

INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

20th May 2008

Subject CT1 – Financial Mathematics

Time allowed: Three Hours (10.00 – 13.00 Hrs)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

1. *Do not write your name anywhere on the answer sheets. You have only to write your Candidate Number on each answer sheet/s.*
2. *Mark allocations are shown in brackets.*
3. *Attempt all questions, beginning your answer to each question on a separate sheet. However, answers to objective type questions could be written on the same sheet.*
4. *Fasten your answer sheet/s together in numerical order of questions. This, you may complete immediately after expiry of the examination time.*
5. *In addition to this paper you should have available graph paper, Actuarial Tables and an electronic calculator.*

Professional Conduct:

“It is brought to your notice that in accordance with provisions contained in the Professional Conduct Standards, If any candidate is found copying or involved in any other form of malpractice, during or in connection with the examination, Disciplinary action will be taken against the candidate which may include expulsion or suspension from the membership of IAI.”

Candidates are advised that a reasonable standard of handwriting legibility is expected by the examiners and that candidates may be penalized if undue effort is required by the examiners to interpret scripts.

AT THE END OF THE EXAMINATION

Please return your answer sheet/s and this question paper to the supervisor separately.

- Q. 1)** The force of interest $\delta(t)$ is:
 $\delta(t) = 0.007t + 0.0003t^2$ for all t
- (i) At $t = 10$, calculate the accumulated value of an investment of Rs 500 made at time $t = 0$. (3)
- (ii) Calculate the constant annual effective rate of interest over the ten-year period. (2)
- [5]

- Q. 2)** On 1 January 2005, a person holds following two annuities in payment from a life insurance company.
- Annuity (a): Rs.5000 per annum payable monthly, the final payment being on 1 February 2020.
- Annuity (b): Rs.6500 per annum payable quarterly on 1 January, 1 April, 1 July and 1 October each year, the final payment being on 1 January 2015
- (i) Calculate present value of each of these annuities on 1 January 2005 at the rate of interest of 8% per annum effective. (3)
- Immediately after receiving the quarterly payment due on 1 January, the person requested that the two annuities be combined into a single annuity payable half-yearly on 1 March and 1 September in each subsequent year including 2005 with the final annuity payment on 1 September 2024.
- (ii) Find the amount of the revised annuity, given that it was calculated on the basis of an interest rate of 8% per annum effective. (2)
- [5]

- Q. 3)** Calculate an equivalent annual effective rate of interest on an investment that is discounted for 3 months at a simple rate of discount of 7% per annum. [3]

- Q. 4)** The market value of assets of a small pension fund at various times in the calendar year 2007 was as follows:

1 January 2007	1 April 2007	1 July 2007	1 October 2007	31 December 2007
1,000,000	1,160,000	1,200,000	1,280,000	1,260,000

There was a fresh investment of 200,000 on 31 March 2007 into the fund. For 2007, calculate:

- (i) The money-weighted rate of return for the fund (4)
- (ii) The time-weighted rate of return for the fund (1)
- (iii) The linked quarterly rate of return for the fund (1)
- [6]

