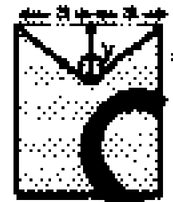
Physics

Time: 2 hours

Note: Question number 1 to 10 carries ***2 marks*** each and 11 to 20 carries ***4 marks*** each

1. A long wire of negligible thickness and mass per unit length λ is floating in a liquid such that the top surface of liquid dips by a distance 'y'. If the length of base of vessel is 2a, find surface tension of the liquid. ($y << a$)

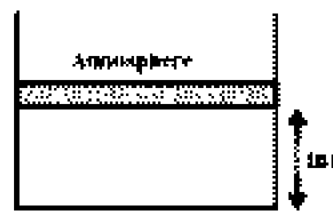


Sol. $\ell (2T \cos \theta) = \lambda \ell g$

$$T = \frac{\lambda g}{2 \cos \theta}$$

$$\Rightarrow T = \frac{\lambda g (a^2 + y^2)^{1/2}}{2y} \approx \frac{\lambda g a}{2y}.$$

2. An ideal diatomic gas is enclosed in an insulated chamber at temperature 300K. The chamber is closed by a freely movable massless piston, whose initial height from the base is 1m. Now the gas is heated such that its temperature becomes 400 K at constant pressure. Find the new height of the piston from the base.
If the gas is compressed to initial position such that no exchange of heat takes place, find the final temperature of the gas.



Sol. Process 1 is isobaric
$T_1 = 300$ K, $T_2 = 400$ K

$\frac{V}{T}$ = constant

$$\frac{A \times 1}{300} = \frac{A \times h}{400} \Rightarrow h = \frac{4}{3} m.$$

Process 2 is adiabatic

$$TV^{\gamma - 1} = \text{constant}, \; 400\left(\frac{A \times 4}{3}\right)^{\gamma - 1} = T_f (A \times 1)^{\gamma - 1} \Rightarrow T_f = 400\left(\frac{4}{3}\right)^{2/5} K.$$

3. In Searle's apparatus diameter of the wire was measured 0.05 cm by screw gauge of least count 0.001 cm. The length of wire was measured 110 cm by meter scale of least count 0.1 cm. An external load of 50 N was applied. The extension in length of wire was measured 0.125 cm by micrometer of least count 0.001 cm. Find the maximum possible error in measurement of young's modulus.

Sol. $$Y = \frac{4F/\pi D^2}{(\Delta L)/L} = \frac{4FL}{\pi D^2 (\Delta L)}$$

Maximum possible relative error

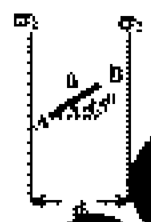
$$\frac{\Delta Y}{Y} = \frac{\Delta L}{L} + \frac{2\Delta D}{D} + \frac{\Delta(\Delta L)}{\Delta L} = \left(\frac{0.1}{110} + \frac{2 \times 0.001}{0.050} + \frac{0.001}{0.125}\right)$$

Percentage error

$$100 \times \frac{\Delta Y}{Y} = \frac{1}{11} + 4 + \frac{4}{5}$$

$= 0.8 + 4 + 0.09 = 4.89\ \%$

4. Two infinitely large sheets having charge densities σ_1 and σ_2 respectively ($\sigma_1 > \sigma_2$) are placed near each other separated by distance 'd'. A charge 'Q' is placed in between two plates such that there is no effect on charge distribution on plates. Now this charge is moved at an angle of 45° with the horizontal towards plate having charge density σ_2 by distance 'a' (a < d). Find the work done by electric field in the process.



Sol. $E = \frac{(\sigma_1 - \sigma_2)}{2\varepsilon_0}$

work done by electric field, $W = q\vec{E} \cdot \vec{d} = E\frac{a}{\sqrt{2}}q = \frac{q(\sigma_1 - \sigma_2)a}{2\sqrt{2}\varepsilon_0}$

5. An α-particle and a proton are accelerated from rest through [illegible] potential difference and both enter into a uniform perpendicular magnetic field. Find the ratio of [illegible] radii of curvature.

