SECTION - A $10 \times 2 = 20$

VERY SHORT ANSWER TYPE QUESTIONS

Answer All questions. Each question carries 2 marks.

1. Find the inverse of the function $f: R \rightarrow (0, \infty)$ defined by $f(x) = 5^x$.
2. Find the domain of the function $\sqrt{9 - x^2}$.
3. The position vectors of $\vec{A}$ and $\vec{B}$ are **a**, **b** respectively. If $\vec{C}$ is a point on the line $\overleftrightarrow{AB}$ such that $\vec{AC} = 5\,\vec{AB}$, then find the position vector of $\vec{C}$.
4. Find the vector equation of the straight line passing through the point (2, 3, 1) and parallel to the vector (4, – 2, 3).
5. Find the area of the parallelogram having $2\mathbf{i} - 3\mathbf{j}$ and $3\mathbf{i} - \mathbf{k}$ as adjacent sides.
6. If $A + B + C = \pi/2$, prove that $\tan A \tan B + \tan B \tan C + \tan A \tan C = 1$
7. Show that $\cos^2 76^\circ + \cos^2 16^\circ - \cos 76^\circ \cos 16^\circ = 3/4$.
8. Show that $\tanh^{-1}\left(\frac{1}{2}\right) = \frac{1}{2}\log_e 3$.
9. If the angles of a triangle are in the ratio 1 : 5 : 6, find the ratio of its sides.
10. Show that the triangle formed with the points in the Argand plane represented by $2 + 2i, -2 - 2i, -2\sqrt{3} + 2\sqrt{3}\,i$ is an equilateral triangle.

SECTION - B $5 \times 4 = 20$

SHORT ANSWER TYPE QUESTIONS

Attempt any 5 questions. Each question carries 4 marks.

11. Determine whether the straight line passing through the points $2\mathbf{a} + 3\mathbf{b} - \mathbf{c}$, $3\mathbf{a} + 4\mathbf{b} - 2\mathbf{c}$ and the straight line passing through the points $\mathbf{a} - 2\mathbf{b} + 3\mathbf{c}$, $\mathbf{a} - 6\mathbf{b} + 6\mathbf{c}$ intersect. If so, find the point of intersection.
12. Find the volume of the tetrahedron having the vertices (1, 2, 1), (3, 2, 5), (2, – 1, 0) and (– 1, 0, 1).
13. For any $\alpha \in R$, prove that $\cos^2(\alpha - 45^\circ) + \cos^2(\alpha + 15^\circ) - \cos^2(\alpha - 15^\circ) = \frac{1}{2}$.

14. Solve $\sqrt{3}\cos\theta + \sin\theta = \sqrt{2}$.

15. Show that $\cos\left(2\,Tan^{-1}\frac{1}{7}\right) = \sin\left(4\,Tan^{-1}\frac{1}{3}\right)$.

16. Show that $\Sigma\, a^3 \sin(B - C) = 0$.

17. Show that $\cos 5\theta = \cos^5\theta - 10\cos^3\theta\,.\sin^2\theta + 5\cos\theta.\,\sin^4\theta$.

SECTION - C

5 × 7 = 35

LONG ANSWER TYPE QUESTIONS

Attempt any 5 questions. Each question carries 7 marks.

18. If $f : A \rightarrow B$ be a bijective function, then prove that $f \circ f^{-1} = I_B$ and $f^{-1} \circ f = I_A$.

19. Using the principles of Mathematical Induction, prove that
$2 + 3\cdot 2 + 4\cdot 2^2 + \ldots\ldots$ up to n terms $= n.2^n, \forall\, n \in N$.

20. By Vector method, prove that $\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$.

21. If $A + B + C = 180^\circ$, prove that
$$\cos\frac{A}{2} + \cos\frac{B}{2} + \cos\frac{C}{2} = 4\cos\left(\frac{\pi - A}{4}\right)\cos\left(\frac{\pi - B}{4}\right)\cos\left(\frac{\pi - C}{4}\right)$$

22. In a ΔABC, show that $\frac{r_1}{bc} + \frac{r_2}{ca} + \frac{r_3}{ab} = \frac{1}{r} - \frac{1}{2R}$.

23. From a point on the slope of a hill, the angle of elevation of the top of the hill is 45°. After walking 200 meters from that point on a slope which makes 15° angle with the horizontal, the same point on the top of the hill, makes an angle of elevation of 60°. Show that the height of the hill is $100(\sqrt{6} + \sqrt{2})$ meters.

24. If n is a positive integer, prove that $(1 + i)^n + (1 - i)^n = 2^{(n/2)+1}\cdot\cos\frac{n\pi}{4}$