2004

DISCRETE MATHEMATICAL STRUCTURE

Time Allotted: 3 hours | *Full Marks: 70*

The questions are of equal value.

Candidates are required to give their answers in their own words as far as practicable.

Answer Question No.1 and any six from the rest

1. Answer any five of the following:-

a) If A, B and C are any three subsets of the universal set S, prove that
$(A' \cap B' \cap C) \cup (B \cap C) \cup (A \cap C) = C$

b) Determine the union of the following two fuzzy sets: $A = \{4/.1,\ 6/.5,\ 8/.6,\ 10/.7\}$ and
$B = \{0/.4,\ 2/.6,\ 4/1,\ 6/1,\ 8/.6,\ 10/.5\}$

c) Use pigeonhole principle to show that in any set of eleven integers, there are two integers whose difference is divisible by *10.*

d) Consider the lattice $L = \{1,2,3,4,6,12\}$, the division of 12 ordered by divisibility. Find the lower and upper bound of *L.*

e) Write an equivalent form for: $\sim (p \leftrightarrow (q \to (r \vee p)))$ which does not contain any conditional $(\leftrightarrow)$, and biconditional $(\leftrightarrow)$.

f) Determine the type of a grammar G which consists of the productions:
i) $S \to aA, A \to aAB, B \to b, A \to a.$
ii) $S \to aAB, AB \to bB, B \to b, A \to aB.$

g) Find the generating function of the sequence.
$\{1,1,0,1,1,1,1,1,1,- - --\}$

2. a) Let $A = R - \{3\}$ and $B = R - \{1\}$ where R is the set of real numbers. Let the function f : $A \to B$ be defined as $f(x) = \dfrac{x-2}{x-3}$ for $x \in A$. Show whether f is bijective. Also, find f^{-1} if it exists.

b) Let ρ be a relation on the set of integers Z defined by $x\ \rho\ y$, if $(x - y)$ is divisible by 6. Show whether ρ is an equivalence relation.

a) Prove, by method of induction,

$$\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{4^2}\right)----\left(1-\frac{1}{(n+1)^2}\right)=\frac{n+2}{2n+2}$$

b) Define a lattice. Prove that a collection of sets closed under union and intersection is a lattice.

4. a) Use generating function to solve the following recurrence relation: $u_{n+2}-2u_{n+1}+u_n=2^n, \qquad u_0=2, u_1=1$

b) Find the complement of the Boolean expression $f=xy'+x'y+x'y'$

5. a) Define a poset. Show that $(P(S),\subseteq)$ is a poset where $P(S)$ denotes the power set of the set $S'=\{x,y,z\}$. Also draw the Hasse diagram for this poset.

b) Prove that in a bounded distributive lattice $(L,\cap,\cup)$ an element connot have more than one complement.

6. Answer <u>any two</u>

a) Under what condition a statement p is said to tautologically imply statement $q(i.e., p\Rightarrow q)$?

Test the validity of the following implication:
$(p\vee q)\wedge(p\rightarrow r)\wedge(q\rightarrow r)\Rightarrow r$

b) Write an equivalent formula for $\sim (p\leftrightarrow(q\rightarrow(r\vee p)))$ which does not contain any conditional $(\rightarrow)$ and biconditional $(\leftrightarrow)$.

c) Determine the validity of the conclusion C from the following premises: $p\rightarrow q\wedge r,\ q\vee s\rightarrow t,\ s\vee p$. Conclusion C :t.

7. a) State the generalized Pigeonhole principle. Suppose a laundry bag contains many red, white and blue socks. Find the minimum number of socks that one needs to choose in order to get two pairs of same colour.

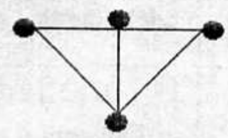
b) How many even numbers of four digits can be made with the digits 0, 3, 5, 4? Find the sum of the numbers.

8. a) A graph G has the following adjacency matrix A(G).

$$A(G)=\begin{bmatrix}0&1&1&0&0\\1&0&2&0&0\\1&2&0&0&0\\0&0&0&1&2\\0&0&0&2&0\end{bmatrix}$$

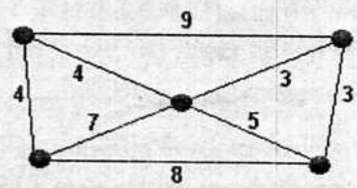
Draw the Graph G.

b) Define spanning tree of a graph G. Find all the spanning trees of the following graph:



9. a) Define Eulerian graph. Show that a nonempty connected graph is Eulerian if and only if all its vertices are of even degree.

b) Apply Kruskal's algorithm of Prim's algorithm to find a minimal spanning tree of the following weighted graph:



10. a) Obtain a Grammar which generates the language $L=\{a^n b^{n+1} : n \geq 0\}$

b) Design a NDFA with three states that accepts the language $L=\{ab,\ abc\}^*$

11. a) Determine a DFA from the NDFA.

$M=(\{q_0,q_1\},\{a,b\},\delta,q_0,\{q_1\})$

With the state transition function δ as given in the following table:

δ	a	b
q_0	$\{q_0,q_1\}$	$\{q_1\}$
q_1	$\varnothing$	$\{q_0,q_1\}$

b) Let $A=\{ab,bc,ca\}$, Find whether the following strings (i) $ababab$, (ii) $bcabbab$ belong to A*.