Sol. $r = \frac{\sqrt{2qVm}}{qB}$

$$\frac{r_\alpha}{r_p} = \sqrt{\frac{m_\alpha}{m_p} \times \frac{q_p}{q_\alpha}}$$

$$= \sqrt{\frac{4}{1} \times \frac{e}{2e}} = \sqrt{2} : 1$$

6. A small ball of radius 'r' is [illegible] in a viscous liquid under gravity. Find the dependency of rate of heat produced in terms of radius [illegible] after the drop attains terminal velocity.

Sol. Rate of heat produ[illegible]ed = F v

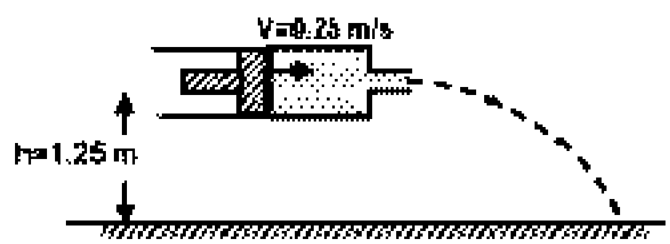
$= 6\pi\eta r v_T \times v_T$

$\frac{dQ}{dt} = 6$[illegible]

[illegible] $\rho) r^2 g \times \eta$

$\frac{dQ}{dt} \propto r^5$

7. A syringe of diameter D = 8 mm and having a nozzle of diameter d = 2 mm is placed horizontally at a height of 1.25 m as shown in the figure. An incompressible and non-viscous liquid is filled in syringe and the piston is moved at speed of 0.25 m/s. Find the range of liquid jet on the ground.



Sol. AV = Constant

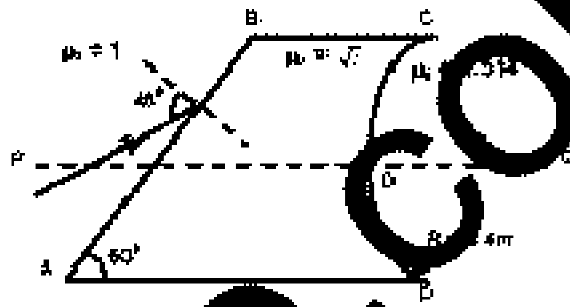
$D^2V = d^2v$

$$v = \frac{D^2}{d^2}V = \left(\frac{8}{2}\right)^2 \times 0.25$$

$$= 16 \times 0.25 = 4\,m/s$$

$$x = v\sqrt{\frac{2h}{g}} = 4\sqrt{\frac{2 \times 1.25}{10}} = 4 \times \frac{1}{2} = 2m$$

8. A light ray is incident on an irregular shaped slab of refractive index $\sqrt{2}$ at an angle of 45° with the normal on the incline face as shown in the figure. The ray finally emerges from the curved surface in the medium of the refractive index $\mu = 1.514$ and passes through point E. If the radius of curved surface is equal to 0.4 m, find the distance OE correct upto two decimal places.



Sol. Using Snell's law

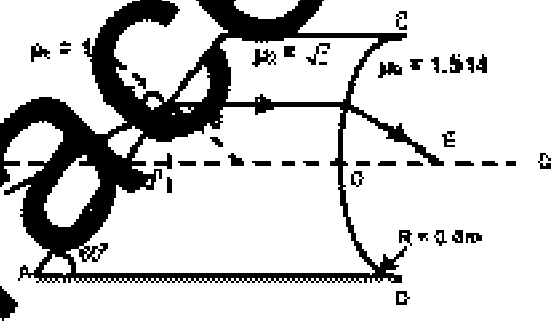
$\mu_1 \sin 45° = \mu_2 \sin \theta$

$\theta = 30°$,

i.e. ray moves parallel to axis

$$\frac{\mu_3}{OE} - \frac{\mu_2}{\infty} = \frac{(\mu_3 - \mu_2)}{R}$$

$OE = 6.056$ m ≈ 6.06 m



9. A screw gauge of pitch 1mm has a circular scale divided into 100 divisions. The diameter of a wire is to be measured by above said screw gauge. The main scale reading is 1mm and 47^{th} circular division coincides with main scale. Find the curved surface area of wire in true significant figures. (Given the length of wire is equal to 5.6 cm and there is no zero error in the screw gauge.)

