Actuarial Society of India

EXAMINATIONS

21st May 2007

Subject CT1 – Financial Mathematics

Time allowed: Three Hours (10.00 – 13.00 Hrs)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

1. ***Do not write your name anywhere on the answer sheets. You have only to write your Candidate's Number on each answer sheets.***

2. ***Mark allocations are shown in brackets.***

3. ***Attempt all questions, beginning your answer to each question on a separate sheet. However, answers to objective type questions could be written on the same sheet.***

4. ***Fasten your answer sheets together in numerical order of questions. This, you may complete immediately after expiry of the examination time.***

5. ***In addition to this paper you should have available graph paper, Actuarial Tables and an electronic calculator.***

Professional Conduct:

"It is brought to your notice that in accordance with provisions contained in the Professional Conduct Standards, If any candidate is found copying or involved in any other form of malpractice, during or in connection with the examination, Disciplinary action will be taken against the candidate which may include expulsion or suspension from the membership of ASI."

Candidates are advised that a reasonable standard of handwriting legibility is expected by the examiners and that candidates may be penalized if undue effort is required by the examiners to interpret scripts.

AT THE END OF THE EXAMINATION

Please return your answer scripts and this question paper to the supervisor Separately.

Q.1)

(i) Define the effective rate of interest over a given time period (2)

(ii) Define a nominal rate of interest (2)

Calculate the nominal rate of interest per annum convertible quarterly which is equivalent to:

(iii) An effective rate of interest of 0.8% per month. (2)

(iv) A nominal rate of interest of 8% per annum convertible every 2 years. (2)

[8]

Q. 2) The 1, 2, 3, 4 and 7-year spot rates are 4.25%, 4%, 3.75%, 3.5% and 3.9% pa respectively. The 3-year forward rate from time 3 is 4.1% pa and the 3-year forward rate from time 2 is 3.6% pa.

Calculate:

(i) The 2-year forward rate from time 5 (2)

(ii) The present value of payments of 100 at the end of each of the 7 years (2)

(iii) The accumulated value at time 6 of a payment of 100 at time 1 (1)

[5]

Q. 3)

(i) Prove by general reasoning the following relationship

$$a^{(p)}_{n\rceil} = i / i^{(p)} a_{n\rceil}$$ (5)

(ii) Sanjay deposits Rs.500000 in a bank account and then withdraws a level amount at the end of each quarterly starting 3 months after the deposit is made. Immediately after 48th quarterly withdrawal, Sanjay withdraws the balance amount of Rs.75000 from the account. Calculate the amount of each withdrawal, given that the annual rate of interest is 6% pa effective. (4)

(iii) A special annuity is payable quarterly in advance for 20 years. The annual amount of the annuity is Rs 400 in year 1 and increases at the beginning of each year thereafter by 6%, compound. Calculate the present value of this annuity at an effective rate of interest of 6% per annum. (5)

[14]

Q. 4)

(i) What does n-year par yield represent? (2)

(ii) Calculate the 5 year par yield if the annual term structure of interest rates is: (5.5%, 5.90%, 6.25%,6.5%, 6.75%,……..) (2)

A portfolio of shares has a current price of Rs. 1,000,000 and a dividend yield of 3.0% pa. Dividends are received continuously.

(iii) What is the forward price for a forward contract to buy the portfolio in 10 years, assume a risk-free force of interest of 4.3%? (2)

(iv) If the shares drop in price so that in 2 year's time they are worth Rs. 976,500 what is the value of the forward contract at that time? (4)

[10]

Q. 5) A bank issued a loan of Rs. 500,000 on 1January 2000. The loan is repayable by regular monthly installments in arrears over 15 years period. The amount of monthly repayment increases by Rs. 500 after five years and by a further Rs.500 after ten years. The amount of monthly repayments was calculated on the basis of a nominal rate of interest of 10% per annum.

(i) Calculate the initial amount of monthly repayment. (6)

(ii) Calculate the amount of capital that was repaid on 1 February 2000. (2)

(iii) Calculate the amount of loan, which will remain outstanding after the monthly repayment due on 1 January 2007 has been made. (4)

(iv) The borrower requests that after the 1 January 2007 payment has been made, future repayments be for a fixed amount payable annually on each 1 January, the last repayment being on 1 January 2011. Calculate the amount of the revised annual payment on this basis if the interest rate remains unchanged. (3)

[15]

Q. 6)

(i) An investor buy a 20-year fixed interest government bond at a price that gives him is a gross redemption yield of 8.5% per annum at date of purchase. Give four reasons why actual return over the period up to the maturity date may be different from the yield at purchase date. (4)

(ii) A stock with a term of 15 years has a coupon of 7% pa payable half-yearly in arrears and is redeemable at 105%. An investor who is not subject to tax purchases the stock at 90 per 100 nominal immediately after the coupon payment. Calculate the yield obtained by the investor. (5)

[9]

Q. 7)

(i) List other considerations in project appraisal process in addition to net present value, internal rate of return and discounted payback period. (3)

(ii) A project involves an initial cost of Rs. 500000 and provides income annually in arrear for 6 year. The amount of annual income at the end of the first year is Rs.30000 and thereafter inflates at rate of 8% pa. The investor will be able to sell the ongoing rights to the project at the end of 6 years after the payment then due for Rs 650000.

The investor has no capital for investing in either project, but can borrow Rs 500000 from a bank at 8% pa interest payable annually in arrear. The loan would be repayable at the end of 6 years at par with no early repayment option. If further loans

are required they will also be granted at 8% pa repayable at par at the end of 6 years with no early repayment option. Interest on further loans is also rolled up to end of 6 years. If the investor has any surplus proceeds after paying interest on the original loan as it becomes due, these can be invested at an interest rate of 6% pa effective up to end of 6 years.
Calculate the accumulated profit on the project at the end of 6 years. (10)
[13]

Q. 8) In any year t, the yield on a fund of investments has mean j_t and standard deviation s_t. In any year, the yield is independent of the value in any other year. The accumulated value, after n years, of a unit sum of money invested at time 0 is S_n.

(i) Derive formulae for the mean and variance of S_n if $j_t = j$ and $s_t = s$ for all years t. (5)

(ii) (a) Calculate the expected value of S_{10} if $j = 0.05$. (1)
(b) Calculate the standard deviation of S_{10} if $j = 0.05$ and $s = 0.07$. (2)
[8]

Q. 9)

(i) State the characteristics that are particular to property investments. (3)

(ii) An investor buys a property contract that gives rental income for 48 years. The value of property at the end of 48 years is expected to be original purchase price increasing every year at compound annual rate expected of 10%pa. The initial rent of the property is Rs.15000 per month payable in advance and will increase by 15% compound every 3 years. What would be the purchase price of the property if the investor wants to achieve a yield of 12% per annum effective? (6)
[9]

Q. 10)

(i) State the conditions to be met under Redington's theory of immunization. (2)

(ii) A fund has two lump sum liability payments due at known times in the future. The first liability is due for payment 4 years before the second and is half of the second. At market interest rate of 8% pa effective, the discounted mean term of the liabilities is 7 years and their present values is Rs.150000. Determine the timing and amount of the payments. (5)

(iii) The fund hold an asset consist of a single zero coupon bond that will mature in 7 years from now and has current market value equal to present value of the above liability payouts. State whether the portfolio satisfy the Redington's immunization conditions. (2)
[9]

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