SATHYABAMA INSTITUTE OF SCIENCE AND TECHNOLOGY
DEEMED UNIVERSITY

Course: B.E./B.Tech. Semester: IV
Title of the paper: Engineering Mathematics – IV Max. Marks: 80
Sub. Code: 401 (2002/2003/2004) Time: 3 Hours

PART – A (10 x 2 = 20)
Answer ALL the Questions

1. State the convergence of Fourier series of a function f(x) in any given interval, (i) when f(x) is continuous throughout and (ii) when f(x) has a point of discontinuity.

2. State the complex form of the Fourier series for a function f(x) in the interval (c, c + 2l).

3. Define singular solution of a partial differential equation.

4. Form the partial differential equation by eliminating the arbitrary constants 'a' and 'b' from the equation $z = (x^2 + a)\ (y^2 + b)$.

5. Derive the one – dimensional wave equation starting from the equation of motion.

6. List the various solutions of the equation $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$.

7. Express $\frac{\partial^2 y}{\partial x^2} + \frac{\partial^2 y}{\partial y^2} = 0$ in its equivalent polar form.

8. List all the solution for a two-dimensional heat equation in steady state in polar coordinates.

9. Show that $F_c\,[f(x)\sin ax] = \frac{1}{2}[F_s(a+s) + F_s(a-s)]$, if $F_s[f(x)] = F_s(s)$ and $F_s[f(x)]$ is called the Fourier sine transform of f(x).

10. State Parseval's identities for Fourier sine and cosine transforms.

PART – B (5 x 12 = 60)

Answer ALL the Questions

11. (a) Find the Fourier series of $f(x) = x + x^2$ in $(-\pi, \pi)$ of periodicity 2π. Hence, show that the sum $\sum_{1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$.

(b) Express $f(x) = |x|$ in the interval $-\pi < x < \pi$ as a Fourier series. Hence, show that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\infty = \frac{\pi^2}{8}$.

(or)

12. (a) Find the Fourier series of periodicity 2 for $f(x) = \pi x$, $0 \le x < 1$ and $f(x) = \pi(2 - x)$, $1 < x \le 2$.

(b) The displacements y of a part of mechanism corresponding to the movements x of the crank are tabulated as follows:

x	0	30	60	90	120	150	180	210	240	270	300	330
y	1.8	1.1	0.3	0.16	0.5	1.5	2.16	1.88	1.25	1.3	1.76	2.0

Express y = f(x) in a Fourier series up to the third harmonic.

13. (a) From the partial differential equation of all spheres of radius 'c' units and having their centers in the X o Y plane.

(b) Solve: $x(y^2 + z^2)\,p + y(z^2 + x^2)\,q = z\,(y^2 - x^2)$.

(or)

14. (a) Solve: $z^2\,(p^2 + q^2) = x^2 + y^2$.

(b) Solve: $(D^2 + 2D\,D' - D'^2)\,z = x^2y$.

15. A uniform elastic string of length 60 cm is subjected to a constant tension of 2 kg. If the ends are fixed and the initial displacement is $y(x, 0) = 60x - x^2$, $0 < x < 60$, while the initial velocity is zero, find the displacement function $y(x, t)$.

(or)

16. An insulated metal rod of length 100 cm has one end A kept at 0°C and the other end Bat 100°C, until steady state conditions prevail. At time $t = 0$, the temperature at A is then suddenly raised to 50° and thereafter maintained while at the same time $t = 0$, the end B is insulated. Find the temperature distribution at any point of the rod at any subsequent time.

17. Find the steady state temperature at any point of a square plate whose two adjacent edges are kept at 0° and the other two edges at 100°C.

(or)

18. A plate with insulated surfaces has the shape of a quadrant of a circle of radius 10 cm. The bounding radii $\theta = 0$ and $\theta = \pi/2$ are kept at 0°C and the temperature along the circular quadrant is kept at $100\ (\pi\theta - 2\theta^2)$°C, for $0 \le \theta \le \pi/2$ until steady state conditions prevail. Find the steady state temperature at any point on the plate.

19. (a) Applying the Fourier sine Transform

$$f(t) = \begin{cases} \sin t\ , & \text{when} \quad 0 < t \le \pi \\ 0\ , & \text{when} \quad t > \pi, \end{cases}$$

(b) Find the Fourier transform of $f(x) = \begin{cases} x^2 & , \text{for } |x| < a \\ 0 & , \text{for } |x| > a \end{cases}$

(or)

20. (a) Find the Fourier sine and cosine transforms of $f(x) = e^{-ax}$.

(b) Use Parseval's identity to evaluate

$$\int_0^\infty \frac{dx}{(x^2 + a^2)^2} \quad \text{and} \quad \int_0^\infty \frac{x^2}{(x^2 + a^2)^2}\, dx \quad (a > 0).$$