Mathematics
Class X
Past Year Paper - 2013

Time: 2½ hour | Total Marks: 80

1. Answer to this paper must be written on the paper provided separately.
2. You will NOT be allowed to write during the first 15 minutes. This time is to be spent in reading the question paper.
3. The time given at the head of this paper is the time allowed for writing the answers.
4. This question paper is divided into two Sections.
Attempt all questions from Section A and any four questions from Section B.
5. Intended marks for questions or parts of questions are given in brackets along the questions.
6. All working, including rough work, must be clearly shown and should be done on the same sheet as the rest of the answer. Omission of essential working will result in loss of marks.
7. Mathematical tables are provided.

SECTION – B (40 marks)

Q. 1

(a) Given $A = \begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix}$, $B = \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$

Find the matrix X such that A + 2X = 2B + C. (3)

(b) At what rate % p.a. will a sum of ₹ 4000 yield ₹ 1324 as compound interest in 3 years? (3)

(c) The median of the following observations

11, 12, 14, (x - 2), (x + 4), (x + 9), 32, 38, 47 arranged in ascending order is 24. Find the value of x and hence find the mean. (4)

Q. 2

(a) What number must be added to each of the numbers 6, 15, 20 and 43 to make them proportional? (3)

(b) If (x - 2) is a factor of the expression $2x^3 + ax^2 + bx - 14$ and when the expression is divided by (x - 3), it leaves a remainder 52, find the values of a and b. (3)

(c) Draw a histogram from the following frequency distribution and find the mode from the graph: (4)

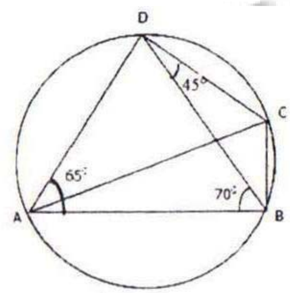
Class	0-5	5-10	10-15	15-20	20-25	25-30
Frequency	2	5	18	14	8	5

Q. 3

(a) Without using tables evaluate:

$3 \cos 80^{o}. \operatorname{cosec} 10^{o} + 2 \sin 59^{o} \sec 31^{o}.$ (3)

(b) In the given figure , $\angle BAD = 65^{o}$
$\angle ABD = 70^{o}$, $\angle BDC = 45^{o}$



(i) Prove that AC is a diameter of the circle.

(ii) Find $\angle ACB$ (3)

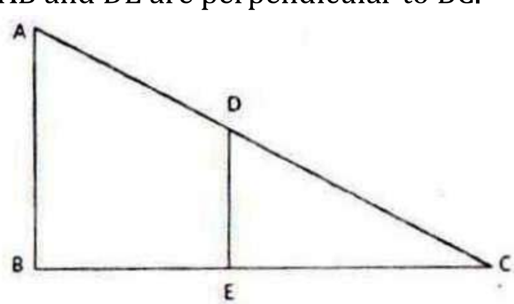
(c) AB is a diameter of a circle with centre C = (-2, 5). If A = (3, -7). Find
(i) the length of radius AC

(ii) the coordinates of B. (4)

Q. 4

(a) Solve the following equation and calculate the answer correct to two decimal places:
$x^2 - 5x - 10 = 0$.

(b) In the given figure, AB and DE are perpendicular to BC.



(i) Prove that $\triangle ABC \sim \triangle DEC$

(ii) If AB = 6 cm; DE = 4 cm and AC = 15 cm. Calculate CD.

(iii) Find the ratio of area of $\triangle$ ABC: area of $\triangle$ DEC.

(c) Using a graph paper, plot the points A(6,4) and B(0,4).

(i) Reflect A and B in the origin to get the images A' and B'.

(ii) Write the co-ordinates of A' and B'.

(iii) State the geometrical name for the figure ABA' B'.

(iv) Find its perimeter.

SECTION – B (40 marks)

Q. 5

(a) Solve the following inequation, write the solution set and represent it on the number line: $-\frac{x}{3} \le \frac{x}{2} - 1\frac{1}{3} < \frac{1}{6}, x \in R$ (3)

(b) Mr. Britto deposits a certain sum of money each month in a Recurring Deposit Account of a bank. It the rate of interest is of 8% per annum and Mr. Britto gets ₹ 8088 from the bank after 3 years, find the value of his monthly instalment. (3)

(c) Salman buys 50 shares of face value ₹ 100 available at ₹ 132.

(i) What is his investment?

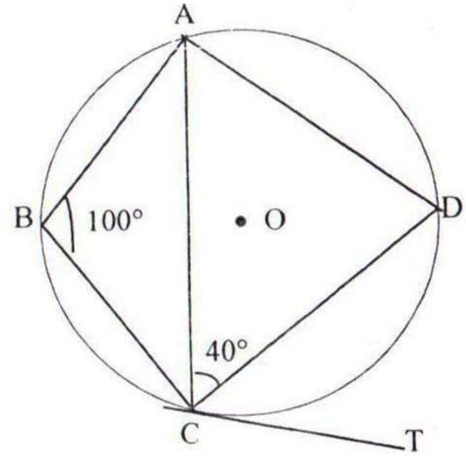
(ii) If the dividend is 7.5%, what will be his annual income?

