# B4.2-R3: DISCRETE STRUCTURES

**NOTE:**

| | |
|---|---|
| 1. | **Answer question 1 and any FOUR questions from 2 to 6.** |
| **2.** | **Parts of the same question should be answered together and in the same sequence.** |

**Time: 3 Hours** **Total Marks: 100**

**1.**

a) Can a relation R in a set A be both symmetric and anti symmetric? Justify your answer.

b) Write the negation of the following by changing the quantifiers:
"Every complete bipartite graph is not planar."

c) Prove absorption law in a Boolean algebra.

d) How many ways can one right and one left shoe be selected from 10 pairs of shoes without obtaining a pair?

e) What is the largest possible number of vertices in a graph with 35 edges and all vertices of degree at least 3?

f) Find a grammar to generate the set
$\{0^m 1^n \mid m$ and $n$ are non negative integers$\}$

g) Let (A,*) be an algebraic system, where * is a binary operation such that for any a and b in A
a*b = a
Show that this operation is associative.

**(7x4)**

**2.**

a) Suppose R is an arbitrary transitive and reflexive relation on a set A. Prove that the relation E defined by "x E y iff x R y and y R x" is an equivalence relation.

b) Prove or disprove the validity of the following argument:
   i) Every living thing is a plant or animal.
   ii) Ram's dog is alive and is not a plant.
   iii) All animals have heart.
   iv) Hence Ram's dog has a heart.

**(9+9)**

**3.**

a) Prove that if R is a partial ordering relation on a set S, then for $n \geq 2$,there can not be a sequence $s_1, s_2, s_3, \ldots s_n$ of distinct elements of S such that $s_1 R s_2 R s_3 \ldots R s_n R s_1$.

b) Minimize following switching function
$\Sigma_m(0, 2, 8, 12, 13)$.

c) Consider the group ($Z_4$, $\oplus$ ): the integer modulo 4 group with respect to the operation $\oplus$ : addition modulo 4. Does H={[0], [2]} form a subgroup of $Z_4$. If yes, is it a normal subgroup? Justify.

**(6+6+6)**

**4.**

a) Solve the following:

$a_n = 2a_{n-1} + 1$

where

$a_0 = 0$

$a_1 = 1$

$a_2 = 3.$

b) Find a generating function to count the number of integral solutions of

$e_1 + e_2 + e_3 = 10$ if for each i, $0 \le e_i$

**(9+9)**

**5.**

a) Show that complement of a regular set is a regular set.

b) Write a grammar/ regular expression for the language on the alphabet{0,1} having all the strings with different first and last symbols.

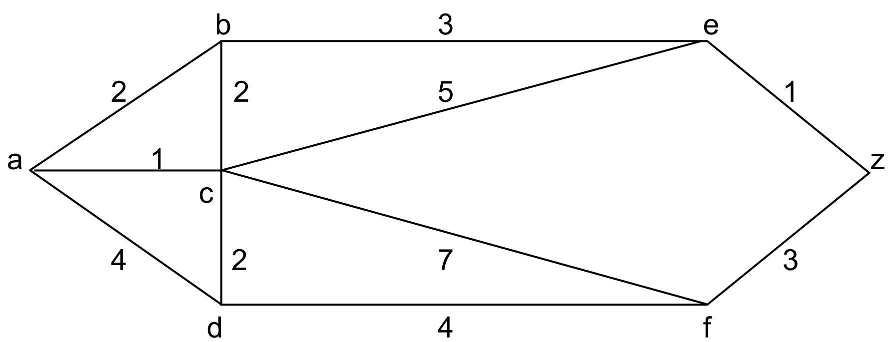
c) Find a deterministic finite state machine that recognizes the set:

$L=\{0^i10^j \mid i \ge 1, j \ge 1\}$

**(6+6+6)**

**6.**

a) Apply Dijkstra's algorithm to determine a shortest path between a and z in the following graph:




The numbers associated with the edges are distances between vertices.

b) Obtain the principal conjunctive normal form and principal disjunctive normal form of the formula S given by

$(\neg P \to R) \wedge (Q \leftrightarrow P)$

c) State Pigeon hole principle. Show that in a sequence of $n^2+1$ distinct integers, there is either an increasing subsequence of length$_{(n+1)}$ or decreasing subsequence of length.

**(6+6+6)**