**Code: A-07**
**Subject: NUMERICAL ANALYSIS & COMPUTER PROGRAMMING**
**Time: 3 Hours**
**Max. Marks: 100**
**NOTE: There are 11 Questions in all.**

- • **Question 1 is compulsory and carries 16 marks. Answer to Q. 1. must be written in the space provided for it in the answer book supplied.**
- • **Answer any THREE Questions each from Part I and Part II.**

**Q.1 Choose the correct or best alternative in the following: (2x8)**

a. a. The number 0.015625 is rounded as 0.0156. Then, the relative error in this approximation is

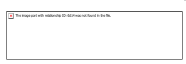
**(A)** -0.0016. **(B)** 0.0016.
**(C)** 0.016. **(D)** 0.000025.

b. b. A real root of f(x) = 0 lies in the interval [0, 1]. Bisection method is applied to find this root. If the permissible error in the approximation is $\in$, then the number of iterations required is greater than or equal to

**(A)** **(A)** $-\log\in/\log 2$. **(B)** $\log\in/\log 2$.
**(C)** $-\log\in$. **(D)** $-\log\in\log 2$.

c. Gauss-Seidel method is applied to solve the system of equations $\begin{bmatrix} 1 & -p \\ -p & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, p real constant. The method converges for

**(A)** **(A)** $|p| > 1$. **(B)** all p.
**(C)** $|p| \le 1$. **(D)** $|p| < 1$.

d. The divided difference  for the function $f(x) = x^3$ is given by

**(A)** 6. **(B)** $2(x_1 + x_2 + x_3)$.
**(C)** $x_1 + x_2 + x_3$. **(D)** $x_1 x_2 x_3$.

e. e. The least squares approximation to the data

| | | | | |
|---|---|---|---|---|
| x | 1 | 2 | 3 | 4 |
| f (x) | 6 | 9 | 14 | 21 |

is given as f(x) = 5x. Then, the least squares error is given as

**(A)** 0.04. **(B)** 4.
**(C)** 6. **(D)** 0.004.

f. f. The error in the numerical differentiation formula

$f''(x_k) = \frac{1}{h^2}[f(x_{k-1}) - 2f(x_k) + f(x_{k+1})]$ is given by $Mf^{(4)}(\xi)$, where the value of M is

**(A)** **(A)** $h^2/12$. **(B)** $h^4/24$.
**(C)** $h^4/12$. **(D)** $h/12$.

g. g. The value of the integral

$$\int_0^1 \frac{x}{1+x^2}\,dx$$

using Simpson's rule is

**(A)** $\log\sqrt{2}$ **(B)** $13/40$

**(C)** $7/20$ **(D)** $7/10$

h. h. The following C program is given

```
# include &lt;stdio.h&gt;
main( )
{
 switch (choice = toupper(getchar( )))
 {
     case 'B':
          printf("BLUE");
          break;
     case 'P':
          printf("PINK");
          break;
     case 'G':
          printf("GREEN");
          break;
     default:
          printf("ERROR");
   }
}
```

If the character 'g' is entered, the output is

**(A)** ERROR **(B)** GREEN

**(C)** PINK **(D)** green

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**PART I**

**Answer any THREE Questions. Each question carries 14 marks.**

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**Q.2** a. Solve the system of equations $\begin{bmatrix} 2 & 1 & -4 & 1 \\ -4 & 3 & 5 & -2 \\ 1 & -1 & 1 & -1 \\ 1 & 3 & -3 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 4 \\ -10 \\ 2 \\ -1 \end{bmatrix}$ using the Gauss elimination method. **(8)**

b. Using the Choleski method, find the solution of the following system of equations $\begin{bmatrix} 8 & -3 & 0 \\ -3 & 8 & -3 \\ 0 & -3 & 8 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 49/16 \\ 1/2 \end{bmatrix}$. **(6)**

**Q.3** a. The error in the Newton-Raphson method for finding a simple root of f(x)=0, can be written as $\epsilon_{k+1} = c\,\epsilon_k^p$. Determine the values of c and p. What is the order of the method? **(6)**

b. The equation f(x)=0 has a simple root in the interval (1, 2). The function f(x) is such that $|f'(x)| \ge 10$ and $|f''(x)| \le 1$ for all x in (1, 2). Assuming that the Newton-Raphson's method converges for all initial approximations in (1, 2), find the number of iterations required to obtain the root correct to $5 \times 10^{-7}$. **(8)**

**Q.4** a. Locate the negative root of smallest magnitude of the equation $7x^4 + x^3 + 6x^2 + 2x - 16 = 0$ in an interval of length 1. Taking the end points of this interval as the initial approximations to the root, perform five iterations using secant method (use five decimal places). **(7)**

b. b. Write a C program to find a simple root of f(x) = 0 by the secant method. Input (i) two initial approximations to the root as a and b, (ii) maximum number of iterations m, that the user wants to be done. (iii) error tolerance epsilon. Evaluate f(x) as a function. Output (i) number of iterations taken to obtain the root, (ii) the value of the root, (iii) value of f(root). If the iterations m, are not sufficient, output that "Number of iterations given are not sufficient". **(7)**