(iii) If he wants to increase his annual income by ₹ 150, how many extra shares should he buy? (4)

Q. 6

(a) Show that $\sqrt{\frac{1-\cos A}{1+\cos A}} = \frac{\sin A}{1+\cos A}$. (3)

(b) In the given circle with centre O, $\angle$ABC = 100°, $\angle$ACD = 40° and CT is a tangent to the circle at C. Find $\angle$ADC and $\angle$DCT. (3)



(c) Given below are the entries in a Saving Bank A/c pass book.

Date	Particulars	Withdrawls	Deposit	Balance
Feb8	B/F	-	-	₹ 8500
Feb 18	To self	₹ 4000	-	
April 12	By cash	-	₹ 2230	
June 15	To self	₹ 5000	-	
July 8	By cash	-	₹ 6000	

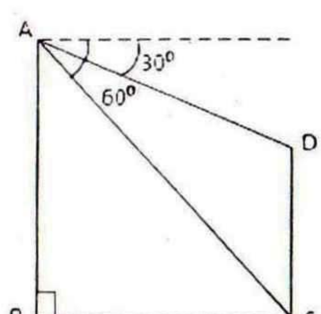
Calculate the interest for six months from February to July at 6% p.a. (4)

Q.7

(a) In Δ ABC, A(3,5), B(7,8) and C(1,-10). Find the equation of the median through A.(3)

(b) A shopkeeper sells an article at the listed price of ₹ 1500 and the rate of VAT is 12% at each stage of sale. If the shopkeeper pays a VAT of ₹ 36 to the Government, what was the price, inclusive to TAX, at which the shopkeeper purchased the article from the wholesaler? (3)

(c) In the figure given, from the top of a building AB = 60 m high, the angles of depression of the top and bottom of a vertical lamp post CD are observed to 30^o and 60^o respectively. Find:



(i) The horizontal distance between AB and CD.

(ii) The height of the lamp post. (4)

Q. 8

(a) Find x and y if $\begin{bmatrix} x & 3x \\ y & 4y \end{bmatrix}\begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 12 \end{bmatrix}$. (3)

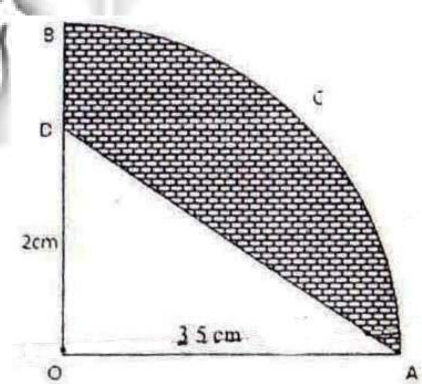
(b) A solid sphere of radius 15 cm is melted and recast into solid right circular cones of radius 2.5 cm and height 8 cm. Calculate the number of cones recast. (3)

(c) Without solving the following quadratic equation, find the value of 'p' for which the given equation has real and equal roots:

$x^2 + (p - 3)x + p = 0$. (4)

Q. 9

(a) In the figure alongside, OAB is a quadrant of a circle. The radius OA = 3.5 cm and OD = 2 cm. Calculate the area of the shaded portion. (Take $\pi = \frac{22}{7}$). (3)



(b) A box contains some black balls and 30 white balls. If the probability of drawing a black ball is two-fifths of a white ball, find the number of black balls in the box. (3)

(c) Find the mean of the following distribution by step deviation method: (4)

Class interval	20-30	30-40	40-50	50-60	60-70	70-80

Frequency	10	6	8	12	5	9

Q. 10

(a) Using a ruler and compasses only:

(i) Construct a triangle ABC with the following data:

AB = 3.5 cm, BC = 6 cm and $\angle ABC = 120^{o}$

(ii) In the same diagram, draw a circle with BC as diameter. Find a point P on the circumference of the circle which is equidistant from AB and BC.

(iii) Measure $\angle BCP$. (4)

(b) The marks obtained by 120 students in a test are given below:

Marks	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
No of students	5	9	16	22	26	18	11	6	4	3

Draw an ogive for the given distribution on a graph sheet.

Use suitable scale for ogive to estimate the following:

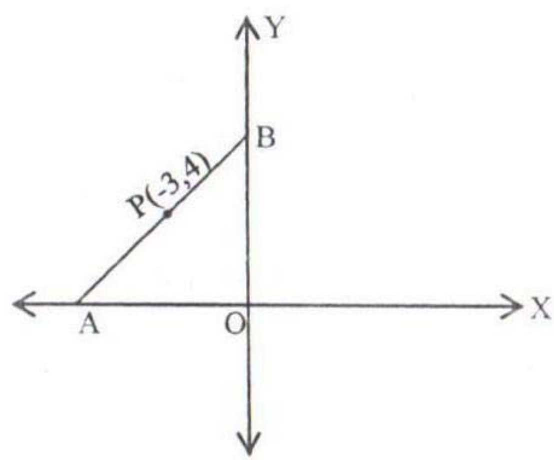
(i) The median.

(ii) The number of students who obtained more than 75% marks in the test.

