SECTION - A

$10 \times 2 = 20$

VERY SHORT ANSWER TYPE QUESTIONS

Attempt **ALL** questions. Each question carries **2** marks.

1. If f and g are real valued functions defined by $f(x) = 2x - 1$ and $g(x) = x^2$, then find i) $(fg)(x)$ ii) $(f + g + 2)(x)$

2. Find the domain and range of the function $f(x) = \dfrac{x}{2 - 3x}$.

3. Let $\mathbf{a} = 2\mathbf{i} + 4\mathbf{j} - 5\mathbf{k}$, $\mathbf{b} = \mathbf{i} + \mathbf{j} + \mathbf{k}$, $\mathbf{c} = \mathbf{j} + 2\mathbf{k}$. Find the unit vector in the opposite direction of $(\mathbf{a} + \mathbf{b} + \mathbf{c})$.

4. $OABC$ is a parallelogram. If $OA = \mathbf{a}$ and $OC = \mathbf{c}$, find the vector equation of the side BC.

5. Find the radius of the sphere whose equation is $\mathbf{r}^2 = 2\mathbf{r} \cdot (4\mathbf{i} - 2\mathbf{j} + 2\mathbf{k})$.

6. $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$. Prove that $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$.

7. Find the maximum and minimum values of $\cos\left(x + \frac{\pi}{3}\right) + 2\sqrt{2}\sin\left(x + \frac{\pi}{3}\right) - 3$.

8. If $\sinh x = \frac{1}{2}$, find the value of $\cosh 2x + \sinh 2x$.

9. If $\dfrac{a}{\cos A} = \dfrac{b}{\cos B} = \dfrac{c}{\cos C}$, then show that ΔABC is an equilateral triangle.

10. If $z_1 = -1, z_2 = i$, then find the value of $Arg\left(\dfrac{z_1}{z_2}\right)$.

SECTION - B

$5 \times 4 = 20$

SHORT ANSWER TYPE QUESTIONS

Attempt **any 5** questions. Each question carries **4** marks.

11. If $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are linearly independent vectors, then show that $\mathbf{a} - 2\mathbf{b} + 3\mathbf{c}$, $-2\mathbf{a} + 3\mathbf{b} - 4\mathbf{c}$, $-\mathbf{b} + 2\mathbf{c}$ are linearly dependent.

12. If $\mathbf{a} = 2\mathbf{i} + \mathbf{j} - \mathbf{k}$, $\mathbf{b} = -\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}$ and $\mathbf{c} = \mathbf{i} + \mathbf{j} + \mathbf{k}$ then find $(\mathbf{a} \times \mathbf{b}) \cdot (\mathbf{b} \times \mathbf{c})$.

13. If A is not an integral multiple of π, prove that

$$\cos A \cos 2A \cos 4A \cos 8A = \frac{\sin 16A}{16 \sin A}.$$

14. Find the values of x in $(-\pi, \pi)$ satisfying the equation

$$8^{1 + \cos x + \cos^2 x + \dots \infty} = 4^3.$$

15. Solve the equation $3\, Sin^{-1}\left(\frac{2x}{1+x^2}\right) - 4\, Cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) + 2\, Tan^{-1}\left(\frac{2x}{1-x^2}\right) = \frac{\pi}{3}.$

16. Prove that $\cot \frac{A}{2} + \cot \frac{B}{2} + \cot \frac{C}{2} = \frac{s^2}{\Delta}.$

17. Show that $\frac{\sin 6\theta}{\sin \theta} = 32 \cos^5 \theta - 32 \cos^3 \theta + 6 \cos \theta$ when $\sin \theta \neq 0.$

SECTION - C

5 × 7 = 35

LONG ANSWER TYPE QUESTIONS

Attempt **any 5** questions. Each question carries 7 marks.

18. If $f : A \to B,\ g : B \to C$ be bijections, then show that $g \circ f : A \to C$ is a bijection.

19. Using mathematical induction, prove that

$$1^2 + (1^2 + 2^2) + (1^2 + 2^2 + 3^2) + \dots \text{ upto } n \text{ terms} = \frac{n(n+1)^2(n+2)}{12}.$$

20. Find the shortest distance between the skew lines $\mathbf{r} = (6\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}) + t(\mathbf{i} - 2\mathbf{j} + 2\mathbf{k})$ and $\mathbf{r} = (-4\mathbf{i} - \mathbf{k}) + s(3\mathbf{i} - 2\mathbf{j} - 2\mathbf{k})$.

21. If $A + B + C = 180°$, then prove that $\frac{\sin A + \sin B + \sin C}{\sin A + \sin B - \sin C} = \cot \frac{A}{2} \cot \frac{B}{2}.$

22. If $r_1 = 2, r_2 = 3, r_3 = 6$ and $r = 1$, prove that $a = 3, b = 4$ and $c = 5$.

23. From a point B on the level ground away from the foot of the hill AD, the top of the hill makes an angle of elevation α. From the point B, the point C is reached by moving a distance 'd' along a slant / slope which makes an angle γ with the horizontal. If β is the angle of elevation of the top of the hill from C, find the height of the hill.

24. If n is an integer, then show that $(1+i)^{2n} + (1-i)^{2n} = 2^{n+1} \cos \frac{n\pi}{2}.$