Code: D-01 / DC-01 **Subject: MATHEMATICS - I**
Time: 3 Hours **Max. Marks: 100**

NOTE: There are 11 Questions in all.

- **Question 1 is compulsory and carries 16 marks. Answer to Q. 1. must be written in the space provided for it in the answer book supplied and nowhere else.**
- **Answer any THREE Questions each from Part I and Part II. Each of these questions carries 14 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or best alternative in the following: (2x8)

a. The co-ordinates of the middle points of the sides of a triangle are (4, 2) (3, 3) and (2, 2). Then the co-ordinates of the centroid are

(A) $\left(3, \frac{7}{3}\right)$. **(B)** (3, 3).
(C) (4, 3). **(D)** (4, 7).

b. If x, 2x +2, 3x + 3 are first three terms of a G.P. then its 4^{th} term is

(A) 27. **(B)** -27.
(C) 13.5. **(D)** -13.5.

c. The angle made by any diameter of a circle at any point on the circumference is

(A) 90° **(B)** 180°
(C) 45° **(D)** 60°

d. If $^{n}P_{r} = 720\ (^{n}C_{r})$ then the value of r is

(A) 6. **(B)** 5.
(C) 4. **(D)** 7.

e. $\int \frac{Cos^2 x - Sin^2 x}{Sin\ x\ Cos\ x} dx$ is equal to

(A) $\log(\sin 2x) + C$. **(B)** $\log(\cot 2x) + C$.
(C) $\log(\cos 2x) + C$. **(D)** $\log(\tan 2x) + C$.

f. If $^{20}C_{r} = {}^{20}C_{r-10}$ then $^{18}C_{r}$ is equal to

(A) 4896. **(B)** 816.
(C) 1632. **(D)** 408.

g. $\lim_{x \to 5} \frac{x^4 - 625}{x^3 - 125}$ is

(A) $\frac{20}{3}$. **(B)** 5.

(C) Not defined. **(D)** $\frac{4}{3}$.

h. $\frac{1 - \tan^2 165^\circ}{1 + \tan^2 165^\circ}$ is equal to

(A) $\frac{1}{2}$. **(B)** $-\frac{\sqrt{3}}{2}$.

(C) $-\frac{1}{2}$. **(D)** $\frac{\sqrt{3}}{2}$.

PART I
Answer any THREE Questions. Each question carries 14 marks.

Q.2 a. If α, β are the roots of the equation $ax^2 + bx + c = 0$. Find the equation whose roots are $(\alpha - \beta)^2$ and $(\alpha + \beta)^2$. **(7)**

b. If the roots of the equation $p(q - r)x^2 + q(r - p)x + r(p - q) = 0$ are equal, show that $\frac{1}{p} + \frac{1}{r} = \frac{2}{q}$. **(7)**

Q.3 a. In a ΔABC show that $c^2 = (a - b)^2 \cos^2 \frac{C}{2} + (a + b)^2 \sin^2 \frac{C}{2}$. **(7)**

b. If $\sin^{-1} \frac{2a}{1 + a^2} + \sin^{-1} \frac{2b}{1 + b^2} = 2 \tan^{-1} x$ then show that $x = \frac{a + b}{1 - ab}$. **(7)**

Q.4 a. Evaluate $\lim_{x \to 0} \frac{\sqrt{a^2 + x^2} - \sqrt{a^2 - x^2}}{x^2}$. **(7)**

b. Differentiate $f(x) = \cos^2 x$ by the first principle. **(7)**

Q.5 a. Find the area bounded by the curve $x^2 = 4y$ and the straight line $x = 4y - 2$. **(7)**

b. Find the equation of tangent to $16x^2 + 9y^2 = 144$ at (x_1, y_1), where $x_1 = 2$ and $y_1 > 0$. **(7)**

Q.6 a. Find the equation of a line passing through $(-2, -4)$ and perpendicular to the line $3x - y + 5 = 0$. **(7)**

b. Find the equation of the circle whose centre lies on the line $x - 4y = 1$ and which passes through the points (3, 7) and (5, 5). **(7)**

PART II
Answer any THREE Questions. Each question carries 14 marks.

Q.7 a. Find the term independent of x in the expansion of $\left(2x - \frac{1}{x}\right)^{10}$. **(7)**

b. Find the equation of the parabola whose focus is (1, -1) and whose vertex is (2,1). **(7)**

Q.8 a. Show that the height of a cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$. Also find the maximum value. **(7)**

b. Evaluate $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}}\, dx$. **(7)**

Q.9 a. Using induction, prove that $2^n > n$ for all n. **(7)**

b. Solve $\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$. **(7)**

Q.10 a. Evaluate $\int \frac{x^2 \tan^{-1} x^3}{1 + x^6}\, dx$. **(7)**

b. Evaluate $\int \frac{x - \sin x}{1 - \cos x} dx$. **(7)**

Q.11 a. Solve $(x^2 - y^2)dx + 2xy\ dy = 0$, given y =1 when x = 1. **(7)**

b. Find the differential equation of which $y = Ae^x + Be^{3x} + Ce^{5x}$ is a solution. **(7)**