SATHYABAMA UNIVERSITY

(Established under section 3 of UGC Act, 1956)

Course & Branch: B.E. / B. Tech – IT/ECE
Title of the paper: Higher Mathematics
Semester: IV Max. Marks: 80
Sub.Code: 112402/113401 Time: 3 Hours
Date: 17-04-2007 Session: FN

PART – A (10 x 2 = 20)

Answer ALL the Questions

1. If f(x) = sinpx is defined in the interval $(-l, l)$ find the value of a_0 and a_n (p is not an integer).

2. Find the half range sine series for f(x) = 2 in $0 < x < 4$.

3. Write the complete integral of Z = px + qy + pq.

4. Find the complete integral of q = 2px.

5. Classify the following equations:
   (i) $U_{xx} + 2\,U_{xy} + U_{yy} = 0$. (ii) $X\,f_{xx} + y\,f_{yy} = 0,\ x > 0,\ y > 0$.

6. In one dimensional heat equation $\frac{\partial u}{\partial t} = \alpha^2\left(\frac{\partial^2 u}{\partial x^2}\right)$, α^2 is termed as - ---------- .

7. Write any two solutions of the Laplace equation obtained by the method of separation of variables.

8. The steady state temperature distribution is considered in a square plate with sides x = 0, y = 0, x = a and y = a. The edge y=0 is kept at a constant temperature T and the other three edges are insulated. The same state is continued subsequently. Express the problem mathematically.

9. Define finite Fourier cosine transform of f(x) in (0, l).

10. If $F_s(s)$ is the Fourier sine transform of f(x), show that $F_s[f(x) \cos ax] = ½ [F_s (S + a) + F_s (s – a)]$.

PART – B (5 x 12 = 60)

Answer All the Questions

11. (a) Find Fourier series for the function

$$f(x) = \begin{cases} 1 + 2x/\pi, & -\pi < x < 0 \\ 1 - 2x/\pi, & 0 < x < \pi \end{cases}$$

(b) Obtain sine series for the function f(x) = x in (0, –l).

(or)

12. (a) Find the Fourier series expansion of the periodic function f(x) of period 2l defined by

$$f(x) = \begin{cases} l + x, & -l \le x \le 0. \\ l - x, & 0 \le x \le l. \end{cases}$$

(b) Find the half range sine series for

$$f(x) = \begin{cases} 2x/l\ , & 0 < x < l/2 \\ 2(l-x)/l, & l/2 < x < l \end{cases}$$

13. (a) Form the partial differential equation by eliminating arbitrary constants from $z = (x^2+a)(y^2+b)$.

(b) Solve $\frac{x^2}{p} + \frac{y^2}{q} = z$.

(or)

14. (a) Solve: $9 (p^2 z + q^2) = 4$.

(b) Solve: $(D^3 - 7 DD^{/2} - 6D^{/3}) z = x^2 y + \sin (x + 2y)$.

15. A string is tightly stretched and its ends are fastened at two points x = 0 and x = $\boldsymbol{l}$. The midpoint of the string is displaced transversely through a small distance 'b' and the string is released from rest in that position. Find an expression for the transverse displacement of the string at any time during the subsequent motion.

(or)

16. A rod 30 cm long, has its ends A and B kept at 20°C and 80°C, respectively until steady state conditions prevail. The temperature at each end is then suddenly reduced to 0°C and kept so. Find the resulting temperature function u (x, t) taking x = 0 at A.

17. A rectangular plate with insulated surfaces is 8 cm wide and so long compared to its width that it may be considered as an infinite plate. If the temperature along short edge y = 0 is $u(x,0)=100 \sin \pi x / 8$, $0 < x < 8$ while two long edges x = 0 and x=8 as well as the other short edge are kept at 0°C. Find the steady state temperature at any point of the plate.

(or)

18. A plate in the form of a ring is bounded by the circles r = 5 and r=10. Its surfaces are insulated and the temperature along the boundary are

$u(5, \theta) = 10\cos\theta + 6\sin\theta$

$u(10, \theta) = 17\cos\theta + 15\sin\theta$

Find the steady steady state temperature in the plate.

19. (a) Find the Fourier cosine transform of e^{-ax}, $a > 0$ and hence deduce the inversion formula.

(b) Find the Fourier transform of

$$f(x) = \begin{cases} 1 - |x|, & \text{if } |x| < 1 \\ 0, & \text{if } |x| > 1 \end{cases}$$

and hence find the value $\int_0^\infty \frac{\text{Sin}^4 t}{t^4}\, dt$.

(or)

20. (a) Using Parseval's identity, calculate $\int_0^\infty \frac{dx}{(a^2 + x^2)^2}$.

(b) Find finite Fourier sine and cosine transforms of $f(x) = e^{ax}$ in $(0, l)$.