(iii) The number of students who did not pass the test if minimum marks required to pass is 40. (6)

Q. 11

(a) In the figure given below, the line segment AB meets X-axis at A and Y-axis at B. The point P(-3,4) on AB divides it in the ratio 2:3. Find the coordinates of A and B. (3)



(b) Using the properties of proportion, solve for x, given

$\frac{x^4+1}{2x^2}=\frac{17}{8}$. (3)

(c) A shopkeeper purchases a certain number of books for ₹ 960. If the cost per book was ₹ 8 less, the number of books that could be purchased for ₹ 960 would be 4 more. Write an equation, taking the original cost of each book to be ₹ x, and solve it to find the original cost of the books. (3)

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Solution

SECTION – A (40 marks)

Sol. 1

(a) A + 2X = 2B + C

$$\begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix} + 2X = 2\begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix} + 2X = \begin{bmatrix} -6+4 & 4+0 \\ 8+0 & 0+2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix} + 2X = \begin{bmatrix} -2 & 4 \\ 8 & 2 \end{bmatrix}$$

$$2X = \begin{bmatrix} -2 & 4 \\ 8 & 2 \end{bmatrix} - \begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix}$$

$$2X = \begin{bmatrix} -4 & 10 \\ 6 & 2 \end{bmatrix}$$

$$X = \frac{1}{2}\begin{bmatrix} -4 & 10 \\ 6 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 5 \\ 3 & 1 \end{bmatrix}$$

(b) P = ₹ 4000, C.I. = ₹ 1324, n = 3 years

Amount, A = P + C.I. = ₹ 4000 + ₹ 1324 = ₹ 5324

$$A = P\left(1 + \frac{r}{100}\right)^n$$

$$5324 = 4000\left(1 + \frac{r}{100}\right)^3$$

$$\left(1 + \frac{r}{100}\right)^3 = \frac{5324}{4000} = \frac{1331}{1000}$$

$$\left(1 + \frac{r}{100}\right)^3 = \left(\frac{11}{10}\right)^3$$

$$1 + \frac{r}{100} = \frac{11}{10}$$

$$\frac{r}{100} = \frac{11}{10} - 1 = \frac{1}{10}$$

$$r = 10\%$$

(c) The observations in ascending order are:

11, 12, 14, (x - 2), (x + 4), (x + 9), 32, 38, 47

Number of observations = 9 (odd)

Median = $\left(\frac{n+1}{2}\right)^{th}$ observation = 5^{th} observation

$\therefore$ x + 4 = 24

$\Rightarrow$ x = 20

Thus, the observations are:

11, 12, 14, 18, 24, 29, 32, 38, 47

Mean = $\frac{11+12+14+18+24+29+32+38+47}{9}$

$= \frac{225}{9}$

= 25

Sol. 2

(a) Let the number added be x.

$\therefore$ (6 + x) : (15 + x) :: (20 + x) (43 + x)

$\frac{6+x}{15+x} = \frac{20+x}{43+x}$

$(6+x)(43+x) = (20+x)(15+x)$

$258 + 6x + 43x + x^2 = 300 + 20x + 15x + x^2$

$49x - 35x = 300 - 258$

$14x = 42$

$x = 3$

Thus, the required number which should be added is 3.

(b) Let $p(x) = 2x^3 + ax^2 + bx - 14$

Given, (x - 2) is a factor of p(x)

$\Rightarrow$ Remainder = p(2) = 0

$\Rightarrow 2(2)^3 + a(2)^2 + b(2) - 14 = 0$

$\Rightarrow$ 16 + 4a + 2b - 14 = 0

$\Rightarrow$ 4a + 2b + 2 = 0

$\Rightarrow$ 2a + b + 1 = 0 ...(1)

Given, when p(x) is divided by (x - 3), it leaves a remainder 52.

$\therefore$ p(3) = 52

$\Rightarrow 2(3)^3 + a(3)^2 + b(3) - 14 = 52$

$\Rightarrow$ 54 + 9a + 3b - 14 - 52 = 0

$\Rightarrow$ 9a + 3b - 12 = 0

$\Rightarrow$ 3a + b - 4 = 0 ...(2)

