INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

27th October 2009

Subject CT1 – Financial Mathematics

Time allowed: Three Hours (15.00 – 18.00 Hrs)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

1. *Please read the instructions on the front page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception*

2. *Mark allocations are shown in brackets.*

3. *Attempt all questions, beginning your answer to each question on a separate sheet. However, answers to objective type questions could be written on the same sheet.*

4. *In addition to this paper you will be provided with graph paper, if required.*

AT THE END OF THE EXAMINATION

Please return your answer book and this question paper to the supervisor separately.

Q1) A loan of Rs.3,00,000 is to be repaid over 12 years by equated monthly installments (EMIs). The rate of interest on the loan is 8% p.a. effective.

    (i) Calculate the EMI. (2)

    (ii) Calculate the principal and interest content of the 57th installment. (4)

    (iii) Calculate the loan outstanding immediately after payment of the 60th installment. (2)

    (iv) If the loan outstanding as calculated in (iii) above is to be repaid over the next 5 years, at the same rate of interest of 8% p.a. effective, through annual installments, increasing at 5% p.a. compound, calculate the initial installment. (4)

    (v) If the loan outstanding as calculated in (iii) above is to be repaid through level monthly payments of Rs.4000/- each, with a residual payment(less than Rs.4000) in the last installment, calculate the number of installments (including the installment of residual payment) in which the loan would be repaid. (4)

    (vi) Calculate the residual payment made in the last installment. (2)

[18]

Q2)   **(i)** Prove by general reasoning that $1 = v^{\wedge}n + i\, a_n$ (2)

    (ii) Prove algebraically that $k\, \ddot{a}_n + (Ia)_{n-1} = (k-1)\ddot{a}_n + (I\ddot{a})_n$
where k is an integer. (4)

    (iii) Sanjay has choice to receive one of the following three payment streams :-
      a) Rs. X at time 12
      b) Rs.1250 at time n, Rs.2500 at time 2n, Rs.3750 at time 3n
      c) Rs. 6500 at time 10

The present value of all the three payment streams are equal at interest rate of i p.a. effective.
Given that $v^{\wedge}n = 0.517541$, Calculate i and X. (4)

    (iv) A Company issues a 10 year annuity payable annually in arrear, with the following features:-

      ? The first annuity payment is of Rs.100, and subsequent annuity payments increase by 5% p.a. compound.

      ? Each time the annuity is paid, the Company incurs expenses, as under:
      First year – 4% of the annuity amount
      Thereafter – 4% of the annuity paid in the respective year, inflating at 4% p.a. compound.

Find the purchase price of the annuity at interest rate of 9.2% p.a. effective. (6)

[16]

Q3)   **(i)**   List four limitations of Redington's immunization theory to apply in practice.   (4)

(ii) State the formula for Convexity of a series of cashflows.   (1)

A Company incurs a liability to pay Rs.1000 $(1+e^{-t/100})$ at the end of year t, where t=1,2,3,…..40. It values these liabilities at interest of 5% p.a.

An amount equal to the total value of the liabilities is immediately invested in two bonds, each bearing interest at 5% p.a. payable annually in arrear, one redeemable at the end of 20 years and the other at the end of 45 years. Both the bonds are issued and redeemable at par.

(iii) What is the present value of the liabilities?   (5)

(iv) What is the discounted mean term (DMT) of the liability outgo?   (7)

(v) If the DMT of the asset proceeds is the same as the DMT of the liability outgo, how much is invested in each of the bonds?   (8)

[25]

Q4)   **(i)**   A finance company accepts deposits on the following terms :

    a) The term of deposit is 4 years

    b) Interest on deposit is 16% p.a. payable quarterly in arrear

    c) At the end of the term, the deposit is returned along with bonus interest equal to 25% of total gross interest paid during the term of deposit as in (b) above.

If a depositor is subject to tax @ 20% on all interest payments including the bonus interest, calculate the net annual effective yield to the depositor.   (5)

(ii) A special bank account has the following features :

    a) Any money invested by the account holder earns interest at 7% p.a. effective.

    b) Any interest credited to the account earns 6% pa effective.

    c) Interest is credited to the account annually in arrear.

If an account holder has a balance of Rs.10,000 now and also makes deposits of Rs.100 every year for a period of 5 years, the first deposit being at the end of one year from now, calculate the accumulation in the account at the end of 6 years.   (4)

(iii) During a particular year, the force of interest d(t) was 10% pa at the start of the year, 7% pa at the end of the 9th month and 5% pa at the end of the year. The function d(t) was linear during the two periods before and after the end of the 9th month.

A sum of Rs.10,000 was deposited at the start of the year.

Calculate the accumulated value at the end of the year.   (4)

[13]

Q5)   **(i)**   Define discounted payback period.   (1)

         (ii)   Define payback period.   (1)

         (iii)   A company is about to set a manufacturing plant to manufacture a new model of car. To incur the cost of setting and running the operation, on 1.1.2007, it borrowed Rs. 500 crores from a bank which will charge an effective rate of interest of 10% p.a. On 1.7.07, it borrowed another Rs. 1500 crores at the same rate of interest. The bank loan is not for a fixed term but may be reduced by repayments at any time.

                The company decides that the price of each car will be Rs. 1,00,000.

                On 1.1.09 the company took a booking advance of Rs. 70,000 per car for 60,000 cars. The delivery of cars will start from 01.07.2009 and 5000 cars will be delivered uniformly over a month, for 1 year. The Company will receive the balance amount of Rs. 30,000 per car on car delivery.

                Thereafter, i.e., from 01.07.2010 onwards, the Company expects to sell 10,000 cars each month, which shall be delivered uniformly during each month and for which there will be no booking advance and the entire amount of Rs. 1,00,000 per car will be paid to the company at the time of sale.

                Calculate the discounted payback period for this project.   (8)

                [10]

Q6)   By adopting a particular investment strategy a company expects that on average the annual yield on its funds will be 8% with a standard deviation of 7%. The yields in different years may be assumed to be independently distributed.

         a)   Find the expected value and standard deviation of the accumulated amount after 15 years of a single investment of Rs.1000 made now.   (3)

                Assume further that each year $1+i_t$ has a lognormal distribution where i_t is the annual yield on company's funds in year t.

         b)   Calculate the parameters of the lognormal distribution.   (4)

         c)   Calculate the probability that a single investment of Re. 1/- made now will be less than 60% of its expected value.   (3)

                [10]

Q7)   **(i)**   What is a forward contract?   (2)

         (ii)   State the formula for calculating the forward price for

                a)   a security with no income

                b)   a security with fixed cash income

                c)   a security with known dividend yield

              Explain all the notations used in the formulae.   (3)

        (iii)   Explain what is meant by term structure of interest rates.   (1)

        (iv)   Explain briefly any two theories that explain the term structure of interest rates.   (2)

                [8]

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