S.E. /Comp/ Sem III / Exam 2010

Con. 3018–10. (REVISED COURSE) AN–2503

Sub. D.S.D.T.

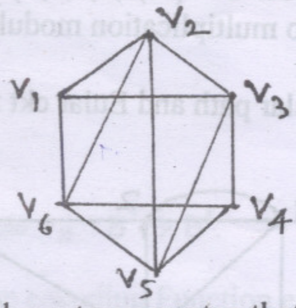
(3 Hours) [Total Marks : 100

Discrete Structure and Graph Theory

N.B. : (1) Question No. **1** is **compulsory**.
(2) Attempt any **four** questions out of remaining **six** questions.
(3) Assumption made should be **clearly stated**.
(4) **Figures** to the **right** indicate **full marks**.

1. (a) Use mathematical induction to prove the following inequality $n < 2^n$ for all positive integers n . 4
   (b) Define a pigeonhole principle. 4
   Show that if seven colours are used to paint 50 bicycles, at least 8 bicycles will be of same colour.
   (c) What is an Universal and existential quantifier? 4
   (d) Define the following terms with the example 4
   (i) Disjoint set (iii) Partial order relation
   (ii) Symmetric difference (iv) Antisymmetric relation.
   (e) How many numbers must be selected from the set {1,2,3,4,5,6} to Guarantee that at least one pair of these numbers add up to 7? 4

2. (a) Prove that if x is a rational number and y is an irrational number, then x + y is an irrational number. 5
   (b) Define following – Power Set, Surjective and Injective function 5
   (c) Is a graph a planar graph? 5



   (d) How many friends must you have to guarantee that at least five of them will have birthdays in same month ? 5

3. (a) Let A={ a, b, c, d} and x be a relation on A whose matrix is 10

$$M_R = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Prove that R is partial order. Draw Hasse diagram of R.

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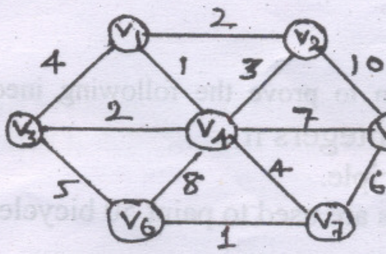
(b) (i) Define group, monoid, semigroup. 5

(ii) Converse of statement is given . 5
Write inverse and contra positive of statement
" If I come early then I can get a car "

4. (a) Write Prims Algorithm. Apply it to following graph. 10



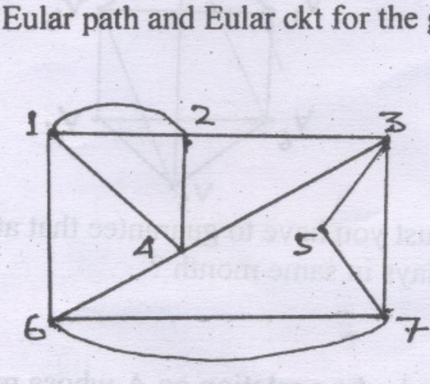
(b) (i) Let A= { 1,2,3,4,5}, P={{1,2} {3}, {4,5}} find equivalence relation Determined by P and draw its diagraph. 5

(ii)Check whether relation is reflexive , irreflexive, symmetric, anti symmetric , transitive. 5

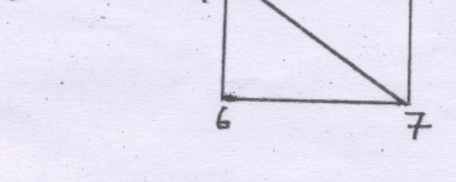
$R_1 = \{(1,1),(1,2),(2,1),(2,2),(3,3),(4,3),(3,4),(4,4)\}$
$R_2 = \{(1,3),(1,1),(3,1),(1,2),(3,3),(4,4)\}$

5. (a) Prove that the set G = { 1,2,3,4,5,6} is a Finite Abelian group of order 6 with respect to multiplication modulo 7 . 10

(b) (i) Find out Eular path and Eular ckt for the graph 5



(ii) Find out Hamiltonian path and Hamiltonian cycle. 5



6. (a). (i) Show that (2,5) encoding function e: $B^2 \rightarrow B^5$ defined by 5

e (00) = 00000
e (01) = 01110
e (10) = 10101
e(11) = 11011

is a group code.

(ii) R = {O,2,4,6,8}.Show that R is a commutative ring under addition and multiplication modulo 10. Verify whether it is field or not. 5

(b) (i) Let L be the bounded distributive lattice. 5
Prove that if complement exist then it is unique.

(ii) Give the exponential generating functions for the sequences given bellow 5

(i) {1,1,1,.....................}
(ii) {0,1,0,-1,0,1,0,-1,..................}

7. (a) In any Ring (R + .) prove that 10

i) The zero element z is unique.
ii) The additive inverse of each ring element is unique.

(b) (i) Let m = 2, n = 5 and 5

$$H=\begin{bmatrix} 1\,1\,0 \\ 0\,1\,1 \\ 1\,0\,0 \\ 0\,1\,0 \\ 0\,0\,1 \end{bmatrix}$$

Determine the group code $e_H : B^2 \rightarrow B^5$.

(ii) Consider the (3, 5) group encoding Function e : $B^3 \rightarrow B^5$ defined by 5

e(000) = 00000	e(100) = 10011
e(001) = 00110	e(101) = 10101
e(010) = 01001	e(110) = 11010
e(011) = 01111	e(111) = 11100

Decode the following words relative to a maximum likelihood decoding functic

(i) 11001 . (ij) 01010 (iii) 00111
