S. E. III 1-7 | CMPN   Discrete Structure   3015706

[REVISED COURSE]

CON/1923-06.   TV-7974

(3 Hours)   [Total Marks : 100

N.B.:
1) Question number 1 is compulsory.
2) Attempt any four questions out of remaining six questions.
3) Assumptions made should be clearly stated.
4) Figures to the right indicate full marks.
5) Assume suitable data wherever required but justify the same.

Q. No.1 a) (i)Let Q be the set of positive rational numbers which can be expressed in the form 2^a3^b. where a and b are integers. prove that algebraic structure (Q .) is a group. Where . is multiplication operation. (06)

(ii)prove the following(use laws of set theory) (06)

$(A \cap B) \cup [B \cap ((C \cap D) \cup (C \cap \overline{D}))] = B \cap (A \cup C)$

b) (i) Let G be the group and let a and b are elements of G. then verify that $(ab)^{-1} = b^{-1}a^{-1}$ (04)

(ii) Let R be the relation represented by the matrix (04)

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

Find the matrix that represents R^4

Q. No.2 a) (i)Determine whether the poset with the following Hasse diagrams are lattices or not. Justify your answer. (06)

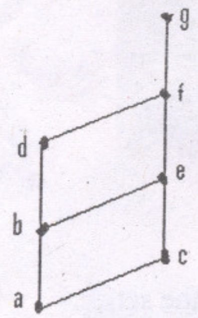

fig 1

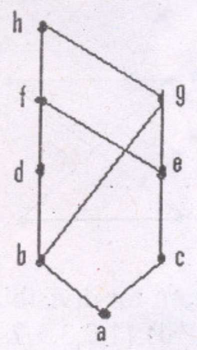

fig 2

(ii) Use induction to prove that (06)

$7^n - 1$ is divisible by 6 for n=1,2,3, - - - -

b) (i) Let A = {1,2,3,4} for thr relation R whose matrix is given bellow (04)
Find the matrix of transitive closure using warshall algorithm.

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(ii) Let R be the relation on set of real numbers such that aRb if and only if a-b is an integer. Prove that R is an equivalence relation. (04)

... is an equivalence relation.

Q. No.3 a) (i)Find the solution of recurrence relation (06)

$a_n = 5a_{n-1} - 6a_{n-2} + 7^n$

(ii)Suppose R and S is the relation from A to B , then prove that (06)

$(R \cap S)^{-1} = R^{-1} \cap S^{-1}$ and $(R U S)^{-1} = R^{-1} U S^{-1}$

b) (i) f: R→R is defined as $f(x) = x^3$ (04)

g: R→R is defined as $f(x) = 4x^2+1$

h: R→R is defined as $h(x) = 7x-1$

find the rule of defining $(h \circ g) \circ f$, $g \circ (h \circ f)$

(ii) Consider the chains of divisors of 4 and 9 i,e $L_1 = \{1,2,4\}$ and (04)

$L_2 = \{1,3,9\}$ and partial ordering relation of division on L_1 and L_2

Draw the lattice $L_1 \times L_2$.

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Q. No.4 a) (i)Show that (06)

$((P \vee Q) \wedge \neg(\neg P \wedge (\neg Q \vee \neg R))) \vee (\neg P \wedge \neg Q) \vee (\neg P \wedge \neg R)$

is tautology. (use laws of logic)

(ii)Prove that if (F, +, .) is a field then it is a integral domain. (06)

b) (i)Show that among any group of five(not necessarily consecutive) integers, there are two with the same remainder when divided by 4. (04)

(ii) Define Eulerian, Hamilton path and circuit with example. What is the necessary and sufficient condition for euler path and circuit? (04)

Q. No.5 a) (i) In a survey of 60 people, it was found that (06)

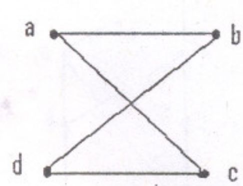
25 read Business India.
26 reads India Today.
26 read Times of India.
11 read both Business India and India Today.
09 read both Business India and Times of India.
08 read both India Today and Times of India.
08 read none of the three.

i) How many read all three?
ii)How many read exactly one ?

(ii)Prove that the set {1,2,3,4,5,6} is group under multiplication modulo 7 (06)

. b) (i) Define universal and existential quantifier with suitable example. (04)

(ii)How many paths of length 4 are there from a to d in simple graph shown bellow. (04)



No.6 a) (i) Draw the Hasse diagram for divisibility on the set (06)

i) {1,2,3,4,5,6,7,8} ii) {1,2,3,5,7,11,13}

(ii) Consider (3,8) encoding function $e: B^3 \rightarrow B^8$ defined by (06)

e(000) = 00000000	e(100) = 10100100
e(001) = 10111000	e(101) = 10001001
e(010) = 00101101	e(110) = 00011100
e(011) = 10010101	e(111) = 00110001

and let d be the (8,3) maximum likelihood decoding function associated with e. How many errors can (e,d) correct?

. b)(i) Conjecture a simple formula for a_n if the first 10 terms of the sequence $\{a_n\}$ are 1, 7, 25, 79,241, 727,2185,65559,19687,59047 (04)

(ii) Prove that in any ring (R + .) the additive inverse of each ring element is unique. (04)

No.7 a) (i) Find the complement of each element in D_{20} and D_{30} . (06)

(ii) Let G be the group of integers under the operation addition, and H be group of all even integers under the operation of addition, show that the function f:G→H is an isomorphism. (06)

b)(i) A connected planar graph has a 9 vertices having degrees2,2,2,3,3,3, 4,4,5. How many edges are there? (04)

(ii)Define with example Reflexive closure and symmetric closure. (04)