Sol. Least count $= \frac{1mm}{100} = 0.01mm$

Diameter = M. S. + No. of division coinciding with main scale × Least count.

= 1mm + 47 × 0.01 mm

= 1.47 mm = 0.147 cm

Curved surface area $= \pi d l = \frac{22}{7} \times 0.147 \times 5.6 = 2.6\ cm^2$

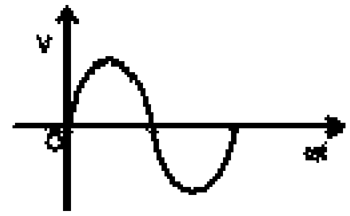
10. The age of a rock containing lead and uranium is equal to 1.5×10^9 yrs. The uranium is decaying into lead with half life equal to 4.5×10^9 yrs. Find the ratio of lead to uranium present in the rock, assuming initially no lead was present in the rock. (Given $2^{1/3} = 1.259$)

Sol. $$\frac{N}{N_0} = \left(\frac{1}{2}\right)^{t/t_{1/2}} = \left(\frac{1}{2}\right)^{1/3} = \frac{1}{1.259}$$

$$\frac{N_U}{N_{Pb} + N_U} = \frac{1}{1.259}$$

$$\frac{N_{Pb}}{N_U} = 0.259$$

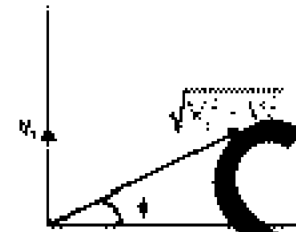
11 An inductor of inductance (L) equal to 35 mH and resistance (R) equal to 11 Ω are connected in series to an AC source. The rms voltage of a.c. source is 220 volts and frequency is 50 Hz.



(a) Find the peak value of current in the circuit

(b) Plot the current (I) vs (ωt) curve on the given voltage vs (ωt) curve. (Given $\pi = \frac{22}{7}$)

Sol. $Z = \sqrt{(\omega L)^2 + R^2}$



$I_0 = \frac{V_0}{Z} = \frac{220\sqrt{2}}{\sqrt{(100\pi \times 35 \times 10^{-3})^2 + (11)^2}} = 20\text{Amp}$

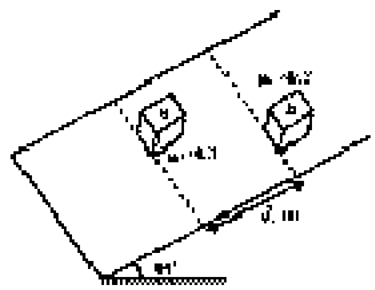
$\tan\phi = \frac{\omega L}{R} = \frac{100\pi \times 35 \times 10^{-3}}{11} = 1$

$\Rightarrow \phi = 45^0$

$I = I_0 \sin(\omega t - \frac{\pi}{4})$

$= 20 \sin(100\pi t - \frac{\pi}{4})$

12 Two identical blocks A and B are placed on a rough inclined plane of inclination 45°. The coefficient of friction between block A and incline is 0.2 and that of between B and incline is 0.3. The initial separation between the two blocks is $\sqrt{2}$ m. The two blocks are released from rest, then find (a) the time after which front faces of both blocks come in same line and (b) the distance moved by each block for attaining above position.



Sol. $a_A = g \sin 45° - 0.2 g \cos 45° = 4\sqrt{2}\ m/s^2$

$a_B = g \sin 45° - 0.3 g \cos 45° = \frac{7}{2}\sqrt{2}\ m/s^2$

$a_{AB} = 0.5\sqrt{2}\ m/s^2$

$S_{AB} = \frac{1}{2} a_{AB} t^2$

$t^2 = \frac{2\sqrt{2}}{0.5\sqrt{2}} = 4$

$t = 2$ s

$S_B = \frac{1}{2} a_B t^2 = 7\sqrt{2}$ m

$S_A = \frac{1}{2} a_A t^2 = 8\sqrt{2}$ m

13 In a photoelectric setup, the radiations from the Balmer series of hydrogen atom are incident on a metal surface of work function 2eV. The wavelength of incident radiations lies between 450 nm to 700 nm. Find the maximum kinetic energy of photoelectron emitted.
(Given hc/e = 1242 eV-nm)

