MATHEMATICS

Option II

1. Which of the following functions is differentiable at the origin :

(a) $f(x) = |x| + |x - 1|$ for all $x \in \mathbf{R}$

(b) $$f(x) = \begin{cases} x \sin \frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

(c) $f(x) = x|x|$

(d) $f(x) = |x|$ for all $x \in \mathbf{R}$

2. The function $f(x) = x - [x]$, for all $x \geq 0$ (where $[x]$ is the greatest integer less or equal to x) is continuous at :

(a) $x = 1$

(b) $x = \frac{3}{2}$

(c) $x = 2$

(d) $x = 3$

3. The angle ϕ between the radius vector and the tangent at the point (r, θ) on the curve $f(r, \theta) = 0$ is given by :

(a) $\tan \phi = \frac{1}{r} \frac{d\theta}{dr}$

(b) $\tan \phi = \frac{1}{r} \frac{dr}{d\theta}$

(c) $\tan \phi = r \frac{dr}{d\theta}$

(d) $\tan \phi = r \frac{d\theta}{dr}$

4. If $z = \sin^{-1}\left(\frac{x^2 + y^2}{x + y}\right)$, then which of the following is *true*.

(a) $x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y} - \tan z = 0$

(b) $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} - \tan z = 0$

(c) $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} + \tan z = 0$

(d) $x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y} + \tan z = 0$

5. The radius of curvature of the lemniscate $pa^2 = r^3$ at $r = \frac{1}{3}$ is :

(a) a^2

(b) $\frac{1}{3}a^2$

(c) $\frac{1}{3}a$

(d) a

6. The equation of the hyperbola having $x + y - 1 = 0$ and $x - y + 2 = 0$ as its asymptotes, and passing through the origin is :

(a) $(x + y - 1)(x - y + 2) + 2 = 0$

(b) $(x + y - 1)(x - y + 2) - 2 = 0$

(c) $(x + y - 1)(x - y + 2) - 1 = 0$

(d) $(x + y - 1)(x - y + 2) + 1 = 0$

7. The modulus of the complex number $\frac{1+7i}{(2-i)^2}$ is :

(a) $\frac{1}{\sqrt{2}}$

(b) $\frac{1}{\sqrt{3}}$

(c) $\sqrt{2}$

(d) $\sqrt{3}$

8. The sum and the product of all the values of $(1)^{\frac{1}{3}}$ is :

(a) 0, 1

(b) 1, 0

(c) 0, –1

(d) –1, 0

9. Which of the following is *not true* ?

(a) $\sin(ix) = -i \sinh x$

(b) $\cos(ix) = \cosh x$

(c) $\tan(ix) = i \tanh x$

(d) $\operatorname{cosec}(ix) = -i \operatorname{cosech} x$

10. The general value of $\log i$ is :

(a) $\frac{1}{2}(4n-1)+i\pi$

(b) $\frac{1}{2}(4n+1)+i\pi$

(c) $\frac{1}{2}(4n+1)-i\pi$

(d) $\frac{1}{2}(4n-1)-i\pi$

11. The equation of the chord to the parabola $y^2 = 8x$ which is bisected at the point $(2, -3)$ is :

(a) $4x - 3y + 1 = 0$

(b) $4x + 3y - 1 = 0$

(c) $4x - 3y - 1 = 0$

(d) $4x + 3y + 1 = 0$

12. The eccentricity of an ellipse when its latus rectum is equal to one half of its minor axis is equal to :

(a) $\sqrt{\frac{3}{2}}$

(b) $\frac{\sqrt{3}}{2}$

(c) $\frac{3}{\sqrt{2}}$

(d) $\frac{3}{2}$

13. The condition that the line $x + 2y + 3 = 0$ touches the ellipse $b^2x^2 + a^2y^2 = a^2b^2$ is :

(a) $a^2 + b^2 = 9$

(b) $a^2 + 4b^2 = 9$

(c) $a^2 - b^2 = 9$

(d) $a^2 - 4b^2 = 9$

14. The number of normals that can be drawn from any point to an ellipse is :

(a) 2

(b) 3

(c) 4

(d) 5

15. The diameters $y = \alpha x$ and $y = \beta x$ of the hyperbola $b^2x^2 - a^2y^2 = a^2b^2$ are conjugate if :

(a) $\frac{\alpha}{b^2} = \frac{1}{\beta a^2}$

(b) $\frac{\alpha}{\beta} = \frac{b^2}{a^2}$

(c) $\alpha\beta = \frac{a^2}{b^2}$

(d) $\frac{\alpha}{\beta} = \frac{a^2}{b^2}$

16. The intercepts made by the plane $2x - 3y + z = 12$ on the coordinate axes are :

(a) 6, –4, 12

(b) 2, –3, 12

(c) 2, –3, –12

(d) 6, 4, –12

17. The magnitude of the line of shortest distance between the lines $x + \alpha = 2y = -12z$ and $x = y + 2\alpha = 6z - 6\alpha$ is :

