

**SECTION - A****10 × 2 = 20****VERY SHORT ANSWER TYPE QUESTIONS**

Answer All questions. Each question carries 2 marks.

1. If  $f: R \rightarrow R$ ,  $g: R \rightarrow R$  are defined by  $f(x) = 3x - 2$  and  $g(x) = x^2 + 1$ , then find  $(g \circ f^{-1})(2)$ .
2. Find domain of the function  $\sqrt{4x - x^2}$ .
3. Let  $\mathbf{a} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$  and  $\mathbf{b} = 3\mathbf{i} + \mathbf{j}$ . Find the unit vector in the direction of  $\mathbf{a} + \mathbf{b}$ .
4. Show that the vectors  $\mathbf{i} + \mathbf{j}$ ,  $\mathbf{j} + \mathbf{k}$ ,  $-\mathbf{k} + \mathbf{i}$  are linearly dependent.
5. If  $\mathbf{a} = \mathbf{i} + \mathbf{j} + \mathbf{k}$  and  $\mathbf{b} = 2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$ , find the length of the projection of  $\mathbf{b}$  on  $\mathbf{a}$  and the length of the projection of  $\mathbf{a}$  on  $\mathbf{b}$ .
6. If  $\tan 20^\circ = \lambda$ , prove that  $\frac{\tan 160^\circ - \tan 110^\circ}{1 + \tan 160^\circ \cdot \tan 110^\circ} = \frac{1 - \lambda^2}{2\lambda}$ .
7. Find the maximum and minimum values of  $\sin x \sin(60^\circ + x) \sin(60^\circ - x)$ .
8. If  $\cosh x = 5/2$ , find the value of  $\cosh 2x$  and  $\sinh 2x$ .
9. In  $\Delta ABC$ , express  $\sum r_1 \cot(A/2)$  in terms of 's'.
10. If the amplitude of  $\left(\frac{z-2}{z-6i}\right)$  is  $\frac{\pi}{2}$ , find the equation of locus of  $z$ .

**SECTION - B****5 × 4 = 20****SHORT ANSWER TYPE QUESTIONS**

Attempt any 5 questions. Each question carries 4 marks.

11. Find the vector equation of the plane passing through the points  $4\mathbf{i} - 3\mathbf{j} - \mathbf{k}$ ,  $3\mathbf{i} + 7\mathbf{j} - 10\mathbf{k}$ ,  $2\mathbf{i} + 5\mathbf{j} - 7\mathbf{k}$ . Show that the point  $\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$  lies in this plane.
12. If  $\mathbf{a} = 2\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$ ,  $\mathbf{b} = \mathbf{i} + \mathbf{j} - \mathbf{k}$  and  $\mathbf{c} = \mathbf{i} - \mathbf{j} + \mathbf{k}$ , compute  $\mathbf{a} \times (\mathbf{b} \times \mathbf{c})$  and verify that it is perpendicular to  $\mathbf{a}$ .
13. If  $\cos \theta > 0$ ,  $\tan \theta + \sin \theta = m$ ,  $\tan \theta - \sin \theta = n$  then show that  $m^2 - n^2 = 4\sqrt{mn}$ .
14. If  $\tan\left(\frac{\pi}{2} \sin \theta\right) = \cot\left(\frac{\pi}{2} \cos \theta\right)$ , then show that  $\sin\left(\theta + \frac{\pi}{4}\right) = \pm \frac{1}{\sqrt{2}}$ .

15. Show that  $\tan^{-1} \frac{1}{8} + \tan^{-1} \frac{1}{2} + \tan^{-1} \frac{1}{5} = \frac{\pi}{4}$ .
16. Show that  $a^3 \cos(B - C) + b^3 \cos(C - A) + c^3 \cos(A - B) = 3abc$ .
17. Show that  $\sin^7 \theta = \frac{1}{64} [35 \sin \theta - 21 \sin 3\theta + 7 \sin 5\theta - \sin 7\theta]$ .

### SECTION - C

5 × 7 = 35

#### LONG ANSWER TYPE QUESTIONS

Attempt any 5 questions. Each question carries 7 marks.

18. Let  $f: A \rightarrow B$  and  $g: B \rightarrow C$  be bijections. Prove that  $g \circ f: A \rightarrow C$  is also a bijection.
19. Using the principles of Mathematical Induction, prove that  
 $2 \cdot 3 + 3 \cdot 4 + 4 \cdot 5 + \dots$  upto  $n$  terms  $= \frac{n(n^2 + 6n + 11)}{3}$ , for all  $n \in N$ .
20. If  $\mathbf{a}, \mathbf{b}, \mathbf{c}$  are three unit vectors such that  $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = \frac{1}{2} \mathbf{b}$  then show that  $(\mathbf{a}, \mathbf{b}) = 90^\circ$ ,  $(\mathbf{a}, \mathbf{c}) = 60^\circ$ .
21. If  $A + B + C = 180^\circ$ , prove that  $\cos A + \cos B - \cos C = -1 + 4 \cos \frac{A}{2} \cos \frac{B}{2} \sin \frac{C}{2}$ .
22. In  $\Delta ABC$ , prove that  $r + r_1 + r_2 - r_3 = 4R \cos C$ .
23. From the top of a tree on the bank of a lake, an aeroplane in the sky makes an angle of elevation  $\alpha$  and its image in the river makes an angle of depression  $\beta$ . If the height of the tree from the water surface is ' $a$ ' and that of the height of the aeroplane is ' $h$ ', show that  $h = \frac{a \sin(\alpha + \beta)}{\sin(\beta - \alpha)}$ .
24. Solve the equation  $x^4 + 1 = 0$