Code: AE07 DECEMBER 2008

Subject: NUMERICAL ANALYSIS & COMPUTER PROGRAMMING
Time: 3 Hours
Max. Marks: 100

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q. 1. must be written in the space provided for it in the answer book supplied and nowhere else.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or best alternative in the following: (2x10)

a. The positive root of the equation $2x^3 + x^2 - 7x - 20 = 0$ lies in the interval

(A) (0, 1). **(B)** (1, 2).
(C) (2, 3). **(D)** (3, 4).

b. Newton-Raphson method, when applied to find a root of the equation $f(x) = 0$, has given the formula $x_{n+1} = \frac{1}{3}\left[2x_n + \frac{N}{x_n^2}\right]$. If the iteration converges, then the quantity that is being determined is

(A) $N^{1/2}$. **(B)** $N^{1/3}$.
(C) $N^{1/4}$. **(D)** $N^{1/5}$.

c. The bound for error in linear interpolation is given by $|\text{Error}| \le A h^2 \max|f''(x)|,\ a \le x \le b$. The value of A is

(A) $1/12$. **(B)** $1/4$.
(C) $1/2$. **(D)** $1/8$.

d. The following data is given

x	-1	-0.5	0
$f(x)$	2.7183	1.6487	1

An approximate value of $f''(-1)$ using forward differences is given by

(A) 1.6836. (B) 1.3832.
(C) 1.7836. (D) 1.3772.

e. The value of the integral $\int_2^4 \frac{dx}{x^2+5x+1}$ evaluated by the trapezoidal rule with $h = 1$, is obtained as

(A) 0.1868. (B) 0.0868.
(C) 0.1736. (D) 0.0846.

f. For the initial value problem $y' = 2x + 3y, y(1) = 2$, an approximation to y(0.1) by Taylor series method of second order with $h = 0.1$, is

(A) 2.52. (B) 2.73.
(C) 2.93. (D) 3.03.

g. What will be the output of the following program?

```
main( ) {
   static int a[5] = { 1,2,3,4,5 };
    int *b,i ;
    b = a;
    for ( i = 0; i<5; i ++ ) {
          printf("%d",*b);
          b++ ; }
}
```

(A) Undefined Output. (B) 1 2 3 4 5.
(C) Error. (D) 5 4 3 2 1.

h. What will be the output of the following program?

```
void main( )     {
    int arr[ ] = {10, 11, 12, 13, 14};
    int i,  *p;
    for (p=arr, i=0; p+i<=arr+4; p++, i++)
         printf("%d", *(p+i));    }
```

(A) 10 11 12 13 14 (B) 10 11 12
(C) 11 13 (D) 10 12 14

i. What will be the output of the following programme?

```
enum month { Illegal month, Jan, Feb, March, April, May, June, July, Aug, Sep,
Oct, Nov, Dec,  };
main ( ) {
    enum month mname;
    mname = Nov;
    printf("%s\n", mname);
}
```

(A) Nov. (B) Undefined Output.
(C) 11. (D) Error.

j. What will be the output of the following programme segment?

```
int m, n=10;
m = n++ * n++;
printf("%d %d %d %d %d", m, n, m++, m--, --m);
```

(A) 100, 12, 100, 101, 99 **(B)** 100, 12, 100, 111, 109
(C) 110, 12, 110, 111, 109 **(D)** 110, 11, 100, 101, 99

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. A root of the equation $\log_{10} x - x + 3 = 0$ is to be determined. Obtain an interval of unit length, in which the root lies. Find this root correct to 4 decimals using the Secant method. (work with 6 places of decimals). **(8)**

b. Write a *C* program to find a simple root of $f(x) = 0$ by the Secant method. Input (i) *a*, *b* (two initial approximations), (ii) *n* (maximum number of iterations) and (iii) error tolerance "tol". Output (i) approximate root, (ii) number of iterations taken. If the inputted value of *n* is not sufficient, the program should write "Iterations are not sufficient". Write the subprogram for $f(x)$ as $f(x) = x^3 + 5x + 1$. **(8)**

