

B. E.
Seventh Semester Examination, May-2009
OPERATIONS RESEARCH

Note : Attempt any five questions. All questions carry equal marks.

Part-A

Q. 1. (a) What is operation research? Account for the growing importance of operation research in business decisions.

Ans. Historical Development of Operations Research (OR) :

While it is difficult to mark the 'beginning' of the operations research/management science, the scientific approach to management can be traced back to the era of Industrial Revolution and even to periods before that. But operations research, as it exists today, was born during the Second World War when the British military management called upon a group of scientists to examine the strategies and tactics of various military operations with the intention of efficient allocation of scarce resources for the war effort. The name operational research was derived directly from the context in which it was used—research activity on operational areas of the armed forces. British scientists spurred the American military management to similar research activities (where it came to be known as operations research). Among the investigations carried out by them were the determination of (i) optimum convoy size to minimise losses from submarine attacks, (ii) the optimal way to deploy radar units in order to maximise potential coverage against possible enemy attacks, and (iii) the invention of new flight patterns and the determination of correct colour of the aircraft in order to minimise the chance of detection by the submarines.

After the war, operations research was adopted by the industry and some of the techniques that had been applied to the complex problems of war were successfully transferred and assimilated for use in the industrialised sector.

The dramatic development and refinement of the techniques of operations research and the advent of digital computers are the two prime factors that have contributed to the growth and application of OR in the post-war period. In the 1950s OR was mainly used to handle management problems that were clear cut, well-structured and repetitive in nature. Typically, they were of a tactical and operational nature such as inventory control, resource allocation, scheduling of construction projects, etc. Since the 1960s, however, formal approaches have been increasingly adopted for the less well-structured planning problems as well. These problems are strategic in nature and are the ones that affect the future of the organisation. The development of corporate planning models and those relating to the financial aspects of the business, for example, are such type of problems. Thus, in the field of business and industry, operations research helps the management to determine their tactical and strategic decisions more scientifically.

Q. 1. (b) State the phases of an OR study and their importance in solving problems.

Ans. Nature and Characteristic Features of OR :

In general terms, operations research attempts to provide a systematic and rational approach to the fundamental problems involved in the control of systems by making decisions which, in a 'sense, achieve the best results considering all the information that can be profitably used. A classical definition of OR is given by Churchman et al, "...Operations Research is the application of scientific methods, techniques and tools to problems involving the operations of systems so as to provide those in control of operations with optimum solutions to the problems."* Thus, it may be regarded as the scientific method employed for problem solving and decision-making by the management.

The significant features of operations research are given below :

1. Decision Making :

Primarily, OR is addressed to managerial decision making or problem solving. A major premise of OR is that decision-making, irrespective of the situation involved, can be considered as a general systematic process that consists of the following steps :

- (a) Define the problem and establish the criterion which will be used. The criterion may be the maximisation of profits, utility and minimisation of costs, etc.
- (b) Select the alternative courses of action for consideration.
- (c) Determine the model to be used and the values of the parameters of the process.
- (d) Evaluate the alternatives and choose the one which is optimum.

2. Scientific Approach :

OR employs scientific methods for the purpose of solving problems. It is a formalised process of reasoning and consists of the following steps :

- (a) The problem to be analysed is defined clearly and the conditions for observations are determined.
- (b) Observations are made under varying conditions to determine the behaviour of the system.
- (c) On the basis of the observations, a hypothesis describing how the various factors involved are believed to interact and the best solution to the problem is formulated.
- (d) To test the hypothesis, an experiment is designed and executed. Observations are made and measurements are recorded.
- (e) Finally, the results of the experiments are analysed and the hypothesis is either accepted or rejected. If the hypothesis is accepted, the best solution to the problem is obtained.

3. Objective :

OR attempts to locate the best or optimal solution to the problem under consideration. For this purpose, it is necessary that a measure of effectiveness is defined which is based on the goals of the organisation. This measure is then used as the basis to compare the alternative courses of action.

