P4-Con No-73 Sem IV / Comp. / 19-06-10 / Applied maths-IV

Con. 3804-10. **(REVISED COURSE)** **AN-3452**

(3 Hours) **[Total Marks : 100**

N.B.: (1) Question no 1 is compulsory.
(2) Attempt any four questions out of the remaining six questions
(3) Figures to right indicate full marks.
(4) Assume any suitable data whenever required and justify the same.

1. a) Find k such that $\frac{1}{2}\log(x^2+y^2)+i\tan^{-1}\frac{kx}{y}$ is analytic. (5)

b) If A= $\begin{bmatrix} \pi & \frac{\pi}{4} \\ 0 & \frac{\pi}{2} \end{bmatrix}$ find $\cos A$. (5)

c) Find all the basic feasible, infeasible, degenerate ,non degenerate solutions of $x_1+2x_2+4x_3+x_4=7$, $2x_1-x_2+3x_3-2x_4=4$. (5)

d) Evaluate the line integral $\int_C (z^2+3z)\,dz$ along the straight line from(2,0) to (2,2) and then from (2,2) to (0,2). (5)

2. a) Find the eigenvalues and eigenvectors of A= $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$. Are the eigenvectors linearly independent? (6)

b) If f(z) =u+iv is analytic and u+v= $\frac{2\sin 2x}{e^{2y}+e^{-2y}-2\cos 2x}$ find f(z) in terms of z. (7)

c) Solve by simplex method Max $z=x_1-x_2+3x_3$ (7)
Subject to the constraints $x_1+x_2+x_3\le 10$
$2x_1-x_3\le 3$,
$2x_1-2x_2+3x_3\le 0$
$x_1,x_2,x_3\ge 0$

3. a) Show that the matrix A= $\begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$ is diagonalizable. Hence find the diagonal and the transforming matrix (6)

b) Use Big M method to solve Max $Z=x_1+4x_2$ (7)
Subject to the constraints $3x_1+x_2\le 3$
$2x_1+3x_2\le 6$
$4x_1+5x_2\ge 20$, $x_1,x_2,\ge 0$

c) State Cauchy's residue theorem and hence evaluate (7)

i. $\oint_C \frac{z-1}{z^2+2z+5}dz$, $C:|z|=1.5$

ii. $\int_0^{2\pi}\frac{d\theta}{5-3\cos\theta}$

4. a) Prove that for the function defined by $f(z)=\dfrac{x^3(1+i)-y^3(1-i)}{x^2+y^2}$ when $z\neq 0$

$= 0$ when $z=0$ (6)

Cauchy Riemann equations are satisfied at the origin yet $f'(0)$ does not exist.

b) Use Principle of Duality to solve Max $z=3x_1+4x_2$ (7)

Subject to the constraints $x_1-x_2 \leq 1$

$x_1+x_2 \geq 4$

$x_1-3x_2 \leq 3$, $x_1,x_2 \geq 0$

c) Find all possible Tailor's and Laurent series expansions of $f(z)=\dfrac{z-1}{z^2-2z-3}$

indicating the regions of convergence. (7)

5. a) Find the maximum or minimum of the function $Z=x_1+2x_3+x_2x_3-x_1^2-x_2^2-x_3^2$ (6)

b) Find the bilinear transformation which maps the points $1,-i,2$ onto the points $0,2,-i$ and hence find the fixed points. (7)

c) Solve by using Dual simplex method Min $z=2x_1+x_2$ (7)

Subject to the constraints $3x_1+x_2 \geq 3$

$4x_1+3x_2 \geq 6$

$x_1+2x_2 \geq 3$, $x_1,x_2 \geq 0$

6. a) Verify Caley Hamilton Theorem for $A=\begin{bmatrix}1 & 2\\ 2 & 2\end{bmatrix}$ and hence

evaluate $2A^4-5A^3-7A+6I$. (6)

b) Evaluate $\int_C \dfrac{\sin z}{4z^2-8iz}\,dz$, C consists of the boundaries of the squares with vertices

$\pm 3, \pm 3i$ (anticlockwise) and $\pm 1, \pm i$ (clockwise) (7)

c) Using Kuhn-Tucker conditions Minimize $z=2x_1+3x_2-x_1^2-2x_2^2$ (7)

Subject to the constraints $x_1+3x_2 \leq 6,\ 5x_1+2x_2 \leq 10,\ x_1,x_2 \geq 0$

7. a) Find the orthogonal trajectories of the family of curves $r^2\cos 2\theta=\alpha$. (6)

b) Find the image of the line $\theta=\dfrac{\pi}{3}$ under the transformation $w=z+\dfrac{1}{z}$. (7)

c) Use the method of Lagrangian multipliers to solve the following problem.

Minimize $Z=6x_1+8x_3-x_1^2-x_2^2$

Such that $4x_1+3x_2=16$ (7)

$3x_1+5x_2=15$, $x_1,x_2, \geq 0$