(a) α

(b) 2α

(c) 3α

(d) 4α

18. The condition for the two spheres :

$$x^2 + y^2 + z^2 + 2x + 2ay + 2z + 2b = 0$$

and $$x^2 + y^2 + z^2 + 2cx + 2y + 2z + 2d = 0$$

to be orthogonal is :

(a) $a + c = b + d$

(b) $a + c = b + d + 1$

(c) $a + c - 1 = b + d$

(d) $a + c + 1 = b + d$

19. The equation of the right circular cone whose vertex is the origin and whose axis is the z-axis is :

(a) $x^2 + y^2 = z^2 \sin^2 \theta$

(b) $x^2 + y^2 = z^2 \cos^2 \theta$

(c) $x^2 + y^2 + z^2 = \tan^2 \theta$

(d) $x^2 + y^2 = z^2 \tan^2 \theta$

20. The enveloping cylinder of the sphere :

$$x^2 + y^2 + z^2 - 2x + 4y = 1$$

having its generators parallel to the line $x = y = z$ is :

(a) $x^2 + y^2 + z^2 - xy - yz - zx - 4x + 5y - z - 2 = 0$

(b) $x^2 + y^2 + z^2 + xy + yz + zx - 4x + 5y - z - 2 = 0$

(c) $x^2 + y^2 + z^2 - xy - yz - .zx + 4x - 5y + z + 2 = 0$

(d) $x^2 + y^2 + z^2 + xy + yz + zx + 4x + 5y + z + 2 = 0$

21. The condition that the plane $lx + my + nz = p$ touches the central conicoid $2x^2 + 2y^2 + 3z^2 = 1$ is :

(a) $3l^2 + 2m^2 + 3n^2 = 6p^2$

(b) $2l^2 + 3m^2 + 3n^2 = 6p^2$

(c) $3l^2 + 3m^2 + 2n^2 = 6p^2$

(d) $3l^2 + 3m^2 - 2n^2 = 6p^2$

22. The sum of the squares of three conjugate semi-diameters to the ellipsoid $x^2 + 2y^2 + z^2 = 1$ is :

(a) $\frac{7}{4}$

(b) $\frac{11}{4}$

(c) $\frac{5}{4}$

(d) $\frac{9}{4}$

23. Which of the following is *not true* ?

(a) $\int \log x \, dx = x \log x - x + c$

(b) $\int \frac{1}{x \log x} dx = \log (\log x) - x + c$

(c) $\int x e^x \, dx = e^x (x - 1) + c$

(d) $\int \frac{3x^2 + 2}{x^3 + 2x + 1} dx = \log \left(x^3 + 2x + 1\right) + c$

24. Which of the following is *not* equal to $\int \frac{dx}{1-x^2}$?

(a) $\frac{1}{2}\log\left(\frac{1+x}{1-x}\right)+c$

(b) $\frac{1}{2}\log\left(\frac{1-x^2}{1+x^2}\right)+c$

(c) $\tanh^{-1} x + c$

(d) None of the above

25. The value of $\int_0^{\pi} \sin^8 \frac{\theta}{2}\, d\theta$ is :

(a) $\frac{35\pi}{128}$

(b) $\frac{53\pi}{128}$

(c) $\frac{35\pi}{138}$

(d) $\frac{53\pi}{138}$

26. Taking $\int_0^{\frac{\pi}{2}} \log \sin x\, dx = \frac{-\pi}{2}\log 2$, the value of $\int_0^{\pi} \log \sin x\, dx$ is :