4. Inter-disciplinary Team Approach :

OR is inter-disciplinary in nature and requires a team approach to a solution of the problem. No single individual can have a through knowledge of the myriad aspects of operations research and how the problems may be addressed. Managerial problems have economic, physical, psychological, biological, sociological and engineering aspects. This requires a blend of people with expertise in the areas of mathematics, statistics, engineering, economics, management, computer science and so on. Of course, it is not always so. Some problem situations may be adequately handled even by one individual.

5. Digital Computer :

Use of a digital computer has become an integral part of the operations research approach to decision-making. The computer may be required due to the complexity of the model, volume of data required or the computations to be made. Many OR techniques are available in the form of 'canned' programmes.

Q. 2. (a) Solve the following LPP by graphical method. Maximize $Z = 2X_1 + 4X_2$

subject to :

$$2X_1 + X_2 \leq 18 ; 3X_1 + 2X_2 \geq 30$$

$$X_1 + 2X_2 = 26 ; X_1, X_2 \geq 0$$

Ans. Maximize $Z = 2X_1 + 4X_2$

Subject to,

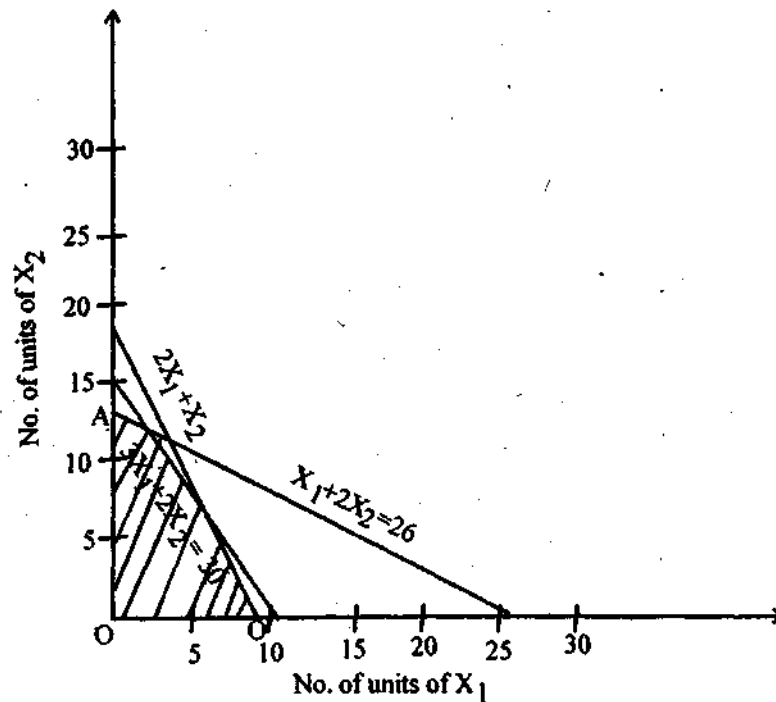
$$2X_1 + X_2 \leq 18$$

$$3X_1 + 2X_2 \geq 30$$

$$X_1 + 2X_2 = 26$$

$$X_1, X_2 \geq 0$$

We can solve the problem graphically. The problem is depicted graphically in fig.



Here the feasible region is shown as the shaded area and therefore points A, B C & D would yield the optimal solution. We have,

At	x_1	x_2	$Z = 2X_1 + 4X_2$
A	0	13	52
B	2	12	52
C	6	6.5	38
D	9	0	18

It is clear that Z, equal to 52, is maximum at point A & B.

Where for $x_1 = 0$ & $x_2 = 13$

For B $x_1 = 2$ & $x_2 = 12$.