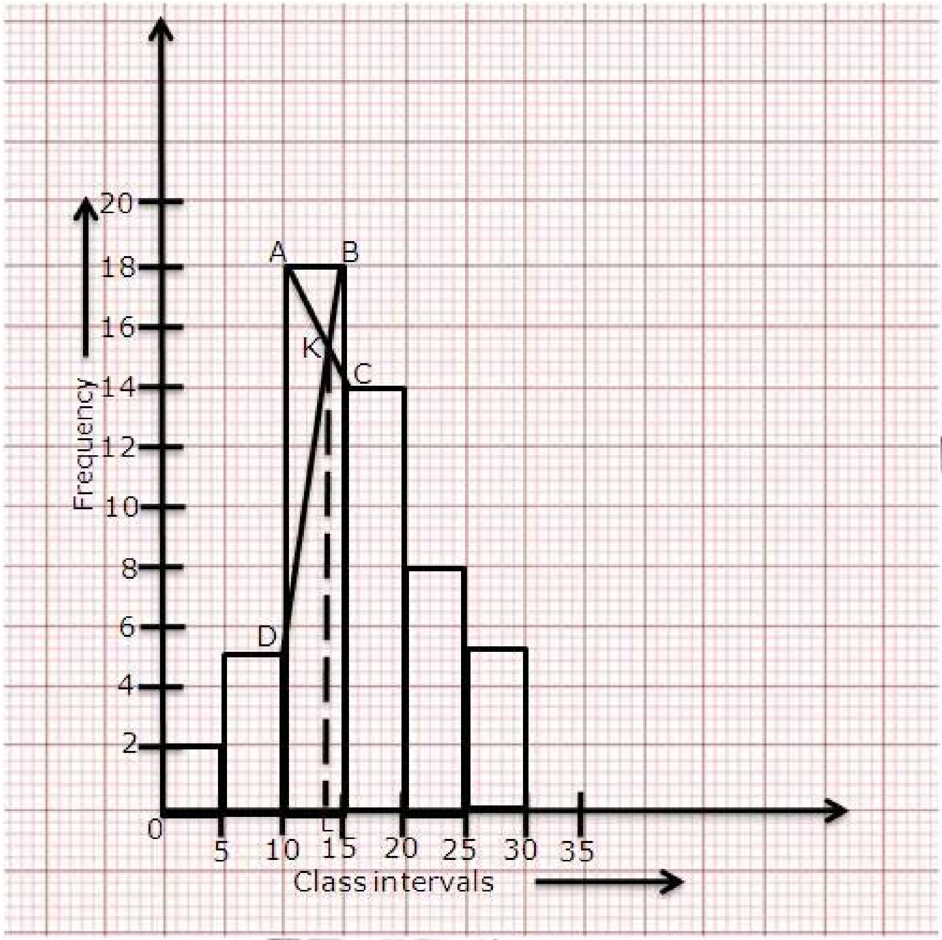
Subtracting (1) from (2), we get,

a - 5 = 0 $\Rightarrow$ a = 5

From (1),

10 + b + 1 = 0 $\Rightarrow$ b = -11

(c)



Steps for calculation of mode.

(i) Mark the end points of the upper corner of rectangle with maximum frequency as A and B.

(ii) Mark the inner corner of adjacent rectangles as C and D.

(iii) Join AC and BD to intersect at K. From K, draw KL perpendicular to x-axis.

(iv) The value of L on x- axis represents the mode.

$\therefore$ Mode = 13

Sol. 3

(a) $3\cos 80^\circ . \text{cosec}10^\circ + 2\sin 59^\circ \sec 31^\circ$

$= 3\cos(90^\circ - 10^\circ)\ \text{cosec}10^\circ + 2\sin(90^\circ - 31^\circ)\sec 31^\circ$

$= 3\sin 10^\circ\ \text{cosec}10^\circ + 2\cos 31^\circ, \sec 31^\circ$

$[\because \sin(90^\circ - \theta) = \cos\theta, \cos(90^\circ - \theta) = \sin\theta]$

$= 3 \times 1 + 2 \times 1$ $\quad [\because \sin\theta.\text{cosec}\,\theta = 1, \cos\theta.\sec\theta = 1]$

$= 3 + 2 = 5$

(b) (i) In Δ ABD,

$\angle DAB + \angle ABD + \angle ADB = 180^\circ$

$\Rightarrow 65^\circ + 70^\circ + \angle ADB = 180^\circ$

$\Rightarrow \angle ADB = 180^o - 70^o - 65^o = 45^o$

Now, $\angle ADC = \angle ADB + \angle BDC = 45^o + 45^o = 90^o$

$\angle ADC$ is the angle of semi-circle so AC is a diameter of the circle.

(ii) $\angle ACB = \angle ADB$ (angle subtended by the same segment)

$\Rightarrow \angle ACB = 45^o$

(c)

(i) Radius AC = $\sqrt{(3+2)^2 + (-7-5)^2}$

$$= \sqrt{(5)^2 + (-12)^2}$$
$$= \sqrt{25+144}$$
$$= \sqrt{169}$$
$$= 13 \text{ units}$$

(ii) Let the coordinates of B be (x, y)

Using mid-point formula, we have

$$-2 = \frac{3+x}{2} \qquad 5 = \frac{-7+y}{2}$$
$$-4 = 3+x \qquad 10 = -7+y$$
$$x = -7 \qquad y = 17$$

Thus, the coordinates of points B are (-7, 17).

Sol. 4

(a) x^2-5x-10 = 0

Use quadratic formula: a = 1, b = -5, c = -10

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-10)}}{2(1)}$$
$$x = \frac{5 \pm \sqrt{25+40}}{2}$$
$$x = \frac{5 \pm \sqrt{65}}{2}$$
$$x = \frac{5 \pm 8.06}{2}$$
$$x = \frac{13.06}{2}, \frac{-3.06}{2}$$
$$x = 6.53, -1.53$$

(b) (i) In ΔABC and ΔDEC,

$\angle ABC = \angle DEC = 90°$ (perpendiculars to BC)

$\angle ACB = \angle DCE$ (Common)

$\therefore \Delta ABC \sim \Delta DEC$ (AA criterion)

