Actuarial Society of India

EXAMINATIONS

20th June 2005

Subject CT1 – Financial Mathematics

Time allowed: Three Hours (10.30 am - 13.30 pm)

INSTRUCTIONS TO THE CANDIDATES

1. ***Do not write your name anywhere on the answer scripts. You have only to write your Candidate's Number on each answer script.***
2. ***Mark allocations are shown in brackets.***
3. ***Attempt all questions, beginning your answer to each question on a separate sheet.***
4. ***Fasten your answer sheets together in numerical order of questions. This, you may complete immediately after expiry of the examination time.***
5. ***In addition to this paper you should have available graph paper, Actuarial Tables and an electronic calculator.***

Professional Conduct:

"It is brought to your notice that in accordance with provisions contained in the Professional Conduct Standards, If any candidate is found copying or involved in any other form of malpractice, during or in connection with the examination, Disciplinary action will be taken against the candidate which may include expulsion or suspension from the membership of ASI."

AT THE END OF THE EXAMINATION

Hand in both your answer scripts and this question paper to the supervisor.

Q.1 **a)** Define real rate of interest. **[1]**

b) Given positive inflation, state whether real rate of interest will be greater than money rate of interest. **[1]**

c) State the relationship between real and money rate of interest. **[2]**

d) An item was priced at Rs 30 on 1st January 2005. If the annual rate of inflation was estimated as on 1st January at 5% p.a., calculate the estimated price of that article on 1st July 2005. Ignore factors other than inflation in estimating its price. **[2]**

e) You have made an investment of Rs 1000 on 1st January 2005 in a fixed deposit with a bank. The rate of return provided by the bank on this investment is 6% p.a., calculate

(i) the maturity value on 1st July 2005

(ii) the expected real rate of return earned on this investment assuming a rate of inflation over this period as 5% per annum. **[2]**

Total [8]

Q.2 The force of interest takes the following values:

$d(t) = 0.04 \qquad 0 < t \le 10$

$d(t) = 0.001(t-10)^2 + 0.04 \quad 10 < t$

a) Calculate the accumulation of Rs150 from time $t = 0$ to time $t = 20$. **[4]**

b) A continuous payment stream of 10 per annum is received from time $t = 5$ to time $t = 10$. Calculate the present value of that payment stream at time $t = 0$. **[3]**

Total [7]

Q.3 An investor is to make payments of Rs100 annually in arrears for six years followed by Rs 200 per annum payable half yearly in arrears for a further six years. The investment earns interest at 8% per annum convertible half yearly.

Calculate the accumulated amount at the end of 12 years. **[4]**

Q.4 Dividends payable on a certain share are assumed to increase at a compound rate of 3% per half-year. A dividend of Rs 2.50 per share has just been paid. Dividends are paid half-yearly. Find the value of the share to the nearest rupee, assuming an effective rate of interest of 8% p.a. **[4]**

Q.5 A loan is repayable over 20 years by level installments of Rs 1,000 per annum made annually in arrear. Interest is charged at the rate of 5% per annum effective for the first 10 years, increasing to 7% per annum effective for the remaining term.

a) Calculate the initial loan amount **[2]**

b) Calculate the effective equivalent flat rate of interest on this loan. **[3]**

Total [5]

Q.6 As a purchaser of a consumer loan, you are offered a choice by a lending institution to avail loans either on a fixed rate basis (the interest charged will be fixed and known in advance) or on a floating rate basis. On floating rate basis, the interest rate applicable for any six month period during the tenure of the loan, would be the rate of return on a pre chosen 5 year gilt security plus 1.5%.

The fixed rate of interest charged by the bank on fixed rate basis is 8%.

Rate of return on the relevant gilt security, which would be used to calculate monthly installments on floating rate basis, as on 1st July 2005 is 6.25%. The following is what you believe as the rate of return on gilt security on the dates when the rate of interest on your loan will be reviewed.

