D 26985 (Pages : 2) Name..................................

Reg. No..................................

FOURTH SEMESTER B.TECH. (ENGINEERING) DEGREE EXAMINATION
DECEMBER 2006

CH 2K 401—ENGINEERING MATHEMATICS—IV

(Common to AI/CE/EE/IC/ME/PM/EC/PE)

Time : Three Hours Maximum : 100 Marks

Answer **all** *questions.*

1. (a) Show that $f(z) = \begin{cases} yx^3 (y - ix) & \text{for } z \neq 0 \\ 0 & \text{for } z = 0 \end{cases}$ is not analytic at $z = 0$.

(b) Find the bilinear transformation which maps 0, 1, ∞ onto $i, -1, -i$.

(c) Using Cauchy's integral formula calculate $\int_C \frac{z+1}{z^3 - 2z^2} dz$ where C is $|z| = \frac{1}{2}$.

(d) Expand $f(z) = \frac{1}{z^2 - 3z + 2}$ in the region $1 < |z| < 2$.

(e) Show that $J_{1/2}(x) = \frac{\sqrt{2}}{\pi x} \sin x$.

(f) Prove that $(2n + 1) x P_n(x) = (n+1) P_{n+1} + n P_{n-1}$.

(g) Classify the PDE $x^2 \frac{\partial^2 u}{\partial x^2} + xy \frac{\partial^2 u}{\partial y^2} + y^2 \frac{\partial^2 u}{\partial x \partial y} = 0$.

(h) By the method of separation of variables, solve $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$.

(8 × 5 = 40 marks)

2. (a) (i) If $f(z)$ is an analytic function of z, prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \log |f(z)| = 0$.

(ii) Prove that under the transformation $w = \frac{z - i}{iz - 1}$ the region $I(z) \geq 0$ is mapped onto the region $|w| \leq 1$. Into what region is $I(z) \leq 0$ mapped by the above transformation.

Or

(b) (i) Determine the analytic function $f(z) = u + iv$ if $u - v = e^x (\cos y - \sin y)$.

(ii) Find the transformation which maps the semi-infinite strip $u \geq 0$, $0 \leq v \leq a$ onto the upper half of the z-plane.

Turn over

3. (a) (i) Find Laurent's series for $f(z) = \dfrac{1}{(z+1)(z+2)^2}$ in the region $1 < |z| < 2$.

(ii) By using contour integration, evaluate $\int_0^{2\pi} \dfrac{d\theta}{97 - 72\cos\theta}$.

Or

(b) (i) Find the residues at its poles $f(z) = \dfrac{z^2}{(z-1)(z-2)^2}$.

(ii) By using contour integration, evaluate $\int_0^{\infty} \dfrac{dx}{x^4 + a^4}$.

4. (a) (i) Prove that when n is a positive integer, $J_n(x)$ is the co-efficient of z^n in the expansion of $e^{x/2}\left(z - \dfrac{1}{z}\right)$.

(ii) Show that $P_n(-x) = (-1)^n P_n(x)$.

Or

(b) (i) Prove that $\int_{-1}^{1} \left[P_n(x)\right]^2 dx = \dfrac{2}{2n+1}$.

(ii) Prove that $J_{-3/2}(x) = -\sqrt{\dfrac{2}{\pi x}}\left(\sin x + \dfrac{1}{x}\cos x\right)$.

5. (a) (i) Derive one dimensional wave equation.

(ii) A string is stretched between two fixed points at a distance of 2 L apart and the points of the string are given initial velocities v where :

$$v = \begin{cases} \dfrac{cx}{L} & \text{in } 0 < x < L \\ \dfrac{c}{L}(2L - x) & \text{in } L < x < 2L \end{cases}$$

x being the distance from an end point. Find the displacement of the string at any subsequent time.

Or

(b) Solve the problem of the vibrating string for the following boundary conditions :—

$$y(0, t) = 0 \,;\, y(L, t) = 0 \quad \frac{\partial y}{\partial t}(x, 0) = x(x - L) \quad 0 < x < L$$

$$y(x, 0) = \begin{cases} x & \text{in } 0 < x < \dfrac{L}{2} \\ L - x & \text{in } \dfrac{L}{2} < x < L \end{cases}$$

(4 × 15 = 60 marks)