INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

21st November 2011

Subject ST6 — Finance and Investment B

Time allowed: Three hours (9.45* – 13.00 Hrs)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

1. *Please read the instructions on the front page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception*

2. ** You have 15 minutes at the start of the examination in which to read the questions. You are strongly encouraged to use this time for reading only, but notes may be made. You then have three hours to complete the paper.*

3. *You must not start writing your answers in the answer sheet until instructed to do so by the supervisor*

4. *The answers are not expected to be any country or jurisdiction specific. However, if Examples/illustrations are required for any answer, the country or jurisdiction from which they are drawn should be mentioned.*

5. *Attempt all questions, beginning your answer to each question on a separate sheet.*

6. *Mark allocations are shown in brackets.*

7. *Please check if you have received complete Question Paper and no page is missing. If so, kindly get new set of Question Paper from the Invigilator.*

AT THE END OF THE EXAMINATION

Please return your answer book and this question paper to the supervisor separately.

Q. 1)	What volatility smile is likely to be observed for 6-month options on a stock when the volatility is uncertain and is positively correlated with the stock price?		
			[2]
Q. 2)	Ramesh is the sole owner and manager of a highly leveraged business entity (debt-equity ratio of 8:1). All the debts will mature in one year. If after one year, the value of the business entity is greater than the face value of the debt, Ramesh will pay off the debt. However, if after one year, the value of the business entity is less than the face value of the debt, Ramesh will declare bankruptcy and the debt holder will own the company.		
	a)	Express the position of Ramesh as an option on the value of the business.	(2)
	b)	Express the position of the debt holders in terms of call options on the value of the business.	(2)
	c)	Express the position of the debt holders in terms of put options on the value of the business.	(2)
	d)	What can Ramesh do to increase the value of his position?	(2)
			[8]
Q. 3)	Given two probability measures P and Q		
	a)	Define the Radon-Nikodym derivative. Under what condition does the Radon-Nikodym derivative apply?	(2)
	b)	You are given that W_T is a Standard Brownian Motion under measure P. Let us define $\tilde{W}_T = W_T + \mu T$. If there exists a measure Q under which $\tilde{W}_T$ is a Standard Brownian motion, then find the Radon – Nikodym derivative of Q with respect to P.	(3)
	c)	Given that W_T is a Standard Brownian Motion under measure P, prove that $\tilde{W}_T$ ~ N(0, T) under Q.	(3)
	d)	State Girsanov's theorem where a variable drift is involved, i.e., $\tilde{W}_t = W_t + \int_o^t \gamma_s ds$.	(2)
			[10]
Q. 4)	W_t is a Standard Brownian Motion. You are given $A_t = W_t^3 + atW_t + bt^2$ and $B_t = W_t^2 + aW_t + ct$		

	a)	Do there exist constants a, b and c such that A_t and B_t are both martingales?	(4)
	b)	If constants a, b and c exist such that A_t and B_t are martingales, then is $D_t = A_t \times B_t$ a martingale for the same set of constants a, b and c?	(5)
	c)	Is $C_t = A_t / B_t$ a martingale, under the same conditions as (b)?	(1)
			[10]
Q. 5)	An investment company sells single premium bonds. In one of its bonds it offers a benefit that accrues over time. The benefit at the beginning of the term of the product is Rs 50,000. At the end of every year the benefit accruing is equal to $A_t = 50{,}000 \times Min(Max(\%\ change\ in\ SENSEX\,(t-1,t),-4\%),10\%)$ so that at maturity after 3 years the maturity amount is $B_t = 50{,}000 + \sum_{1}^{3} A_t$ On 1 October 2010 the SENSEX was at 10000 and on 1 October 2011, the SENSEX is at 10400. On 1 October 2011 the market expects the SENSEX to increase by 4% with a probability of 0.6 and expects it to decrease by 4% with a probability of 0.4.		
	a)	What should the bank price this single premium bond at on 1 October 2011, if the risk free rate in the market is 1% per annum?	(6)
	b)	If the bank can invest in SENSEX and a risk free asset, how much should the bank hold of each of the SENSEX and the risk free asset on 1 October 2011 to replicate the payoffs from the bond?	(3)
	c)	On 1 October 2012, the SENSEX turns out to be at 10920. The market's future expectation changes to an increase of 5% in SENSEX with probability 0.6 and a decrease of 5% in SENSEX with probability 0.4. If as a buyer of the single premium bond, you plan to sell it in the secondary market, would you make a profit or a loss and how much?	(6)
			[15]

Q. 6) According to the Hull and White model, the short rate is described by the process

$$dr = [\theta(t) - ar]dt + \sigma dz$$

where a and σ are constants and $\theta(t)$ is a function of time. dz is a Wiener process.

Zero-coupon bond prices (with face value Re. 1) at time t in the Hull and White model are given by:

$$P(t,T) = A(t,T)e^{-B(t,T)r(t)}$$

where $B(t,T) = \dfrac{1 - e^{-a(T-t)}}{a}$ and

$$\ln A(t,T) = \ln \frac{P(0,T)}{P(0,t)} + B(t,T)F(0,t) - \frac{1}{4a^3}\sigma^2 (e^{-aT} - e^{-at})(e^{2at} - 1)$$

F(0, t) is the instantaneous forward rate for maturity t as seen at time 0.

