Code: D-23 / DC-23 Subject: MATHEMATICS - II
Time: 3 Hours Max. Marks: 100

NOTE: There are 11 Questions in all.

Question 1 is compulsory and carries 16 marks. Answer to Q. 1. must be written in the space provided for it in the answer book supplied and nowhere else.
Answer any THREE Questions each from Part I and Part II. Each of these questions carries 14 marks.
Any required data not explicitly given, may be suitably assumed and stated.

Q.1 Choose the correct or best alternative in the following: (2x8)

a. If $z_1 = r_1(\text{Cos}\,\theta_1 + i\,\text{Sin}\,\theta_1)$, $z_2 = r_2(\text{Cos}\,\theta_2 + i\,\text{Sin}\,\theta_2)$ then $z_1 z_2$ is equal to

(A) $\left(\frac{r_1}{r_2}\right)\{\text{Cos}(\theta_1+\theta_2) + i\,\text{Sin}(\theta_1+\theta_2)\}$.
(B) $r_1 r_2\{\text{Cos}(\theta_1+\theta_2) + i\,\text{Sin}(\theta_1+\theta_2)\}$.
(C) $r_1 r_2\{\text{Cos}(\theta_1\theta_2) + i\,\text{Sin}(\theta_1\theta_2)\}$.
(D) $r_1 r_2\{\text{Cos}(\theta_1-\theta_2) + i\,\text{Sin}(\theta_1-\theta_2)\}$.

b. If ω is cube root of unity then $1+\omega+\omega^2$ is equal to

(A) 0. **(B)** 1.
(C) -1. **(D)** 3.

c. The roots of $x^2 - x - 12 = 0$ are
(A) 2, 3. **(B)** 3, 2.
(C) 4, -3. **(D)** 4, 3.

d. If $A = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}$ then AB is equal to

(A) $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$. **(B)** $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.
(C) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$. **(D)** $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$.

e. If A and B are invertible matrices of the same size then $(AB)^{-1}$ is equal to

(A) AB. **(B)** BA.
(C) $B^{-1}A^{-1}$. **(D)** $A^{-1}B^{-1}$.

f. If A and B are the points (3, 4, 5) and (6, 8, 9) then the vector $\overrightarrow{AB}$ is

(A) $3\vec{i}+4\vec{j}+4\vec{k}$. **(B)** $3\vec{i}+4\vec{j}$.
(C) $3\vec{i}-4\vec{j}-4\vec{k}$. **(D)** $3\vec{i}-4\vec{j}$.

g. The function f (x) = Sin x is
(A) non periodic. **(B)** periodic with period π.

(C) periodic with period 2π. **(D)** periodic with period $\pi/2$.
h. The Laplace transform of Sinh (at) is

(A) $\frac{1}{s^2-a^2}$. **(B)** $\frac{a}{s^2-a^2}$.
(C) $\frac{s}{s^2+a^2}$. **(D)** $\frac{s}{s^2-a^2}$.

PART I
Answer any THREE Questions. Each question carries 14 marks.

Q.2 a. If z is any complex number and $\bar{z}$ is its complex conjugate then show that $z\bar{z}=|z|^2$. **(7)**

b. Find the square root of the complex number 3 + 4i. **(7)**

Q.3 a. If $z=\text{Cos}\,\theta+i\,\text{Sin}\,\theta$ then find $z^n+\frac{1}{z^n}$. **(7)**

b. If $a_r=\text{Cos}\left(\pi/2^r\right)+i\,\text{Sin}\left(\pi/2^r\right)$ $r=1,2,3,....$ then show that $a_1\,a_2\,a_3 ...\text{ad inf}=-1$. **(7)**

Q.4 a. If a square matrix A is invertible then show that A^T (transpose of A) is also invertible and $\left(A^T\right)^{-1}=\left(A^{-1}\right)^T$. **(7)**

b. Compute the inverse of the matrix $A = \begin{pmatrix} 3 & -4 & 2 \\ 0 & 5 & 9 \\ -4 & 8 & 1 \end{pmatrix}$. **(7)**

Q.5 a. Evaluate $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega^2 & 1 & \omega \\ \omega & \omega^2 & 1 \end{vmatrix}$ where ω is a complex cube root of unity. **(7)**

b. Show without evaluating that determinant $\begin{vmatrix} 1 & x & y+z \\ 1 & y & x+z \\ 1 & z & x+y \end{vmatrix} = 0$. **(7)**

Q.6 a. Find the position vector of a point which divides the line joining two given points in three dimensional space. **(7)**

b. Show that the vectors $2\vec{i} - \vec{j} + \vec{k}$, $\vec{i} - 3\vec{j} - 5\vec{k}$ and $3\vec{i} - 4\vec{j} - 4\vec{k}$ form the sides of a right angled triangle. **(7)**

PART II

Answer any THREE Questions. Each question carries 14 marks.

Q.7 a. State Cayley Hamilton Theorem and verify it for the square matrix $\begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}$. **(7)**

b. Show that the system of equations

$2x - 3y + z = 0$

$x + 2y - 3z = 0$

$4x - y - 2z = 0$

has only the trivial solution. **(7)**

Q.8 Find the Fourier Series for the function,

$f(x) = x,\ 0 < x < 2\pi$. **(14)**

Q.9 a. Distinguish between even and odd functions. Give one example for each of these functions. **(7)**

b. Forces $2\vec{i} + 7\vec{j}$, $2\vec{i} - 5\vec{j} + 6\vec{k}$, $-\vec{i} + 2\vec{j} - \vec{k}$ act on a point P having position vector $4\vec{i} - 3\vec{j} - 2\vec{k}$. Find the vector moment of the resultant of three forces acting at P about the point Q whose position vector is $6\vec{i} + \vec{j} - 3\vec{k}$. **(7)**

Q.10 a. Define Laplace transform of a function. Obtain the Laplace transform of Cosh (at). **(7)**

b. Find the inverse Laplace transform of $\frac{s-1}{s^2-6s+25}$. **(7)**

Q.11 a. Solve the differential equation $\frac{d^2y}{dx^2}-7\frac{dy}{dx}+12y=0$. **(7)**

b. Solve by using Laplace transform, the differential equation
$\frac{d^2y}{dt^2}+4y=\sin t,\ y(0)=1,\ y'(0)=0.$ **(7)**