Q. 2. (b) Write the dual of the above problem.

Ans. For the dual of the above problem. Here only first equation in the right direction (being \leq type with a maximisation type of objective function) while the second equation is not. Multiplying both sides by -1 , this can be written as $-3x_1 - 2x_2 \geq -30$. Now, we can write the dual as follows :

Dual,

$$\text{Minimize } G = 18y_1 - 30y_2 + 26y_3$$

Subject to,

$$2y_1 + 3y_2 + y_3 \leq 2$$

$$y_1 - 3y_2 - 2y_3 \leq 4$$

$$y_1, y_2, y_3 \geq 0.$$

Q. 3. (a) Describe the transportation problem and give its mathematical model.

Ans. The classical transportation problem can be stated mathematically as follows :

Let a_i = quantity of product available at origin i

b_j = quantity of product required at destination j

c_{ij} = the cost of transporting one unit of product from source/origin i to destination j

x_{ij} = the quantity transported from origin i to destination j

Assume that $\sum_{i=1}^m a_i = \sum_{j=1}^n b_j$ which means that the total quantity available at the origins is precisely equal

to the total amount required at the destinations.

With these, the problem can be stated as a linear programming problem as :

Minimise Total Cost,

$$Z = \sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij}$$

Subject to

$$\sum_{j=1}^n x_{ij} = a_i \quad \text{for } i = 1, 2, \dots, m$$

$$\sum_{i=1}^m x_{ij} = b_j \quad \text{for } j = 1, 2, \dots, n$$

and $x_{ij} \geq 0$ for all $i = 1, 2, \dots, m$, and $j = 1, 2, \dots, n$

The transportation model can also be portrayed in a tabular form by means of a transportation tableau, shown in Table 5.1.

This tableau can be thought of as a matrix within a matrix, of the dimension $m \times n$. The one is the per unit cost matrix which represents the unit transportation costs for each of the possible transportation routes. Its elements are given by C_{ij} indicating the cost of shipping a unit from the i th origin to the j th destination. Superimposed on this matrix is the matrix in which each cell contains a transportation variable—that is, the number of units shipped from the row-designated origin to the column-designated destination. Each such variable, is represented by x_{ij} , the amount shipped from i th source to j th destination. Right and bottom sides of the transportation tableau show, respectively, the amount of supplies a_i available at source i and the amount demanded b_j at each destination j . The a_i 's and b_j 's represent the supply and demand constraints.

The aggregate transportation cost is determined by multiplying the various x_{ij} 's with corresponding c_{ij} 's and then adding them up all.

Origin	Destination (j)				Supply, a_i
	1	2	...	n	
1	x_{11} c_{11}	x_{12} c_{12}	x_{1n} c_{1n}	a_1
2	x_{21} c_{21}	x_{22} c_{22}	x_{2n} c_{2n}	a_2
....
m	x_{m1} c_{m1}	x_{m2} c_{m2}	x_{mn} c_{mn}	a_m
Demand, b_j	b_1	b_2	b_n	$\sum a_i = \sum b_j$

Q. 3. (b) A company has four factories supplying 15, 20, 30 and 35 units respectively and three warehouses demanding 25, 26 and 50 units respectively. The following table gives transportation cost per unit (in rupees) from factories to warehouses :

	W_1	W_2	W_3
F_1	10	8	9
F_2	5	2	3
F_3	6	7	4
F_4	7	6	8

Ans.

	W_1	W_2	W_3	Supply
F_1	10	8	9	15
F_2	5	2	3	20
F_3	6	7	4	30
F_4	7	6	8	35
Demand	25	26	50	

Iteration 1 :

W_1	W_2	W_3	Supply
	(25)	(-10)	
F_1	10	-8	9
		(36)	(-10)
F_2	5	2	3
	(5)		(-16)
F_3	6	7	4
		(5)	
F_4	7	6	8
			(2)

Demand	25	26	50
V_j	10	5	2

$$\text{Total cost} = 10 \times 25 + 8 \times 10 + 2 \times 36$$

$$= 402$$

W_1	W_2	W_3	Supply	
	(25)		(-10)	
F_1	10	8	9	15
	(5)	(26)		
F_2	5	2	3	20
		(4)		
F_3	6	7	4	30
	(5)	(4)	(1)	
F_4	7	6	8	35
Demand	25	26	50	
V_j	10	8	9	

$$\text{Total Cost} : 10 \times 25 + 26 \times 2 + 10 \times 9$$

$$= 392$$

The optimality test indicates that the solution is optimal.

