Total number of printed pages - 6

MCA

SCM 2006

2ND SEMESTER EXAMINATION – 2006

NUMERICAL METHODS

Full Marks – 70

Time : 3 Hours

The figures in the right hand margin indicate full marks for the questions.

*Answer questions No. **1** which is compulsory and any **five** from the rest.*

1. Answer the following questions : 2×10

   (a) Find the relative percentage error in the approximation of 1/3 (accurate upto four decimal places).

   (b) Which method does have higher order of convergence ? – the Newton-Raphson method or the Secant method ; Justify.

P.T.O.

(c) Design a scheme of iteration for finding an approximate value of $2^{1/3}$ by using the secant method.

(d) Can you find a real root of $x^3 + x^2 - 1$ in the interval [0, 1] by the fixed point method ; justify your answer.

(e) Find $l_k(x_j)$ where x_j is the j-th node and $l_k(x)$ is the Lagrange's fundamental polynomial of $(k-1)$th degree.

(f) Find $\Delta^2 f(x)$ if $f(x) = ab^{cx}$, where a, b, c are constants. (Take step size of 1).

(g) Show that $\Delta + \nabla = \frac{\Delta}{\nabla} - \frac{\nabla}{\Delta}$, where Δ and ∇ have their usual meaning.

(h) Write down the Simpson's 1/3rd rule for finding the approximate area of a curve.

(i) Show that $E = e^{hD}$, where E, h and D have their usual meaning.

(j) Find the second divided difference of the function $f(x) = \frac{1}{x}$. (Taking nodes x_0, x_1 and x_2).

2. (a) Find the approximate value of $\sqrt{2}$, correct to 3 decimal places, using the fixed point method. 5

(b) Use Newton-Raphson method to find an approximate real root of $xe^x - 2 = 0$ correct to 3 decimal places. 5

3. (a) Approximate $2^{2.5}$ by Newton's divided quadratic interpolation with nodes $x_0 = 1$, $x_1 = 2$, $x_3 = 3$ and $x_4 = 4$. 5

(b) Use Newton's forward difference formula to determine the missing value in the following data : 5

x	0	1	2	3	4
y	1	3	9	–	81

4. (a) Solve $e^{2x} = 2.14$ by inverse interpolation using $x = F(y) = \frac{1}{2} \ln y$ with $F(2) = 0.34$, $F(2.2) = 0.394$, $F(2.4) = 0.438$. 5

(b) Show that

$$f'(a) = \frac{1}{h}\left[\Delta f(a) - \frac{1}{2}\Delta^2 f(a) + \frac{1}{3}\Delta^3 f(a) - \ldots\ldots\right]$$

where h is the step size and

$\Delta f(x) = f(x+h) - f(x)$. 5

5. (a) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = 1.1$ from the follwing data : 5

x	1.0	1.1	1.2	1.3	1.4	1.5	1.6
$y = f(x)$	7.989	8.403	8.781	9.129	9.451	9.750	10.031

(b) Calculate $\ln_e 2$ by evaluating the integral $\int_0^1 \frac{dx}{1+x}$ using Simpson's 1/3rd rule, taking step size of $h = 0.2$. 5

6. (a) Evaluate $\int_0^1 \frac{x\,dx}{1+x^2}$ by using the trapezoidal rule with step size $h = 0.2$ 5

(b) Solve the initial value problem 5

$$\frac{dy}{dx} = x + y, \quad y(0) = 1$$

using Euler's method and taking $h = 0.2$

7. (a) Solve the initial value problem 5

$$\frac{dy}{dx} = x - y^2, \text{ with } y(0) = 1$$

to find out $y(0.3)$, by using Runge-Kutta method.

(b) Use modified Euler's method to obtain approximate the value of y at $x = 0.2$ for the initial value problem 5

$$\frac{dy}{dx} = 2y + 3e^x, \quad y(0) = 0$$

8. (a) Use Gauss-Siedel iteration to find approximate solutions for the following system of equations : 5

$$5x + y + 2z = 19$$

$$2x + 3y + 8z = 39$$

$$x + 4y + 8z = -2$$

(b) Find the approximate eigenvalues of the matrix

$$\begin{pmatrix} 9 & 4 \\ 4 & 3 \end{pmatrix}$$

taking initial value $v_0 = (1, 1)^t$, using the power method. 5