**Q.5** a. Solve the system of equations $\begin{bmatrix} 4 & -1 & 0 & 0 \\ -1 & 4 & -1 & 0 \\ 0 & -1 & 4 & -1 \\ 0 & 0 & -1 & 4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 2.5 \\ -2.75 \\ 1.75 \\ -1.25 \end{bmatrix}$ using the Gauss-Jacobi method, with the starting approximations taken as $x_1 = 0.4$, $x_2 = -0.6$, $x_3 = 0.3$, $x_4 = -0.3$. Perform three iterations. **(5)**

b. For the system in 5 (a) above, write the Gauss-Jacobi method in matrix form. Hence, find the rate of convergence of the method. **(9)**

**Q.6** a. The system of equations $x^2 + y^2 = 4.82$, $xy + yz + zx = 0.59$, $yz^2 + y^2z = -1.33$ has a solution near (x, y, z) = (1, 2, -0.5). Derive the Newton's method for solving this system. Iterate once using the given initial approximations. **(7)**

b. Write a C program to rearrange a given set of integer numbers into ascending order. Use the following:

(ii) (ii) Define initially an array f as an 100 element array.
(iii) (iii) Read n, the number of given integer numbers followed by the numerical values.
(iv) (iv) Write a function prototype called "reordering", whose arguments are n and f.
(v) (v) The program for reordering in ascending order is to be given in "reordering". **(7)**

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## PART II

**Answer any THREE Questions. Each question carries 14 marks.**

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**Q.7** a. Construct an interpolating polynomial that fits the data

| x | 0 | 1 | 2 | 5 | 7 | 10 |
|---|---|---|---|---|---|---|
| f(x) | -2.5 | -0.5 | 10.5 | 187.5 | 515.5 | 1502.5 |

Hence, or otherwise interpolate the value of f (8). **(7)**

b. A table of values for f (x)= $e^{x+1}$ in [0, 1] is to be constructed with step size h = 0.1. Find the maximum total error if quadratic interpolation is to be used to interpolate in this interval. **(7)**

**Q.8** a. A mathematical model of a periodic process in an experiment is taken as f (t) = a + b cos (t) and a data of N points $(x_i, f_i)$, i = 1, 2, ……, N is given. If the parameters a and b are to be determined by the method of least squares, find the normal equations. Use these equations to find a, b for the following data (keep four decimal accuracy). **(3+5)**

| t(radians) | 0.5 | 1.0 | 1.5 | 2.0 | 2.5 |
|---|---|---|---|---|---|
| f(t) | 0.9082 | 0.6552 | 0.3031 | -0.0621 | -0.3509 |

b. Write a C program for interpolation using Lagrange interpolation. Input the following (i) Limit to number of points as 10. (ii) Number of points for any application as n. (iii) (Abscissas, Ordinates) = $(x_i, y_i)$. (iv) The value of x for which interpolation is required. Output x and the interpolated value. **(6)**

**Q.9** a. A differentiation rule of the form $hf'(x_2) = af(x_0) + bf(x_1) + cf(x_2)$; $x_j = x_0 + jh, j = 1,2$ is given. (i) Determine a, b, c so that the rule is exact for polynomials of degree 2. (ii) Find the error term. (iii) If the roundoff errors in computing $f(x_0)$, $f(x_1)$, $f(x_2)$ are $\in_1$, $\in_2$, $\in_3$ where $|\in_i| \le \in$, i =1, 2, 3, then obtain the expression for the bound of roundoff error in computing $f'(x_2)$. **(8)**

b. Use the formula $f''(x_0)=\frac{1}{h^2}[f(x_0-h)-2f(x_0)+f(x_0+h)]$, to compute $f''(0.6)$ from the following table of values with h=0.4 and h=0.2. Perform Richardson extrapolation to compute a better estimate for $f''(0.6)$. **(6)**

| X | 0.2 | 0.4 | 0.6 | 0.8 | 1.0 |
|---|---|---|---|---|---|
| f(x) | 1.3016 | 2.5256 | 3.8296 | 5.3096 | 7.1 |

**Q.10** a. Find the values of a,b,c such that the numerical integration formula $\int_{-1}^{1} f(x)dx = af(-1)+bf(c)$ is of as high order as possible. Find the error term. **(7)**

b. Write a C program for solving the initial value problem $y' = f(x, y),\ y(x_0) = y_0$, by Euler's method. (i) Input the initial values $x_0, y_0$; the final value x = xf and step length h. (ii) Use a subprogram for evaluating $f(x, y) = x^2 + y^2$. (iii) Create a file named "result" and put the computed values, for each value of x, in it. **(7)**

**Q.11** a. Evaluate the integral $\int_{-1}^{1} (1 - x^2) \cos x \, dx$ by (i) two point Gauss-Legendre formula, (ii) two point Gauss-Chebyshev formula. **(6)**

b. An approximate value of u(0.2) for the initial value problem $u' = u^2 + t^2, u(0) = 1;$ with h = 0.2 is to be obtained. Find this value using (i) Euler's method, (ii) Taylor series method of order four. **(8)**