- Q. 5)**
- (i) List main features of Eurobonds (3)
 - (ii) List main features of Index linked government bonds (3)
 - (iii) Describe how cashflows are exchanged in an interest rate swap. (2)
 - (iv) Describe briefly the risks faced by counterparty to an interest rate swap. (2)
- [10]**
- Q. 6)** An investor purchased a bond with exactly 15 years to redemption. The bond is redeemable at par and has a gross redemption yield of 5% per annum effective. It pays coupons of 4% per annum, half yearly in arrear. The investor pays tax at 25% on the coupons only.
- (i) Calculate the price paid for the bond. (3)
 - (ii) After exactly eight years, immediately after the payment of the coupon then due, this investor sells the bond to another investor who pays income tax at a rate of 25% and capital gains tax at a rate of 40%. The bond is purchased by the second investor to provide a net return of 6% per annum effective.
- Calculate the price paid by the second investor. (5)
- [8]**
- Q. 7)**
- (i) State the three conditions for Redington's immunization against small changes in the rate of interest. (3)
 - (ii) A pension fund has liabilities of Rs.5 million due in 17 years time and Rs. 8 million in 20 years' time. The fund manager of the fund has only two zero coupon assets to invest, one maturing in 15 years and another in 23 years. The current interest rate is 8 % per annum effective.
- Calculate the amount the manager will invest in each fund so that first two conditions for immunization against small changes in the rate of interest is met. (6)
- (iii) List four limitations of Redington's immunization theory to apply in practice. (4)
- [13]**
- Q. 8)** An investor purchases an ordinary share of a company at the price of Rs.100. The company pays annual dividends. The next dividend is expected to be of Rs.6 per share and is due in exactly one year's time. It is expected that subsequent dividends will grow at rate of 8% per annum and that inflation will be 5% per annum. The investor is expected to sell the security just after receiving 5th dividend at Rs.150.
- Calculate the expected effective real rate of return per annum for the investor. (9)
- [9]**
- Q. 9)** Rs.10, 000 is invested for 10 years. In any year the yield on the investment will be 5% with probability 0.4, 7% with probability 0.2 and 9% with probability 0.4 and is independent of the yield in any other year.

- (i) Calculate the mean accumulation at the end of 10 years. (2)
- (ii) Calculate the standard deviation of the accumulation at the end of 10 years. (5)
- (iii) Without carrying out any further calculations, explain how your answers to (i) and (ii) would change (if at all) if:
the yields had been 6%, 7% and 8% instead of 5%, 7% and 9% per annum, respectively; (2)
- [9]**

Q. 10) A housing finance company loans Rs. 1,500,000 to an individual. The loan is to be repaid by level monthly installments in arrears over a period of 20 years. After 10 years, the amount of level installment increases by 20% of that in first 10 years. The installments are such that the borrower pays interest at an interest rate of 12% per annum convertible quarterly on the loan.

- (i) Calculate the level monthly installment payable in first 10 years and that in last 10 years. (4)
- (ii) Calculate the total capital repayment and total interest payment that will be made from the monthly installments paid during the period from 11th years to 15th years. (6)
- [10]**

Q. 11) (i) State what is meant by a “forward contract”. Your answer should include reference to the terms “short forward position” and “long forward position”. (3)

- (ii) An investor bought a 5-year forward contract on 1 June 2004 to buy Rs.400 nominal of a stock that pays coupons of 5 % pa payable half yearly on 31 March, 30 September. The stock is also expected to pay out a lump sum of Rs.50% of nominal value on 1 June 2008. The stock is expected to yield 5.5% pa effective if purchased on 1 June 2004 and held forever.

Calculate the forward price for the contract, given that the risk free rate of interest is 6% per annum. (5)

[8]

Q. 12) (i) Explain what is meant by the following terms:

- (a) Equation of value (1.5)
- (b) Discounted payback period from an investment project (1.5)

- (ii) A consortium of investors is considering starting a major sport venture of hosting twenty-twenty cricket league in India where players from different cricket playing nations will participate through six teams representing big cities of India. The venture is expected to start on 1 July 2008. The consortium estimates that the following cash flows will be generated by the event (all figures in Rs. Crores):

Costs

Initial Setup costs and initial costs of enrolling the players:

Rs. 390 crore on 1 July 2008

Cost of building infrastructure:

To be incurred continuously at the rate of 20 crore per month for next 6 months starting from 1 July 2008

Running costs of the event:

To be incurred at the end of the month of Rs. 1 crore per month for 5 years starting on 1 January 2009 increasing every month by 0.5% per month.

Revenue

Sale of television rights:

To be received continuously at a rate of 25 crore per month for 6 months starting on 1 July 2008.

Other revenue from sale of tickets, merchandise, marketing rights etc.:

Assumed to be received in the each month from January 2009 to December 2013 at the end of month. The revenue from this source is expected to start at Rs. 4 crore per month and increase each month by 1% per month.

Revenue from sale of venture:

This will be received on 1 January 2014, the expected time to sell the venture.

The project will be regarded as viable if it provides a positive net present value at a rate of interest of 18% per annum convertible monthly. Determine how much sales proceeds would have to be achieved from selling the venture for the project to be considered viable.

(11)
[14]