Sol. $\Delta E = 13.6\left[\frac{1}{4} - \frac{1}{n^2}\right] = \frac{hc}{e\lambda} = \frac{1242}{\lambda}$

$$\Rightarrow \quad \lambda = \frac{1242 \times 4n^2}{13.6(n^2 - 4)}$$

λ_{min} which lies between 450 nm and 700 nm is for transition from n = 4 to n = 2 and is equal to 487.05 nm

For maximum K.E. of photoelectron

$$\frac{hc}{\lambda_{min}} = \phi + K.E._{max}$$

$$K.E._{max} = \frac{13.6 \times 12}{4 \times 16} - 2 = 0.55\,eV$$

14. A spherical ball of radius R is floating in a liquid with half of its volume submerged in the liquid. Now the ball is displaced vertically by small distance inside the liquid. Find the frequency of oscillation of ball.

Sol. Restoring force $= \pi R^2 x \rho g$ (for small x)

$$\Rightarrow -m\frac{d^2x}{dt^2} = \pi R^2 x \rho g$$

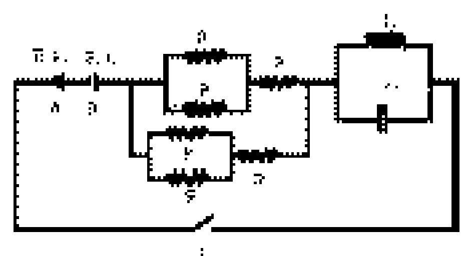
$$\frac{d^2x}{dt^2} = -\frac{3}{2}\frac{g}{R}x, \quad \left(\text{as } \frac{4}{3}\frac{\pi R^3}{2}\rho g = mg\right)$$

$\therefore$ Motion is SHM

$$\Rightarrow \omega^2 = \frac{3}{2}\frac{g}{R}$$

$$\Rightarrow f = \frac{1}{2\pi}\sqrt{\frac{3g}{2R}}$$

15. The two batteries A and B, connected in given circuit, have equal e.m.f. E and internal resistance r_1 and r_2 respectively ($r_1 > r_2$). The switch S is closed at t = 0. After long time it was found that terminal potential difference across the battery A is zero. Find the value of R.



Sol. Since average voltage across capacitor and inductor for D.C. sources will be zero at steady state.

$$i = \frac{2E}{(R_{eq} + r_1 + r_2)} = \frac{2E}{\left(r_1 + r_2 + \frac{3R}{4}\right)} \quad \text{...(i)}$$

P.D. across the battery A $= E - ir_1 = 0$

$$\Rightarrow E = ir_1 \quad \text{...(ii)}$$

From (i) and (ii),

$$R = \frac{4(r_1 - r_2)}{3}$$

16. A point object is moving with velocity 0.01 m/s on principal axis towards a convex lens of focal length 0.3 m. When object is at a distance of 0.4 m from the lens, find

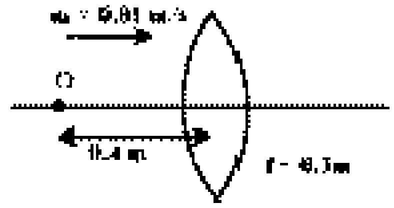
(a) rate of change of position of the image, and

(b) rate of change of lateral magnification of image.

Sol. $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

$-\frac{1}{v^2}\frac{dv}{dt} + \frac{1}{u^2}\frac{du}{dt} = 0$

$\Rightarrow \frac{dv}{dt} = \frac{v^2}{u^2}\frac{du}{dt}$ .... (i)



$\frac{1}{30} = \frac{1}{v} - \frac{1}{-40}$

$\Rightarrow v = 120$ cm.