According to HJM model, the price of a zero-coupon bond follows the following process

$$dP(t,T) = r(t)P(t,T)dt + v(t,T,\Omega_t)P(t,T)dz(t)$$

Ω_t indicates that the zero-coupon bond's volatility can be interpreted in the most general form of the model.

The forward rate in HJM model follows the following stochastic process

$$dF(t,T) = m(t,T,\Omega_t)dt + s(t,T,\Omega_t)dz$$

where:

$$m = s(t,T,\Omega_t)\int_t^T s(t,\tau,\Omega_t)d\tau$$

$$s(t,T,\Omega_t) = v_T(t,T,\Omega_t) = \frac{\partial v(t,T,\Omega_t)}{\partial T}$$

Show that when the forward rate volatility $s(t,T)$ in HJM model is $\sigma e^{-a(T-t)}$, the Hull-White model results.

			[7]
Q. 7)	Consider the Merton model where a company's equity is an option on the assets of the company. Merton's model gives the value of the firm's equity at time T as: $E_T = Max(V_T - D, 0)$ where V_T : Value of the company's asset at time T and D; Face value of the company's debt which is a zero coupon bond repayable at time T Show that under Merton's model the spread between the yield on a T-year zero-coupon riskless bond and a similar T-year zero-coupon issued by the company is: $-\frac{\ln[\Phi(d_2) + \frac{\Phi(-d_1)}{L}]}{T}$, where, $L = \frac{De^{-rT}}{V_0}$, $d_1 = \frac{\ln(\frac{V_0}{D}) + (r + \frac{\sigma^2}{2})T}{\sigma\sqrt{T}}$, $d_2 = d_1 - \sigma\sqrt{T}$, σ is the volatility of V_t , and r is the risk-free rate of interest. The function $\Phi(x)$ is the cumulative probability distribution function for a standardized normal distribution.		
			[10]
Q. 8)	ICICI Bank's position in options on US dollars has delta of 100,000 and a gamma of -5000.		
	a)	Explain how these values of gamma and delta are interpreted.	(3)
	b)	The current exchange rate is \$1 = Rs. 50. What position would you take to make the position of ICICI Bank delta neutral?	(1)
	c)	After a short period of time, the exchange rate moves to \$1 = Rs. 53. Estimate the new delta of ICICI Bank position.	(2)
	d)	What additional trade is necessary to keep ICICI Bank's position delta neutral?	(2)
	e)	Assuming the bank did set up a delta neutral position originally, has ICICI Bank gained or lost money from the exchange-rate movement?	(2)
			[10]

Q. 9) Suppose that f_1 and f_2 are the prices of two derivatives dependent only on θ and t.

where $d\theta = m\theta + s\theta dz$

dz is a Wiener process. The parameters m and s are the expected growth rate in θ and the volatility of θ respectively. m and s depend only on θ and time t.

Assume that during the period under consideration f_1 and f_2 provide no income. The process followed by f_1 and f_2 are

$$df_1 = \mu_1 f_1 dt + \sigma_1 f_1 dz$$

$$df_2 = \mu_2 f_2 dt + \sigma_2 f_2 dz$$

where μ_1, μ_2, σ_1, and σ_2 are functions of θ and time t.

a) Show that $\frac{\mu_1 - r}{\sigma_1} = \frac{\mu_2 - r}{\sigma_2}$, where r is the risk-free rate of interest (risk-free rate of interest per unit of time). (6)

b) Assume that the price of f_1 is measured in units of f_2 rather than in rupees (that is, security price f_2 is numeraire). Show that $\frac{f_1}{f_2}$ is a martingale if $\frac{\mu_1 - r}{\sigma_1} = \frac{\mu_2 - r}{\sigma_2} = \sigma_2$. (7)

[13]

Q. 10) The following are the prices of zero coupon bonds on 1 January 2011.

Duration in years	Price in INR as % of face value
1	94.7%
2	90.4%
3	87.6%
4	73.5%
5	73.0%

a) Draw the spot curve and the forward curve. (5)

b) Given the above data

i) Calculate the price of a 5 year government bond which is to be redeemed at a premium of 5% and pays an annual coupon in arrears of 8% per annum. (2.5)

ii) Is buying this 5 year government bond better than buying a 5 year term deposit with a quoted interest rate of 6% per annum? (2.5)

		A bank is considering selling a fixed term annuity that pays x% of the single premium at the end of each year for 5 years. There is no maturity value. The bank can only invest in zero coupon bonds.	
		iii) What should x be so that the bank makes a profit margin of 20%. The profit margin is defined as present value of profits divided by present value of premiums?	(2)
		iv) How much of each of the zero coupon bonds should the bank buy with the single premium if it wants to realise equal profits each year?	(3)
			[15]
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