SECTION - A $10 \times 2 = 20$

VERY SHORT ANSWER TYPE QUESTIONS

Answer All questions. Each question carries 2 marks.

1. If $f : R - \{0\} \to R$ is defined by $f(x) = x^3 - \frac{1}{x^3}$, then show that $f(x) + f(1/x) = 0$.
2. Find the domain of $\sqrt{x+2} + \frac{1}{\log_{10}(1-x)}$
3. $\mathbf{a}, \mathbf{b}$ are noncollinear vectors. If $\mathbf{R} = (x + 4y)\,\mathbf{a} + (2x + y + 1)\,\mathbf{b}$, $\mathbf{S} = (y - 2x + 2)\,\mathbf{a} + (2x - 3y - 1)\,\mathbf{b}$ then find x, y such that $3\mathbf{R} = 2\mathbf{S}$.
4. Show that the vectors $\mathbf{i} + \mathbf{j} + 2\mathbf{k}, \mathbf{j} + 2\mathbf{k}, \mathbf{i} + 2\mathbf{j} + 4\mathbf{k}$ are linearly dependent.
5. Find the area of the parallelogram whose diagonals are $3\mathbf{i} + \mathbf{j} - 2\mathbf{k}, \mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$
6. Show that $\cos^4 \alpha + 2 \cos^2 \alpha \left(1 - \frac{1}{sec^2 \alpha}\right) = 1 - \sin^4 \alpha$.
7. Show that $\frac{\sin (n+1)\,\alpha - \sin (n-1)\,\alpha}{\cos (n+1)\,\alpha + 2 \cos n\alpha + \cos (n-1)\,\alpha} = \tan \frac{\alpha}{2}$
8. Prove that $Tanh^{-1} x = \frac{1}{2} \log_e \left(\frac{1+x}{1-x}\right)$ for $x \in (-1, 1)$.
9. Show that $(b + c) \cos A + (c + a) \cos B + (a + b) \cos C = a + b + c = 2s$.
10. Show that $z_1 = \frac{2 + 11i}{25}, z_2 = \frac{-2 + i}{(1 - 2i)^2}$ are conjugate to each other.

SECTION - B $5 \times 4 = 20$

SHORT ANSWER TYPE QUESTIONS

Attempt any 5 questions. Each question carries 4 marks.

11. Show that the points with position vectors $-2\mathbf{a} + 3\mathbf{b} + 5\mathbf{c}, \mathbf{a} + 2\mathbf{b} + 3\mathbf{c}, 7\mathbf{a} - \mathbf{c}$ are collinear, where $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are three noncoplanar vectors.
12. $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are vectors of lengths 2, 3, 4 respectively. If $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are perpendicular to $\mathbf{b} + \mathbf{c}, \mathbf{c} + \mathbf{a}, \mathbf{a} + \mathbf{b}$ respectively then show that $|\mathbf{a} + \mathbf{b} + \mathbf{c}| = \sqrt{29}$.
13. If $\frac{\sin (\alpha + \beta)}{\cos (\alpha - \beta)} = \frac{1 - m}{1 + m}$, prove that $\tan\left(\frac{\pi}{4} - \alpha\right) = m \cot\left(\frac{\pi}{4} - \beta\right)$.

14. If $x + y = 2\pi/3$ and $\sin x + \sin y = 3/2$ then find x and y.

15. Solve $Sin^{-1}\frac{5}{x} + Sin^{-1}\frac{12}{x} = \frac{\pi}{2}$.

16. If A, A_1, A_2, A_3 are the areas of incircle and excircles of a triangle respectively, prove that $\frac{1}{\sqrt{A_1}} + \frac{1}{\sqrt{A_2}} + \frac{1}{\sqrt{A_3}} = \frac{1}{\sqrt{A}}$.

17. Show that $2^6 \cos^3\theta \sin^4\theta = \cos 7\theta - \cos 5\theta - 3\cos 3\theta + 3\cos\theta$.

SECTION - C

5 × 7 = 35

LONG ANSWER TYPE QUESTIONS

Attempt any 5 questions. Each question carries 7 marks.

18. If $f : A \to B$, $g : B \to A$ and $f = \{ (1, a), (2, c), (4, d), (3, b) \}$, $g^{-1} = \{ (2, a), (4, b), (1, c), (3, d) \}$ verify $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$.

19. Prove by induction $1\cdot2\cdot3 + 2\cdot3\cdot4 + 3\cdot4\cdot5 + \ldots n$ terms $= n(n+1)(n+2)(n+3)/4$.

20. Show that the points $2\mathbf{a} + 3\mathbf{b} - \mathbf{c}$, $\mathbf{a} - 2\mathbf{b} + 3\mathbf{c}$, $3\mathbf{a} + 4\mathbf{b} - 2\mathbf{c}$, $\mathbf{a} - 6\mathbf{b} + 6\mathbf{c}$ are coplanar.

21. In ΔABC, prove that $\cos^2\frac{A}{2} + \cos^2\frac{B}{2} + \cos^2\frac{C}{2} = 2 + 2\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$.

22. In ΔABC, prove that $\Delta = r_1 r_2 \sqrt{\left[\frac{4R - r_1 - r_2}{r_1 + r_2}\right]}$.

23. The angles of depression of two milestones along a road, when viewed from an aeroplane are α, β. Show that the aeroplane is flying at a height of $\frac{\tan\alpha \tan\beta}{\tan\alpha + \tan\beta}$ from the road level taking the two mile stones to be on opposite side.

24. If n is a positive integer, prove that $(1 + \cos\theta + i\sin\theta)^n + (1 + \cos\theta - i\sin\theta)^n = 2^{n+1}\cos^n\frac{\theta}{2}\cos\frac{n\theta}{2}$.