(a) $\frac{-\pi \log 2}{2}$

(b) $\pi \log 2$

(c) $-\pi \log 2$

(d) $\frac{\pi \log 2}{2}$

27. The sum of the areas of all the loops of the curve $r = \sin 2\theta$ is :

(a) $\frac{\pi}{3}$

(b) $\frac{\pi}{4}$

(c) $\frac{\pi}{2}$

(d) π

28. The area of the surface of the sphere of radius 3 is :

(a) 36π

(b) 12π

(c) 18π

(d) 9π

29. The integrating factor of the differential equation $x\,dy = y\,(1 + xy)\,dx$ is :

(a) $\log x$

(b) e^x

(c) x

(d) $\frac{1}{x}$

30. The value of $\frac{1}{D^2 + 4} \sin 2x$ is :

(a) $\frac{-1}{4} x \cos 2x$

(b) $\frac{-1}{4} x \sin 2x$

(c) $\frac{1}{4} x \cos 2x$

(d) $\frac{1}{4} x \sin 2x$

31. The particular integral of the differential equation $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 2\cosh x$ is :

(a) $\frac{e^x}{4} + \frac{x^2 e^{-x}}{3}$

(b) $\frac{e^x}{4} + \frac{x^2 e^{-x}}{2}$

(c) $\frac{e^x}{4} + \frac{e^{-x}}{2}$

(d) $\frac{e^x}{4} + \frac{e^{-x}}{3}$

32. The solution of the differential equation $y = xp + f(p)$, where $p = \frac{dy}{dx}$, is :

(a) $y = cx + f(c)$

(b) $x = cy + f(c)$

(c) $y = cx^2 + f(c)$

(d) $x = cy^2 + f'(c)$

33. If $\vec{a} = 3\hat{i} - \hat{j} + 2\hat{k}$, $\vec{b} = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{c} = \hat{i} - 2\hat{j} + 2\hat{k}$, then the value of $\left(\vec{a} \times \vec{b}\right) \times \vec{c}$ is :

(a) $24\hat{i} + 7\hat{j} + 5\hat{k}$

(b) $24\hat{i} - 7\hat{j} + 5\hat{k}$

(c) $24\hat{i} + 7\hat{j} - 5\hat{k}$

(d) None of the above

34. If $\phi(x, y, z) = 3x^2y - y^3z^2$, then the value of gradient ϕ at the point $(1, -2, -1)$ is :

(a) $-12\hat{i} + 9\hat{j} - 16\hat{k}$

(b) $-12\hat{i} - 9\hat{j} + 16\hat{k}$

(c) $-12\hat{i} - 9\hat{j} - 16\hat{k}$

(d) $12\hat{i} - 9\hat{j} - 16\hat{k}$

35. The diagonal elements of a skew-hermitian matrix are :

(a) non-zero integers

(b) non-zero real

(c) either purely imaginary or zero

(d) none of the above

36. The value of k so that the matrix $\begin{bmatrix} 1 & 2 & k \\ -1 & 0 & 1 \\ 2 & 1 & 1 \end{bmatrix}$ does not have the inverse is :

(a) 4

(b) 5

(c) 6

(d) –4

37. If A and B are equivalent matrices, then which of the following is *true* ?

(a) rank A – rank B = 0

(b) rank A – rank B = 1

(c) rank A + rank B = 0

(d) rank A + rank B = 1

38. The condition for the two vectors $\begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$ and $\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$ to be linearly dependent is :

(a) $a_1 a_2 - b_1 b_2 = 0$

(b) $a_1 b_1 - a_2 b_2 = 0$

(c) $a_1 b_2 - a_2 b_1 = 0$

(d) $a_1 b_2 + a_2 b_1 = 0$

39. If A is a non-singular matrix of order n, and X and B are both of order $n \times 1$, then the number of solutions of the system of equations AX = B is :

(a) infinite

(b) n

(c) 1

(d) $n - 1$

40. The value of k so that the equation $x^3 - 3x^2 + 3x - k = 0$ has three equal roots is :

(a) +1

(b) –1

(c) –2

(d) +2

41. To remove the second term of the equation $x^3 + 3x^2 + 4x - 10 = 0$ its roots are to be diminished by :

(a) –1

(b) 1

(c) –2

(d) 2

42. If α, β and γ are the roots of the equation $x^3 + px^2 + qx + r = 0$, then the value of $\alpha^2 + \beta^2 + \gamma^2$ is :

(a) $p^2 - 2r$

(b) $r^2 - 2p$

(c) $q^2 - 2p$

(d) $p^2 - 2q$

43. If k is the number of complex roots of the equation $x^9 - x^5 + x^4 + x^2 + 1 = 0$, then k equals :

(a) 8

(b) 6

(c) 4

(d) 2

44. Which of the following is *not* true for the infinite set S = {$x : 0 \leq x \leq 1$, x belongs to the set of rational numbers} ?