(ii) Since $\Delta ABC \sim \Delta DEC$,

$$\frac{AB}{DE} = \frac{AC}{CD}$$

$$\Rightarrow \frac{6}{4} = \frac{15}{CD}$$

$$\Rightarrow 6 \times CD = 60$$

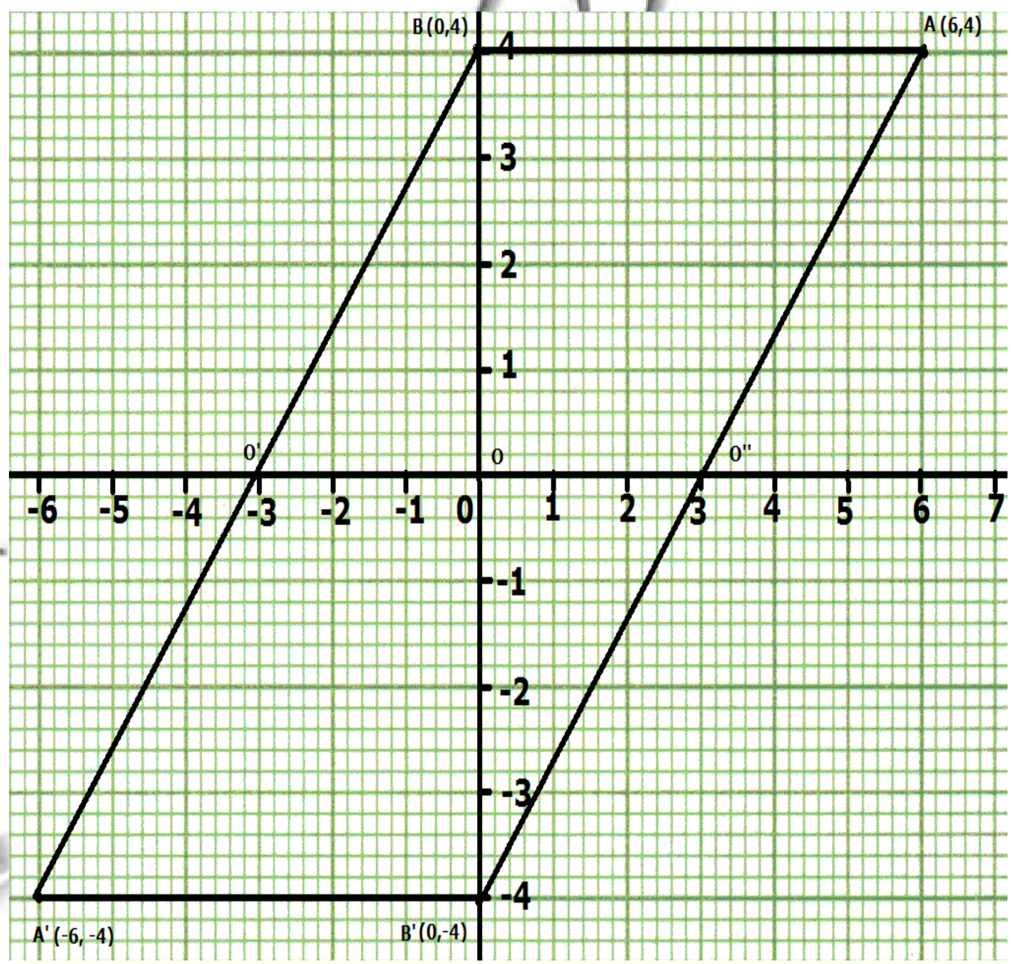
$$\Rightarrow CD = \frac{60}{6} = 10 \text{ cm}$$

(iii) It is known that the ratio of areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

ar(Δ ABC): ar(Δ DEC) = $AB^2 : DE^2 = 6^2 : 4^2 = 36 : 16 = 9 : 4$

(c)

(i)



(ii) Co-ordinates of A' = (-6, -4)

Co-ordinates of B' = (0, -4)

(iii) ABA'B' is a parallelogram.

(iv) AB = A'B' = 6 units

In $\triangle OBO'$,

$OO' = 3$ units

$OB = 4$ units

$$BO' = \sqrt{OB^2 + OO'^2}$$
$$= \sqrt{4^2 + 3^2}$$
$$= \sqrt{25}$$
$$= 5 \text{ units}$$

Since BO' = 5 units

BA' = 10 units = AB'

Perimeter of ABA'B' = (6 + 10 + 6 + 10) units = 32 units

SECTION – B (40 marks)

Sol. 5

(a) The given inequation is $-\frac{x}{3} \le \frac{x}{2} - 1\frac{1}{3} < \frac{1}{6}, x \in R$

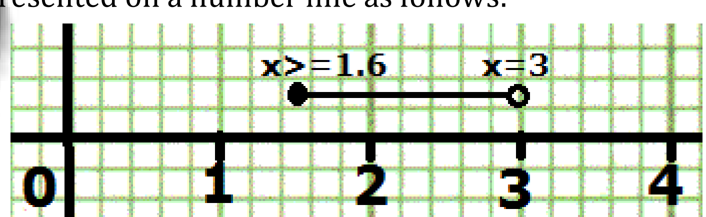
$$-\frac{x}{3} \le \frac{x}{2} - 1\frac{1}{3}$$
$$-\frac{x}{3} - \frac{x}{2} \le -\frac{4}{3}$$
$$\frac{2x + 3x}{6} \ge \frac{4}{3}$$
$$\frac{5x}{6} \ge \frac{4}{3}$$
$$5x \ge 8$$
$$x \ge \frac{8}{5}$$
$$x \ge 1.6$$

and

$$\frac{x}{2} - 1\frac{1}{3} < \frac{1}{6}$$
$$\frac{x}{2} < \frac{1}{6} + \frac{4}{3}$$
$$\frac{x}{2} < \frac{1+8}{6}$$
$$\frac{x}{2} < \frac{9}{6}$$
$$x < \frac{18}{6}$$
$$x < 3$$

The solution set is {x: x $\in$R and 1.6 $\le$ x < 3}

It can be represented on a number line as follows:



(b) Maturity amount = ₹ 8088

Period (n) = 3 yrs = 36 months

Rate = 8% p.a.