Q. 4. (a) Differentiate between PERT and CPM.

Ans. The use of the PERT and CPM techniques is made in both planning and controlling of the projects. Planning in this context implies developing the overall layout of the project with estimates of the time and resources required and the detailed scheduling of the timing and sequence of various jobs to be performed. The control, on the other hand, takes place during the work on the project. Gradually as actual resource use and completion times are obtained, project management techniques can be used to reallocate, if necessary, the resources, according to the revised criticality rankings of the jobs remaining to be done. In particular, these techniques help the project managers to determine the expected project completion date; the scheduled start and completion time for the different activities comprising the project; the key activities of the project which must be completed at the scheduled time; the time period by which the non-key activities may be delayed without causing a delay in the completion of the whole project, etc.

Both the techniques use similar terminology and have the same general purpose, but they were developed independently of each other, during the late 1950s. PERT was developed and used in conjunction with the planning and designing of the Polaris Submarine System. It was credited with saving substantial time and cost on Polaris, which caused its use to be made mandatory on all the significant development projects undertaken for and on behalf of the Department of Defence of the United States. The CPM, on the other hand, was developed by the Du Pont Company and the Univac Division of Remington Rand Corporation as a device to control the maintenance of chemical plants.

While both PERT and CPM have similarities in terms of concepts and methodology, a basic difference exists between the two techniques. PERT is useful for analysing project schedule problems in which the completion time of the different activities, and therefore the whole project, is not certain. It thus emphasizes the uncertainties of the completion times of the activities. On the other hand, CPM is most appropriately used in projects in which the activity durations are known with certainty. Not only the amount of time needed to complete the various facets of the project but also the amounts of resources required for performing each of the activities are assumed to be known. This technique is basically concerned with obtaining the trade-offs between the project duration and cost. Thus, whereas variation in the project time is inherent in the project where PERT is used, the time is systematically varied (using additional resources) where CPM is employed. In essence, then, while PERT is probabilistic in nature and as such is used more in research and development projects, the CPM is a deterministic technique and thus finds application mostly in the construction project.

As seen earlier, widely diverse kind of projects can be analyzed by the techniques of PERT and CPM. In fact they are suitable for any situation where :

- (a) The project consists of well-defined collection of activities or tasks.
- (b) The activities can be started and terminated independently of each other, even if the resources employed on the various activities are not independent.
- (c) The activities are ordered so that they can be performed in a technological sequence. Thus precedence relationships exist which preclude the start of certain activities until others are completed. For instance, road levelling cannot start unless the road bed is laid.

Q. 4. (b) Solve the assignment problem for optimal solution. Figures in the matrix indicate profits :

	A	B	C	D	E
1	30	37	40	28	40
2	40	24	27	21	36
3	40	32	33	30	35
4	25	38	40	36	36
5	29	62	41	34	39

Ans.

30	37	40	28	40
40	24	27	21	36
40	32	33	30	35
25	38	40	36	36
29	62	41	34	39

Iteration 1 :

2	9	12	0	12
19	3	6	0	15
10	2	3	0	5
0	13	15	11	11
0	33	12	5	10

Iteration 2 :

0	7	10	0	10
16	0	3	0	12
8	0	1	0	3
0	2	4	0	0
0	28	7	0	5

Since the number of lines covering all zeros is less than the number of columns/rows, we modify the iteration-2. The least of the uncovered all value is 3. Accordingly, the new table would be as iteration 3.

Iteration 3:

0	7	10	3	7
10	0	3	8	9
7	8	1	8	0
8	2	0	8	8
8	28	7	0	2

The optimal assignment can be made as the least number of lines covering zeros equal to 5. Considering rows and columns, the assignments can be made in the order as indicated above. The total cost associated with the optimal. Machinist job assignment pattern.

