(3 Hours) **[Total Marks : 100**

N. B. : (1) Question No.1 is compulsory.

(2) Attempt any four questions out of remaining six questions.

(3) Assume suitable data if required.

1. (a) Time displacement in a periodic function has no effect on the magnitude spectrum, but changes the phase spectrum. Justify. (20)

(b) Consider a continuous time system with input $x(t)$ and output $y(t)$ related by $y(t) = x(\sin(t))$

(i) Is this system causal ? (ii) Is this system linear ?

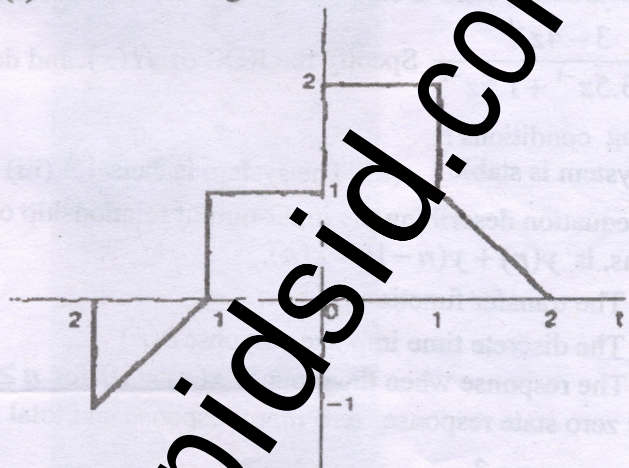
(c) Determine whether the following signals are energy signals or power signals and evaluate their normalized energy and power

(i) $x(t) = e^{-at}u(t), \quad a > 0$ (ii) $x(t) = \cos(\omega_0 t + \theta)$

(d) Determine which of the following signals are periodic.

(i) $x(t) = \left[\cos\left(2t - \frac{\pi}{3}\right)\right]^2$ (ii) $x(t) = Ev\{\cos(4\pi t)u(t)\}$

(e) A continuous-time signal $x(t)$ is shown in Figure.



Sketch and label carefully each of the following signals.

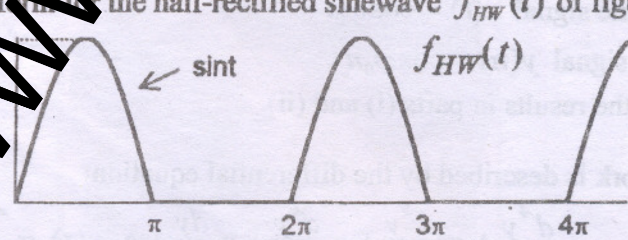
(i) $[x(t) + x(-t)]u(t)$ (ii) $x(t)\left[\delta\left(t + \frac{3}{2}\right) - \delta\left(t - \frac{3}{2}\right)\right]$

2. (a) Given the system $\dot{y} + 3y = 5x$, find the output $y(t)$ when $x(t) = \sin(6t)u(t)$ (8)

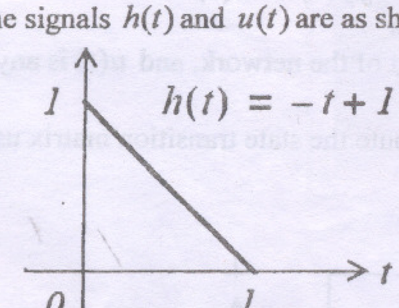
(i) Using Laplace transform (ii) Using Fourier transform (iii) Comment on the above results.

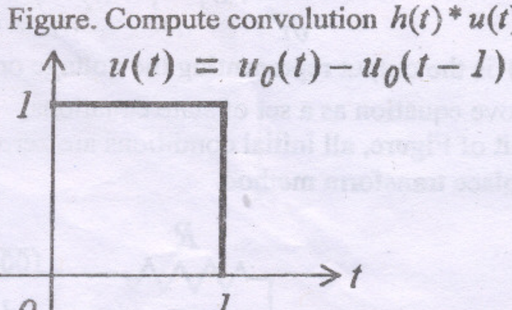
(b) Explain the relation between Laplace Transform and Fourier Transform. (4)

(c) Find the Laplace transform for the half-rectified sinewave $f_{HW}(t)$ of figure. (8)



3. (a) The signals $h(t)$ and $u(t)$ are as shown in Figure. Compute convolution $h(t) * u(t)$ (8)

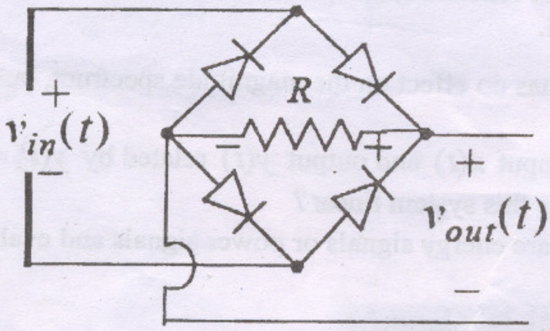




(b) A series RL circuit in which R = 5 ohms and L = 0.02 H has an applied voltage v = (100 + 50 sin ωt + 25 sin 3ωt) volts where ω = 500 rad/s. Find the current and the average power. (8)