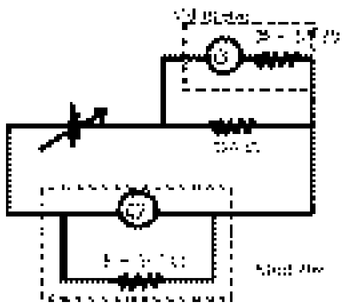
$\Rightarrow m = \frac{dv}{du} = \frac{v^2}{u^2} = \left(1 - \frac{v}{f}\right)^2$

$\frac{dm}{dt} = -\frac{2}{f}\left(1 - \frac{v}{f}\right)\frac{dv}{dt}$

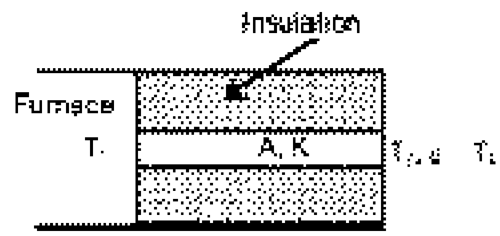
$= \frac{-2}{0.3}\left(1 - \frac{120}{30}\right) \times 0.09 = 1.8\ s^{-1}$

17. An experiment is performed to verify Ohm's law using a resistor of resistance R = 100 Ω, a battery of variable potential difference, two galvanometers and two resistances of 10^6 Ω and 10^{-3} Ω are given. Draw the circuit diagram and indicate clearly position of ammeter and voltmeter.

Sol.



18. A uniform rod of length L, conductivity K is connected from one end to a furnace at temperature T_1. The other end of rod is at temperature T_2 and is exposed to atmosphere. The temperature of atmosphere is T_s. The lateral part of rod is insulated. If $T_2 - T_s << T_s$, $T_2 = T_s + \Delta T$ & $\Delta T \propto (T_1 - T_s)$, find proportionality constant of given equation. The heat loss to atmosphere is through radiation only and the emissivity of the rod is e.



Sol. $\frac{KA(T_1 - T_2)}{L} = e\sigma A\left(T_2^4 - T_s^4\right)$

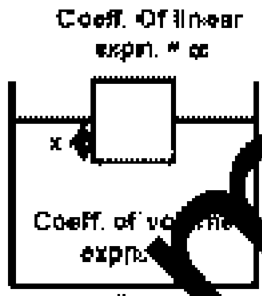
$= e\sigma A\left[(T_s + \Delta T)^4 - T_s^4\right] = 4e\sigma A T_s^3 \Delta T$

$\Rightarrow \frac{K(T_1 - T_s)}{L} = \Delta T\left[4e\sigma T_s^3 + \frac{K}{L}\right]$

$\Rightarrow \Delta T = \frac{K(T_1 - T_s)}{L\left[4e\sigma T_s^3 + \frac{K}{L}\right]}$

∴ Proportionality constant $= \frac{K}{L\left[4e\sigma T_s^3 + \frac{K}{L}\right]}$

19. A cubical block is floating inside a bath. The temperature of system is increased by small temperature ΔT. It was found that the depth of submerged portion of cube does not change. Find the relation between coefficient of linear expansion (α) of the cube and volume expansion of liquid (γ).



Sol. At initial temperature for the equilibrium of the block

$AL\rho_b g = Ax\rho_\ell g$

$L\rho_b = x\rho_\ell$ . . . (i)

At final temperature

$A' = A(1 + 2\alpha\Delta T)$

$\rho'_\ell = \rho_\ell(1 - \gamma\Delta T)$

For the equilibrium of the block

$A(1 + 2\alpha\Delta T)(x)\rho_\ell(1 - \gamma\Delta T) = AL\rho_b = Ax\rho_\ell$

$\Rightarrow 1 + 2\alpha\Delta T - \gamma\Delta T = 1$

$\Rightarrow \gamma = 2\alpha$

20. In a Young's double slit experiment light consisting of two wavelengths $\lambda_1 = 500$ nm and $\lambda_2 = 700$ nm is incident normally on the slits. Find the distance from the central maxima where the maximas due to two wavelengths coincide for the first time after central maxima. (Given $\frac{D}{d} = 1000$) where D is the distance between the slits and the screen and d is the separation between the slits.

Sol. $y_1 = \frac{nD\lambda_1}{d}$

$y_2 = \frac{mD\lambda_2}{d}$

$y_1 = y_2 \Rightarrow \frac{n}{m} = \frac{7}{5}$

For the first location, $m = 5, n = 7$

$\therefore y = 7 \times 1000 \times 5 \times 10^{-7} = 35 \times 10^{-4} = 3.5$ mm.