is from box II.

24. In a book of 450 pages, there are 400 typographical errors. Assuming Poisson law for the number of errors per page, find the probability that a random sample of 5 pages will contain no error.