(c) State and prove frequency shifting property of Fourier transform. State its application in communication engineering. (4)

4. (a) Figure shows a full-wave rectifier circuit with input the sinusoid $v_{in}(t) = A\sin\omega t$. The output of that circuit is $v_{out}(t) = |A\sin\omega t|$. Express $v_{out}(t)$ as a trigonometric Fourier series. Assume $\omega = 1$ (8)



(b) Derive the Fourier transform of the periodic time function $f(t) = A\sum_{n=-\infty}^{\infty}\delta(t-nT)$ (8)

(c) Explain mapping between s-plane and z-plane. (4)

5. (a) A linear time-invariant system is characterized by the system function (8)

$$H(z) = \frac{3-4z^{-1}}{1-3.5z^{-1}+1.5z^{-2}}$$ Specify the ROC of $H(z)$ and determine $h(n)$

for the following conditions :

(i) The system is stable (ii) The system is causal (iii) The system is anticausal

(b) The difference equation describing the input-output relationship of a discrete time system with zero initial conditions, is $y(n) + y(n-1) = x(n)$. (8)

Compute : (i) The transfer function $H(z)$

(ii) The discrete time impulse response $h(n)$

(iii) The response when the input is $x(n) = 10$ for $n \geq 0$.

(c) Explain what is zero state response, zero input response and total response. (4)

6. (a) Consider $X(s) = \dfrac{s+3}{(s+1)\ (s-2)}$ Show all the possible ROC conditions and obtain inverse laplace transform for each case of the ROC conditions. (8)

(b) For the given signal $x(n) = 1 + \sin\left(\frac{2\pi}{N}\right)n + 3\cos\left(\frac{2\pi}{N}\right)n + \cos\left(\frac{4\pi}{N} + \frac{\pi}{2}\right)n$ (6)

Sketch (i) Real and imaginary parts of the Fourier series coefficients

(ii) Magnitude and phase of the same coefficients.

(c) Determine Fourier transform of (6)

(i) Continuous time signal $x(t) = \cos\omega_0 t$

(ii) Discrete time signal $y(n) = \cos\omega_0 n$

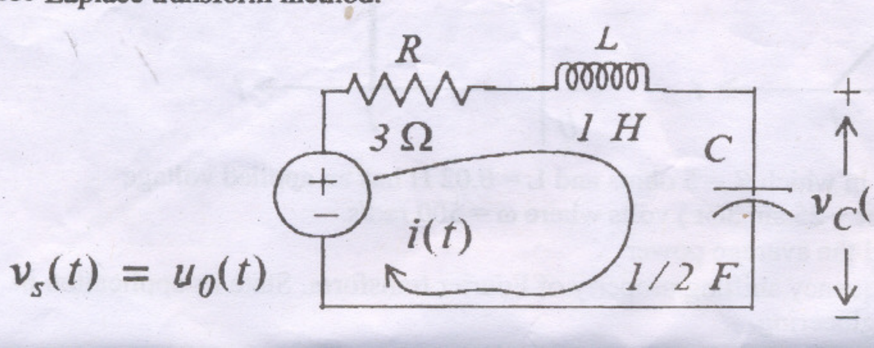
(iii) Comment on the results in parts (i) and (ii).

7. (a) A fourth-order network is described by the differential equation (8)

$$\frac{d^4y}{dt^4} + a_3\frac{d^3y}{dt^3} + a_2\frac{d^2y}{dt^2} + a_1\frac{dy}{dt} + a_0 y(t) = u(t)$$

where $y(t)$ is the output representing the voltage or current of the network, and $u(t)$ is any input. Express above equation as a set of state equations.

(b) In the circuit of Figure, all initial conditions are zero. Compute the state transition matrix using the Inverse Laplace transform method. (8)



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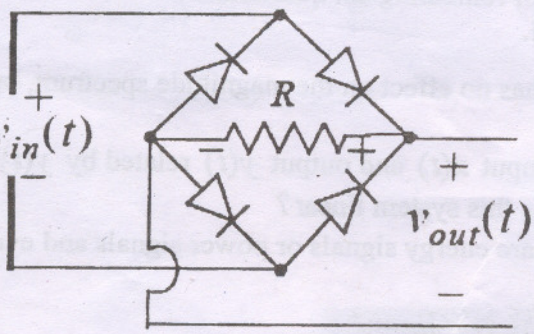

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