Let x be the monthly deposit.

$$S.I. = P \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$$

$$= P \times \frac{36 \times 37}{24} \times \frac{8}{100} = 4.44x$$

Total amount of maturity $= 36x + 4.44x = 40.44x$

Now,

$40.44x = 8088$

$\Rightarrow x = 200$

Thus, the value of monthly installment is ₹ 200.

(c)

(i) No. of shares = 50

Market value of one share = ₹ 132

Salman's investment = ₹ (132 x 50) = ₹ 6600

(ii) Dividend on one share = 7.5% of ₹ 100 = ₹ 7.50

His annual income = 50 × ₹ 7.50 = ₹ 375

(iii) Salman wants to increase his income by ₹ 150.

Income on one share = ₹ 7.50

No. of extra shares he buys $= \frac{150}{7.50} = 20$

Sol. 6

(a)

LHS

$$= \sqrt{\frac{1 - \text{Cos}A}{1 + \text{Cos}A}}$$

$$= \sqrt{\frac{(1 - \text{Cos}A)(1 + \text{Cos}A)}{(1 + \text{Cos}A)(1 + \text{Cos}A)}}$$

$$= \sqrt{\frac{1 - \text{Cos}^2 A}{(1 + \text{Cos}A)^2}}$$

$$= \sqrt{\frac{\text{Sin}^2 A}{(1 + \text{Cos}A)^2}}$$

$$= \frac{\text{Sin}A}{1 + \text{Cos}A} = \text{RHS}$$

(b) In cyclic quadrilateral ABCD,

$\angle B + \angle D = 180^\circ$ (opp. angles of cyclic quad are supplementary)

$\Rightarrow 100^\circ + \angle ADC = 180^\circ$

$\Rightarrow \angle ADC = 80^\circ$

Now, in ΔACD,

$\angle ACD + \angle CAD + \angle ADC = 180^\circ$

$40^\circ + \angle CAD + 80^\circ = 180^\circ$

$\angle CAD = 180^\circ - 120^\circ = 60^\circ$

Now $\angle DCT = \angle CAD$ (angles in the alternate segment)

$\therefore \angle DCT = 60^\circ$

(c)

On completing the given table, we get:

Date	Particulars	Withdrawls	Deposit	Balance
Feb8	B/F	-	-	₹ 8500
Feb 18	To self	₹ 4000	-	₹ 4500
April 12	By cash	-	₹ 2230	₹ 6730
June 15	To self	₹ 5000	-	₹ 1730
July 8	By cash	-	₹ 6000	₹ 7730

Principal for the month of Feb = ₹ 4500

Principal for the month of March = ₹ 4500

Principal for the month of April = ₹ 4500

Principal for the month of May = ₹ 6730

Principal for the month of June = ₹ 1730

Principal for the month of July = ₹ 7730

Total Principal for 1 month

= ₹ (4500+4500+4500+6730+1730+7730) = ₹ 29690

P = ₹ 29690, T = $\frac{1}{12}$ years, R = 6%

$$\therefore \text{Interest} = \frac{P \times R \times T}{100} = \frac{29690 \times 6 \times 1}{100 \times 12} = \text{Rs } 148.45$$

Sol. 7

(a) The vertices of $\triangle$ ABC are A(3, 5), B(7,8) and C(1, -10).

Coordinates of the mid-point D of BC are

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$=\left(\frac{7+1}{2}, \frac{8+(-10)}{2}\right)$$

$$=\left(\frac{8}{2}, \frac{-2}{2}\right)$$

$$=(4,-1)$$

$$\text{Slope of AD} = \frac{y_2-y_1}{x_2-x_1}$$

$$=\frac{-1-5}{4-3}$$

$$=\frac{-6}{1}=-6$$

Now, the equation of median is given by:

$y - y_1 = m(x - x_1)$

$y - 5 = -6(x - 3)$

$y - 5 = -6x + 18$

$6x + y - 23 = 0$

(b) Since, the shopkeeper sells the article for ₹ 1500 and charges sales-tax at the rate of 12%.

$\therefore$ Tax charged by the shopkeeper = 12% of ₹ 1500 = ₹ 180

VAT = Tax charges - Tax paid

₹ 36 = ₹ 180 - Tax paid

Tax paid = ₹ 144

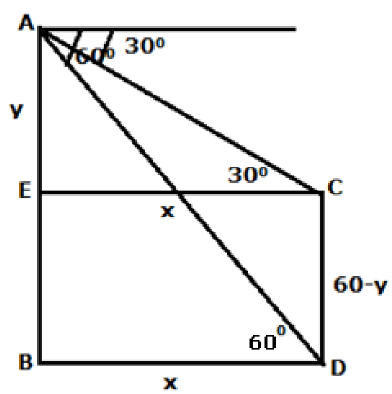
If the shopkeeper buys the article ₹ x.

Tax on it = 12% on ₹ x = ₹ 144

$\Rightarrow x = ₹\ 144 \times \frac{100}{12} = ₹\ 1200$

Thus, the price (inclusive of tax) paid by the shopkeeper = ₹ 1200 + ₹ 144 = ₹ 1344.