Q.3 a. The system of equations $3x^2 + 5y^2 - 3xy = 12$, $x^2 - 3y^2 + 5xy = 5$ has a solution near *x* =1.3, y = 1.6. Perform two iterations to improve the solution, using the Newton's method. **(9)**

b. Find the Cholesky factorization of the matrix.

$$\begin{bmatrix} 9 & -6 & 0 & 0 \\ -6 & 13 & -6 & 0 \\ 0 & -6 & 13 & -6 \\ 0 & 0 & -6 & 13 \end{bmatrix}$$

(7)

Q.4 a. Using Gauss elimination, determine whether the following system of equations has a solution. If it has, then find all the solutions. **(8)**

$$\begin{aligned} 4x + y + z + w &= 3 \\ 2x + 6y - 3z - 2w &= 12 \\ 16x + 15y - 3z - w &= 33 \\ 2x - 5y + 4z + 3w &= -9 \end{aligned}$$

b. Solve the following system of equations using the Gauss-Seidel method

$$\begin{array}{ccccccccc} 6x_1 & + & 2x_2 & + & 4x_3 & + & 3x_4 & = & 7 \\ 2x_1 & + & 9x_2 & + & 2x_3 & + & 6x_4 & = & 16 \\ x_1 & + & 3x_2 & - & 9x_3 & + & x_4 & = & 7 \\ 3x_1 & + & 2x_2 & + & x_3 & + & 6x_4 & = & 12 \end{array}$$

Assume the initial solution vector as $[0.3, 0.7, -0.3, 1.6]^T$ and obtain the result correct to 2 decimal places.
(8)

Q.5 a. For the function $f(x) = 1/(3+5x),\ 0 \le x \le 2$, a table of equispaced data values is to be constructed. If quadratic interpolation is proposed to be used, find the step length *h* such that $|\text{Error in quadratic interpolation}| < 10^{-6}$. **(7)**

b. If $\sum_{k=0}^{n-1} \Delta^2 f_k = a\Delta f_n + b\Delta f_0$, then find the values of *a* and *b*. **(3)**

c. Write a C program for estimating the value of a function *f(x)* using Lagrange interpolation with 10 data values. Input the value of *x* as xin and output the value of *y* as yout. **(6)**

Q.6 a. Construct the forward difference table for the data

x	0	0.2	0.4	0.6	0.8	1.0
f(x)	-0.5	-0.476	-0.308	0.148	1.036	2.5

Hence, approximate f(0.3) using forward differences.
(7)

b. A given data is to be approximated by the quadratic polynomial $f(x) = a + bx + cx^2$. Derive the normal equations using the least squares approximation. Hence, find the least squares approximation to the data

x	-2	-1	0	1	3
$f(x)$	8.0	5.2	2.6	4.2	24.2

(3+6)

Q.7 a. The following data for the function $f(x) = x^4$ is given.

x:	0.4	0.6	0.8
$f(x)$:	0.0256	0.1296	0.4096

Find $f'(0.8)$ and $f''(0.8)$ using quadratic interpolation. Compare with the exact solution. Obtain the bound on the truncation error. **(9)**

b. Find the approximate value of

$$I = \int_0^1 \frac{dx}{1+x}$$

using trapezoidal rule. Obtain a bound for the errors. **(7)**

Q.8 a. Write a C program to evaluate $\int_a^b f(x)dx$ by Simpson's rule of integration based on 2n+1 points. Input the values of the limits *a*, *b* and *n*. Write $f(x) = x/(x^2 + x + 1)$ as a function program. Output all the data and the computed value. **(8)**

b. Evaluate the integral $I = \int_0^1 \frac{dx}{1+x}$ using Composite Simpson's rule with 2, 4 and 8 equal subintervals. **(8)**

Q.9 a. Find the value of the integral

$$I = \int_2^3 \frac{\cos 2x}{1 + \sin x} dx.$$

using Gauss-Lagendre two and three point integration rules. **(8)**

b. Given the initial value Problem

$$u' = -2tu^2, \quad u(0) = 1$$

with h=0.2 on the interval [0, 0.4] use the fourth order classical Runge-Kutta Method to calculate y(0.4). **(8)**