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SATHYABAMA UNIVERSITY

(Established under section 3 of UGC Act, 1956)

Course & Branch: B.E/B.Tech-Common to ALL Branches
Title of the Paper: Engineering Mathematics – IV Max. Marks: 80
Sub. Code: 401-6C0054 Time: 3 Hours
Date: 16/11/2010 Session: AN

PART - A (10 X 2 = 20)
Answer ALL the Questions

1. State the sufficient conditions for a function *f(x)* to be expanded as a Fourier series.

2. Define RMS value and hence find the RMS value of $f(x) = x^2$ in $(-\pi, \pi)$.

3. Form the pde by elminating arbitrary constants from the relation $z = ax^n + by^n$.

4. Find the complete integral of $\frac{z}{pq} = \frac{x}{q} + \frac{y}{p} + \sqrt{pq}$.

5. State the assumptions in derving one-dimensional wave equation.

6. Write the possible solutions of the one-dimensional heat flow(un steady state) equation $u_t = \alpha^2 u_{xx}$.

7. Define steady state.

8. Write the possible solutions of $r^2 u_{rr} + ru_r + u_{\theta\theta} = 0$.

9. Find the fourier sine transform of $\frac{1}{x}$.

10. State convolution theorem on Fourier transforms.

PART – B (5 x 12 = 60)

Answer All the Questions

11. (a) Express $f(x) = (\pi - x)^2$ as a Fourier series of periodicity 2π in $0 < x < 2\pi$ and hence deduce the sum $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots.\infty$.

(b)Using six ordinates analyse harmonically the following data upto two harmonics.

x	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$
y	10	12	15	20	17	11

(or)

12. (a) Expand *f(x)* a series of sines if

$$F(x) = \begin{cases} \sin x & for \quad 0 \le x \le \frac{\pi}{4} \\ \cos x & for \quad \frac{\pi}{4} \le x \le \frac{\pi}{2} \end{cases}$$

(b)Expand $f(x) = e^{-ax}, -\pi < x < \pi$ as a complex form series.

13. (a) Form the PDE by eliminating F from the relation $xy + yz + zx = f\left(\frac{z}{x+y}\right)$.

(b) Solve $(2z - y)\,p + (x + z)\,q = -(2x + y)$.

(or)

14. (a) Solve $p^2 + q^2 = x^2 + y^2$.

(b) Solve $\left(D^2 - 3DD^1 + 2D^{1^2}\right)z = (2 + 4x)e^{x+2y}$.

15. A taut string of length '$2l$' is fastened at both ends. The mid point of the string is taken to a height 'h' and then released from rest in that position. Find the displacement of the string.

(or)

16. Solve $\frac{\partial u}{\partial t}\alpha^2 \frac{\partial^2 u}{\partial x^2}$ subject to the conditions
(i) u is not infinite as $t \to \infty$
(ii) u = 0 for x = 0 and x = π for all t
(iii) $u = \pi x - x^2$ for t = 0 in (0, π).

17. A long rectangular plate with insulated surfaces is π cm wide. The two long edges as well as one of the short edges are kept at 0°C while 0 the short edge y = 0 is kept at a temperature u_0°C. Find the steady state temperature distribution in the plate.

(or)

18. A semi circular plate of radius a cm has insulated faces and heat flows in plane curves. The bounding diameter is kept at 0°C and the semi circumference is maintained at temperature given by

$$u(a,\theta) = \begin{cases} \frac{k\theta}{\pi}, & 0 \le 0 \le \frac{\pi}{2} \\ \frac{k}{\pi}(\pi - \theta). & \frac{\pi}{2} \le \theta \le \pi \end{cases}$$

Find the steady state temperature distribution.

19. (a) Find the Fourier transforms of $f(x) = \begin{cases} a - |x| & for \quad |x| < a \\ 0 & for \quad |x| > a > 0 \end{cases}$ and hence deduce the value of $\int_0^\infty \frac{\sin^2 t}{t^2} dt$.

(b) Solve the integral equation $\int_0^\infty f(x) \cos \lambda x dx = e^{-\lambda}, \lambda > 0$.

(or)

20. (a) Find the Fourier cosine transform of e^{-x^2}

(b) Find the finite fourier sine transform of $f(x) = \left(1 - \frac{x}{\pi}\right)^2, 0 < x < \pi$.