(c)



(i) In $\triangle$ AEC,

$\tan 30^0 = \frac{AE}{EC}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{y}{x}$

$\Rightarrow y = \frac{x}{\sqrt{3}}$......(1)

In $\triangle$ DBA,

$\cot 60^0 = \frac{BD}{AB}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{x}{60}$

$\Rightarrow x = \frac{60}{\sqrt{3}} = 20\sqrt{3} = 34.64 \text{ m}$

Therefore, the horizontal distance between AB and CD = 34.64 m.

(ii) Substituting the value of x in (1),

$\Rightarrow y = \frac{20\sqrt{3}}{\sqrt{3}} = 20 \text{ m}$

$\therefore$ Height of the lamp post = CD = (60 - 20) m = 40 m

Sol. 8

(a) $\begin{bmatrix} x & 3x \\ y & 4y \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 12 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2x + 3x \\ 2y + 4y \end{bmatrix} = \begin{bmatrix} 5 \\ 12 \end{bmatrix}$

$\Rightarrow$ 2x + 3x = 5 $\Rightarrow$ x = 1

and 2y + 4y = 12 $\Rightarrow$ y = 2

(b) Sphere : R = 15 cm

Cone : r = 2.5 cm, h = 8 cm

Let the number of cones recasted be n.

$\therefore$ n $\times$ Volume of one cone = Volume of solid sphere

$\Rightarrow n \times \frac{1}{3}\pi r^2 h = \frac{4}{3}\pi R^3$

$\Rightarrow n \times (2.5)^2 \times (8) = 4 \times (15)^3$

$\Rightarrow n = \frac{4 \times 15 \times 15 \times 15}{2.5 \times 2.5 \times 8}$

$\Rightarrow n = 270$

Thus, 270 cones were recasted.

(c) $x^2 + (p - 3)x + p = 0$

Here, A = 1, B = (p - 3), C = p

Since, the roots are real and equal, D = 0

$\Rightarrow B^2 - 4ac = 0$
$\Rightarrow (p - 3)^2 - 4(1)(p) = 0$
$\Rightarrow p^2 + 9 - 6p - 4p = 0$
$\Rightarrow p^2 - 10p + 9 = 0$
$\Rightarrow (p - 1)(p - 9) = 0$
$\Rightarrow p = 1$ or $p = 9$

Sol. 9

(a) Area of the quadrant OACB =

$\frac{1}{4} \times \pi r^2$

$= \frac{1}{4} \times \frac{22}{7} \times 3.5 \times 3.5$

$= 9.625 \text{ cm}^2$

Area of the triangle OAD =

$\frac{1}{2} \times \text{base} \times \text{height}$

$= \frac{1}{2} \times 3.5 \times 2$

$= 3.5 \text{ cm}^2$

Shaded Area = Area of quadrant OACB - area of triangle OAD

$= 9.625 - 3.5 \text{ cm}^2$

$= 6.125 \text{ cm}^2$

(b) Number of white balls in the bag = 30

Let the number of black balls in the box be x.

$\therefore$ Total number of balls = x + 30

P(drawing a black ball) = $\frac{x}{x+30}$

P(drawing a white ball) = $\frac{30}{x+30}$

It is given that:

P(drawing a black ball) = $\frac{2}{5}$ x P(drawing a white ball)

$\Rightarrow \frac{x}{x+30} = \frac{2}{5} \times \frac{30}{x+30}$

$\Rightarrow \frac{x}{x+30} = \frac{12}{x+30}$

$\Rightarrow x = 12$

Therefore, number of black balls in the box is 12.

(c)

Class interval	Frequency (f)	Class mark (x)	$d=\frac{x-A}{h}$ (A = 55)	fd
20-30	10	25	-3	-30
30-40	6	35	-2	-12
40-50	8	45	-1	-8
50-60	12	A=55	0	0
60-70	5	65	1	5
70-80	9	75	2	18
Total	50			-27

Here, A = 55, h = 10

$$\text{Mean} = A + \frac{\sum fd}{\sum f} \times h$$
$$= 55 + \frac{-27}{50} \times 10$$
$$= 55 - 5.4$$
$$= 49.6$$

Sol. 10

(a)

(i) Steps of constructions:

(a) Draw a line segment BC = 6 cm.

(b) At B, draw a ray BX making an angle of 120^{o} with BC.

(c) From point B cut an arc of radius 3.5 cm to meet ray BX at C.

(d) Join AC.

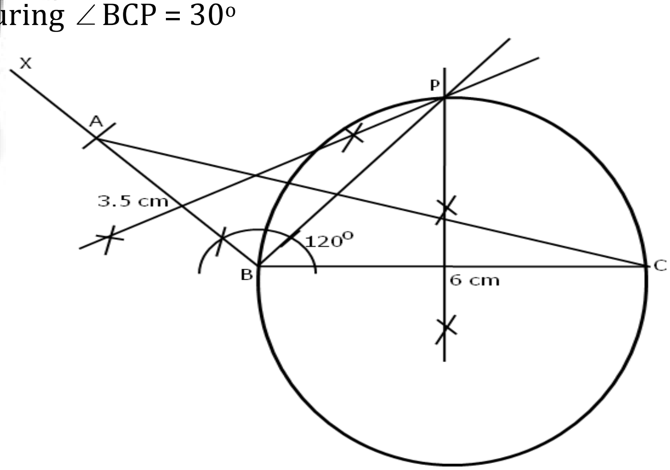
ABC is the required triangle.

(ii)

(a) Bisect BC and draw a circle with BC as diameter.

