Code: DE23/DC23 **CEMBER 2008** **Subject: MATHEMATICS - II**
Time: 3 Hours **Max. Marks: 100**

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q. 1. must be written in the space provided for it in the answer book supplied and nowhere else.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or best alternative in the following: (2x10)

a. If $\left(\frac{1+i}{1-i}\right)^n = 1,$ then n is equal to

(A) -1 (B) 1
(C) 2 (D) 4

b. If $\sin\theta = \tanh\phi$, then $\tan\theta$ is equal to

(A) $\sinh\phi$ (B) $\cosh\phi$
(C) $\operatorname{sech}\phi$ (D) $\operatorname{cosech}\phi$

c. $\bar{A} \times (\bar{B}+\bar{C}) + \bar{B} \times (\bar{C}+\bar{A}) + \bar{C} \times (\bar{A}+\bar{B})$ is equal to

(A) $\vec{A}$ (B) $\vec{B}$
(C) $\vec{C}$ (D) 0

d. If $|\vec{A}+\vec{B}| = |\vec{A}-\vec{B}|$, the angle between the $\vec{A}$ and $\vec{B}$ is

(A) 0° (B) 45°
(C) 90° (D) 135°

e.
$$\begin{vmatrix} 1+a_1 & a_2 & a_3 \\ a_1 & 1+a_2 & a_3 \\ a_1 & a_2 & 1+a_3 \end{vmatrix}$$ is equal to

(A) $(1+a_1)(1+a_2)(1+a_3)$ (B) $1+a_1+a_2+a_3$
(C) $3+a_1+a_2+a_3$ (D) $1+a_1a_2+a_2a_3+a_3a_1$

f. If inverse of $\begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ is $\begin{bmatrix} 3 & 2 & K \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$, then K is equal to

(A) 2 **(B)** 4
(C) 6 **(D)** 8

g. The characteristic equation of $\begin{bmatrix} 1 & 1 \\ 2 & 5 \end{bmatrix}$ is

(A) $\lambda^2 - 6\lambda + 3 = 0$ **(B)** $\lambda^2 + 6\lambda - 3 = 0$
(C) $\lambda^2 + 5\lambda + 2 = 0$ **(D)** $\lambda^2 - 5\lambda - 2 = 0$

h. The period of $|\sin x|$ is

(A) $\frac{\pi}{2}$ **(B)** π
(C) $\frac{3\pi}{2}$ **(D)** 2π

i. The inverse Laplace transform of $\frac{1}{s(s+2)}$ is

(A) $\frac{1-e^{-2t}}{2}$ **(B)** $\frac{1+e^{-2t}}{2}$
(C) $\frac{1-e^{2t}}{2}$ **(D)** $\frac{1+e^{2t}}{2}$

j. The solution of the differential equation $\frac{d^2y}{dx^2} + 4y = e^{2x}$ is

(A) $y = c_1 \cos 4x + c_2 \sin 4x + e^{2x}$ **(B)** $y = c_1 \cos 4x + c_2 \sin 4x + \frac{e^{2x}}{8}$
(C) $y = c_1 \cos 2x + c_2 \sin 2x + \frac{e^{2x}}{8}$ **(D)** $y = c_1 \cos 2x + c_2 \sin 2x + e^{2x}$

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. Show that

$$(1+i)^n + (1-i)^n = 2^{\left(\frac{n}{2}+1\right)} \cos\frac{n\pi}{4}$$ **(8)**

b. If $\tan(x+iy) = \sin(u+iv)$, show that

$$\frac{\sin 2x}{\sinh 2y} = \frac{\tan u}{\tan hv}$$ **(8)**

Q.3 a. The centre of a regular hexagon is at the origin and one vertex is given by 1+i on the Argand diagram. Find the remaining vertices. **(8)**

b. Show that the vectors $\vec{A} = 3i - 2j + k, \vec{B} = i - 3j + 5k, \vec{C} = 2i + j - 4k$ form a right angled triangle. **(8)**

Q.4 a. The vertices of a quadrilateral are

$\bar{A}(i+2j-k), \bar{B}(-4i+2j-2k), \bar{C}(4i+j-5k), \bar{D}(2i-j+3k)$ At the point A the forces of magnitudes 2, 3, 2 gm wt. act along the line AB, AC, AD respectively. Find their resultant. **(8)**

b. Find a unit vector perpendicular to the plane of $\bar{A} = 4i+3j+k$ and $\bar{B} = 2i-j+2k$. **(8)**

Q.5 a. Evaluate

$$\begin{vmatrix} 1 & a & a^2-bc \\ 1 & b & b^2-ca \\ 1 & c & c^2-ab \end{vmatrix}.$$

(8)

b. Use Cramer's rule to solve the equations

$$x+y+z=4$$
$$x-y+z=0$$
$$2x+y+z=5$$

(8)

Q.6 a. Investigate for what values of λ and μ, the equations

$$x+y+z=6$$
$$x+2y+3z=10$$
$$x+2y+\lambda z=\mu$$

have (i) no solution, (ii) a unique solution, (iii) an infinite number of solutions. **(8)**

b. Find the characteristic equation of the matrix

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$$ and hence find the inverse of the matrix A.

(8)

Q.7 a. Find the Laplace transform of

$$e^{-3t} \cdot \sin 5t \cdot \sin 3t \cdot$$

(8)

b. Find the inverse Laplace transform of

$$\frac{s+2}{s^2-4s+13}$$

(8)

Q.8 a. Use Laplace transform technique to solve

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 3y = e^{-t}$$

given that y = 0, $\frac{dy}{dt} = 0$ at t = 0 **(8)**

b. Solve the differential equation

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = e^{2x} + \cos 2x$$

. **(8)**

Q.9 a. Define even and odd functions. Give two examples of each. **(4)**

b. Find the Fourier series expansion for the function

$f(x) = x - x^2$ for $-\pi < x < \pi$. **(12)**