(a) S is bounded

(b) S has supremum 1

(c) S is infimum 0

(d) S is not bounded

45. Which of the following is an open set ?

(a) Set of real numbers

(b) Set of rationals

(c) Closed interval $[a, b]$

(d) $\left\{\frac{1}{n} : n \text{ is a natural number}\right\}$

46. Which of the following is uncountable ?

(a) Set of all integers

(b) Set of all real numbers

(c) Set of all ordered pairs of integers

(d) Set of all rational numbers

47. Which of the following is *not* true ?

(a) A bounded sequence has a limit point

(b) A convergent sequence is bounded

(c) A bounded sequence with a unique limit point is not convergent

(d) None of the above

48. Which of the following series is *not* convergent ?

(a) $\sum_{n=1}^{\infty} \frac{1}{n!}$

(b) $\sum_{n=1}^{\infty} \frac{1}{n^2}$

(c) $\sum_{n=1}^{\infty} \frac{\sin nx}{n^2}$

(d) $\sum_{n=0}^{\infty} 2^n$

49. If $f(x)$ is bounded and integrable on $[a, b]$, which of the following is *not* true ?

(a) $|f(x)|$ is bounded

(b) $|f(x)|$ is integrable

(c) $\left|\int_a^b f(x)\,dx\right| \leq \int_a^b |f(x)|\,dx$

(d) $\left|\int_a^b f(x)\,dx\right| > \int_a^b |f(x)|\,dx$

50. Which of the following functions is *not* integrable ?

(a) $f(x) = \begin{cases} 1, & x \text{ rational} \\ -1, & x \text{ irrational} \end{cases}$

(b) $f(x) = [x]$ in $[0, 3]$

(c) $f(x) = |x|$ in $[-1, 1]$

(d) None of the above

51. $\lim\limits_{(x,y)\to(0,0)} \left(\frac{1}{|x|} + \frac{1}{|y|}\right)$ equals :

(a) 0

(b) ∞

(c) 1

(d) -1

52. What is the number of groups of order 6 upto isomorphism ?

(a) 2

(b) 3

(c) 4

(d) 6

53. $\int_0^\infty e^{-x^2}\,dx$ equals :

(a) $\frac{\sqrt{\pi}}{2}$

(b) $\sqrt{\frac{\pi}{2}}$

(c) $\sqrt{\pi}$

(d) $\sqrt{\frac{2}{\pi}}$

54. What is the order of the element 3 in the additive group of integers ?

(a) 1

(b) 3

(c) 9

(d) infinite

55. What is the number of generators in the cyclic group of order 8 ?

(a) 2

(b) 4

(c) 6

(d) 8

56. Which of the following groups is non-Abelian ?

(a) Group of order 9

(b) Group of order 49

(c) Group of order 121

(d) None of the above

57. Which of the following sets does not form a ring with respect to the operations ordinary addition and multiplication ?

(a) $A = \{b\sqrt{2} : b \text{ is a rational number}\}$

(b) The set of even integers

(c) $B = \{a + b\sqrt{2} : a, b \text{ are rational numbers}\}$

(d) The set of all real numbers

58. If W is a subspace of a finite dimensional vector space V(F) of dimension n, then dimension m of W is given by :

(a) $m > n$

(b) $m \leq n$

(c) $m \geq n + 1$

(d) none of the above

59. Which of the following sets of vectors does *not* form a basis of R^3 ?

(a) $\{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$

(b) $\{(1, 0, 0), (1, 1, 0), (1, 1, 1)\}$

(c) $\{(1, 2, 1), (2, 1, 0), (1, -1, 2)\}$

(d) None of the above

60. If a finite dimensional vector space V(F) is a direct sum of its two subspaces V_1 and V_2, then dimension of V equals :

(a) $\dim V_1 - \dim V_2$

(b) $\dim V_1 + \dim V_2 - 1$

(c) $\dim V_1 + \dim V_2$

(d) $\dim V_1 - \dim V_2 + 1$