(b) Draw perpendicular bisectors of AB. Let the two bisectors meet the ray of angle bisector of $\angle$ABC at point P. P is equidistant from AB and BC.

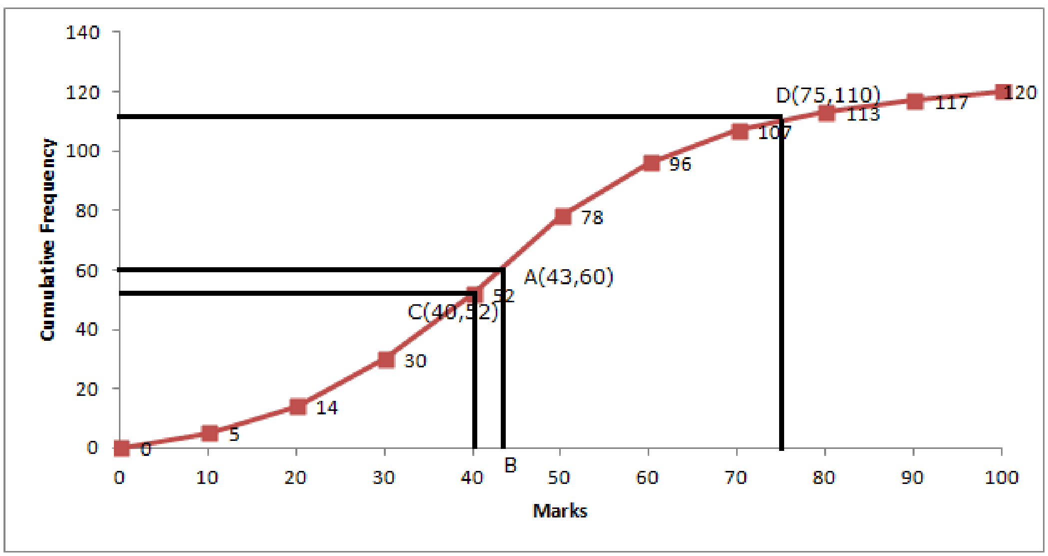
(iii) On measuring $\angle$BCP = 30^{o}



(b)

Marks	No. of Students	Cumulative Frequency
0-10	5	5
10-20	9	14
20-30	16	30
30-40	22	52
40-50	26	78
50-60	18	96
60-70	11	107
70-80	6	113
80-90	4	117
90-100	3	120

$n = \frac{120}{2} = 60$



(i) Through marks 60, draw a line segment parallel to x-axis which meets the curve at A. From A, draw a line perpendicular to x-axis meeting at B.

Median = 43

(ii) Through marks 75, draw a line segment parallel to y-axis which meets the curve at D. From D, draw a line perpendicular to y-axis which meets y-axis at 110.

Number of students getting more than 75% = 120 - 110 = 10 students

(iii) Through marks 40, draw a line segment parallel to y-axis which meets the curve at C. From C, draw a line perpendicular to y-axis which meets y-axis at 52.

Number of students who did not pass = 52.

Sol. 11

(a) Let the coordinates of A and B be (x, 0) and (0, y) respectively.

Given P divides AB is the ratio 2:3,

Using section formula, we have :

$$-3 = \frac{2\times 0 + 3\times x}{2+3} \qquad 4 = \frac{2\times y + 3\times 0}{2+3}$$

$$-3 = \frac{3x}{5} \qquad 4 = \frac{2y}{5}$$

$$-15 = 3x \qquad 20 = 2y$$

$$x = -5 \qquad y = 10$$

Thus, the coordinates of A and B are (-5, 0) and (0, 10) respectively.

(b) $\frac{x^4+1}{2x^2} = \frac{17}{8}$

Using componendo and dividendo,

$$\frac{x^4+1+2x^2}{x^4+1-2x^2} = \frac{17+8}{17-8}$$

$$\Rightarrow \frac{(x^2+1)^2}{(x^2-1)^2} = \frac{25}{9}$$

$$\Rightarrow \left(\frac{x^2+1}{x^2-1}\right)^2 = \left(\frac{5}{3}\right)^2$$

$$\Rightarrow \frac{x^2+1}{x^2-1} = \frac{5}{3}$$

$$\Rightarrow \frac{x^2+1+x^2-1}{x^2+1-x^2+1} = \frac{5+3}{5-3} \text{ (Using componendo and dividendo)}$$

$$\Rightarrow \frac{2x^2}{2} = \frac{8}{2}$$

$$\Rightarrow x^2 = 4$$

$$\Rightarrow x = \pm 2$$

(c) Original cost of each book = ₹ x

$\therefore$ Number of books brought for ₹ 960 = $\frac{960}{x}$

In 2nd case:

The cost of each book = ₹ (x - 8)

Number of books bought for ₹ 960 = $\frac{960}{x-8}$

From the given information, we have:

$$\frac{960}{x-8}-\frac{960}{x}=4$$

$$\Rightarrow \frac{960x-960x+960\times 8}{x(x-8)}=4$$

$$\Rightarrow x(x-8)=\frac{960\times 8}{4}=1920$$

$\Rightarrow x^2 - 8x - 1920 = 0$

$\Rightarrow (x - 48)(x + 40) = 0$

$\Rightarrow x = 48$ or -40

But x can't be negative.

$\therefore x = 48$

Thus, the original cost of each book is ₹ 48.