SATHYABAMA UNIVERSITY

(Established under section 3 of UGC Act, 1956)

Course & Branch: B.E. / B. Tech

Title of the paper: Engineering Mathematics - IV

Semester: IV — Max. Marks: 80

Sub.Code: 401 (2002/2003/2004/2005) — Time: 3 Hours

Date: 17-04-2007 — Session: FN

(Common to ALL Branches Except Bio Groups)

PART – A (10 x 2 = 20)

Answer ALL the Questions

1. State the necessary conditions for the existence of Fourier series of a given function f(x).
2. Define Root Mean Square value of f(x) in the interval (a, b).
3. Find the particular integral of $(D^2 - 4D^2D' + 4DD'^2)Z = 6\ Sin(3x+6y)$.
4. Eliminate f from $Z = x + y + f(xy)$.
5. What is meant by steady state condition in heat flow?
6. Explain Boundary value problem.
7. Obtain one dimensional heat flow equation from two dimensional heat flow equation for the unsteady case.
8. Write the different solutions of Laplace's equation in Cartesian coordinates.
9. Define Convolution of two functions in Fourier transforms.
10. Find f(x) if its Finite Fourier sine transform is $\dfrac{1-\cos p\pi}{p^2\pi^2}, p = 1,2,3...$ and $0 < x < \pi$.

PART – B (5 x 12 = 60)

Answer All the Questions

11. Find Fourier series of periodically 2 for

$$f(x) = \begin{Bmatrix} x & in -1 < x < 0 \\ x+2 & in 0 < x < 1 \end{Bmatrix}$$ and hence deduce the sum of

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + ... \text{to } \infty.$$

(or)

12. Compute the first three harmonics of the Fourier series of f(x) given by the following table.

x	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	π
f(x)	10.	1.4	1.9	1.7	1.5	1.2	1.0

13. (a) Solve $Z = px + qy + \sqrt{1 + p^2 + q^2}$.
(b) Solve $(D^2 + DD' - 6D'^2)Z = y\cos x$.

(or)

14. (a) Solve $z(x-y) = px^2 - qy^2$.
(b) Solve $(D^2 - DD')Z = \sin x \cos 2y$.

15. The points of trisection of a tightly stretched string of Length 'L' with fixed ends are pulled aside through a distance 'd' on opposite sides of the position of equilibrium, and the string is released from rest. Obtain an expression for the displacement of the string at any subsequent time and show that the mid point of the string always remains at rest.

(or)

16. A rod of length L cm long, with insulated sides, has its ends A and B kept at 10 centigrade and 50 centigrade respectively until steady state conditions prevail. The temperature at A is then suddenly raised to 30 centigrade and at the same that at B is lowered to 20 centigrade. Find the temperature distribution u(x, t) subsequently.

17. A square plate is bounded by the lines x = 0, y = 0, x = 20 and y=20 and its faces are insulated. The temperature along the upper horizontal edge is given by u(x, 20) = x(20-x) for 0 < x < 20, while the other three edges are maintained at 0º centigrade. Find the steady state temperature distribution in the plate.

(or)

18. Find the steady state temperature in a Circular plate of radius 'a' which has one half of its circumference at 0º centigrade and the other half at Kº centigrade (K ≠ 0).

19. Find the Fourier transform of $\left(\frac{\sin ax}{x}\right)$ and hence evaluate

$$\int_{-\infty}^{\infty}\left(\frac{\sin^2 ax}{x^2}\right)dx.$$

(or)

20. (a) State and prove Convolution theorem on Fourier transform.
(b) Find the Finite Fourier sine transform of $f(x)=x(\pi-x)$ in (0, π).