1 Jan 2006	1 July 2006	1 Jan 2007
6.50%	7.00%	6.75%

You are planning to avail a loan on 1st July 2005. The loan amount is intended to be Rs 10,000/- and you intend to avail this loan for a period of 2 years. Assume that loan and interest repayments would be made by 24 equated monthly installments and such monthly installments would be payable in arrear (i.e. the first monthly installment would be payable on 1st August 2005)

On a floating rate basis, monthly installments would be revised once in every six months, i.e. on each 1st Jan and 1st July. The method adopted would be as follows:

Loan outstanding on that date would be calculated. Interest rate applicable would be the return on the relevant gilt security on that date plus 1.5%. For calculation of the revised monthly installment it would be assumed that the rate applicable then would remain fixed for the outstanding term of the loan. For example on 1st Jan 2006, monthly installment would be calculated with the following parameters.

Loan amount = Loan outstanding as on 1st January 2006
Interest applicable = 6.50% + 1.50% = 8%
Term of the loan = Outstanding term on the loan as on 1st January 2006 (i.e. 18 months).

Such calculated installments would remain at the same level till the next review, when this process would be repeated.

a) Calculate:

i) The monthly installment payable under fixed rate basis. **[2]**

ii) The monthly installments payable in each of the half year periods under floating rate basis. **[14]**

iii) The equivalent flat rate of interest paid by a borrower who opts for floating rate basis. **[3]**

b) Assuming that your expectations on the interest rate holds, determine which basis, fixed or floating, should you choose and calculate the profit that you make in choosing such a basis. **[4]**

Total [23]

Q.7 An investor purchases a fixed interest security. The security pays coupons at a rate of 10% p.a. half-yearly in arrear, and is to be redeemed at 110 in 20 years. The investor is subject to tax on the coupon payments at a rate of 25%.

a) Show that the price paid by the investor to obtain a rate of return of 10% p.a. effective is Rs 81.76 for every 100 nominal purchased. **[6]**

b) Calculate the effective duration (or volatility) of the security at 10% p.a. effective for this investor at the purchase date. **[5]**

Total [11]

Q.8 An investment provides income of Rs 1000/- payable at the end of each year for the next ten years. There is no capital repayment. If the interest rate is 7% per annum effective, show that the discounted mean term of the investment is 4.946 years. **[5]**

Q.9 A one-year forward contract is issued on 1 April 2005 on a share with a price of Rs. 6.00 at that date. Dividends of Rs 0.30 per share are expected on 30 September 2005 and 31 March 2006. The 6-month and 12-month spot risk-free interest rates are 4% and 4.5% per annum effective respectively on 1 April 2005.

Calculate the forward price at issue, assuming no arbitrage. **[5]**

Q.10 **a)** Define Interest rate swaps. **[2]**

b) Describe how cash flows are exchanged in a interest rate swap. **[2]**

c) Describe briefly the two kinds of risk facing each counterparty to the swap. **[3]**

Total [7]

Q.11 Explain briefly what do we mean by term structure of interest rate and list the three popular theories to explain term structure of interest rates. **[2]**

Q.12 In a particular bond market, the two-year par yield at time $t = 0$ is 5.25% and the issue price at time $t = 0$ of a two-year fixed interest stock, paying coupons of 8% annually in arrears and redeemed at 98, is £105.40 per £100 nominal.

Calculate:

(a) the one-year spot rate

(b) the two-year spot rate **[6]**

Q.13 State the conditions to be met under Redington's theory of immunization. **[2]**

Q.14 Let i_t denote the rate of interest earned in the year t -1 to t. Each year the value of i_t is 8% with probability 0.625, 4% with probability 0.25 and 2% with probability 0.125. In any year, it is independent of the rates of interest earned in previous years.

Let S_3 denote the accumulated value of 1 unit for 3 years.

Calculate the mean and standard deviation of S_3 .

[5]

Q.15 An investment bank models the expected performance of its assets over a 5-day period. Over that period, the return on the bank's portfolio, i, has a mean value of 0.15% and standard deviation 0.3%. $(1 + i)$ is lognormally distributed.

Calculate the value of j such that the probability that i is greater than or equal to j is 0.9. **[6]**

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