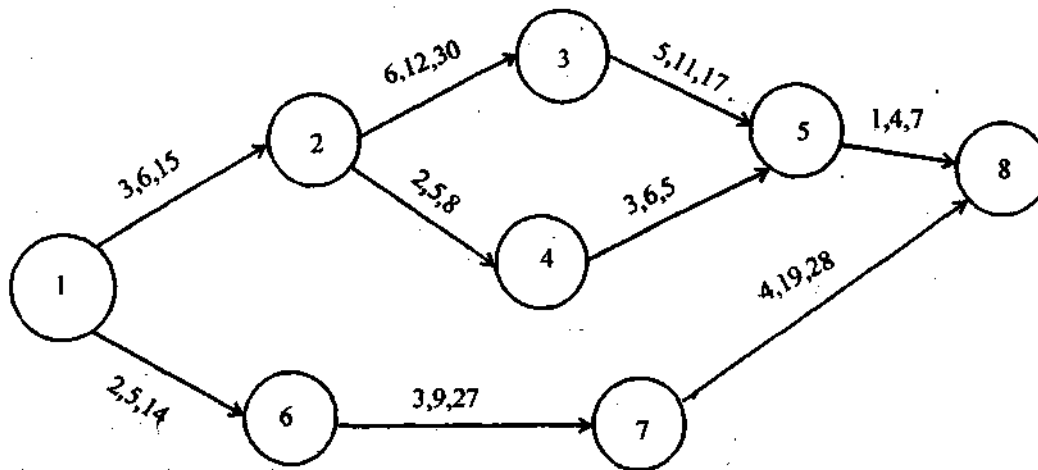
$$\text{Total cost} = 30 + 24 + 36 + 40 + 34 = 164$$

Q. 5. Based on the activities given below :

- Draw the network diagram
- Determine the Critical path
- Determine project completion time
- Determine total float for each activity

Activity	1-2	1-6	2-3	2-4	3-5	4-5	5-8	6-7	7-8
Optimistic Time	3	2	6	2	5	3	1	3	4
Most Likely Time	6	5	12	5	11	6	4	9	19
Pessimistic Time	15	14	30	8	17	15	7	27	28

Ans. The arrow diagram corresponding to the given information is depicted in fig. Also given are the expected times of the various activities as shown circulated in the table.



Activity	a	m	b	$t_{ei} = \frac{a+4mtb}{6}$	$\sigma_i = \frac{b-a}{6}$	σ_i^2
1-2	3	6	15	7	2	4
1-6	2	5	14	6	2	4
2-3	6	12	30	14	4	16
2-4	2	5	8	5	1	1
3-5	5	11	17	11	2	4
4-5	3	6	15	7	2	4
5-8	1	4	7	4	1	1
6-7	3	9	27	11	4	16
7-8	4	19	28	18	4	16

Using the expected times of activity duration, we obtain the critical path as 1-2-3-5-8. Thus, we have the expected project length,

$$T_e = 3+6+5+1=15$$

And the variance of the project length,

$$V_T = 4 + 16 + 4 + 1 = 25$$

Now, the project duration being normally distributed with mean = 15 and standard deviation $\sigma = (\sqrt{V_T}) = 5$, we can determine the probability of the project.

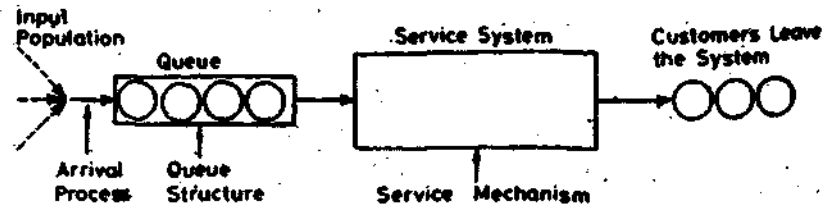
Q. 6. (a) Give the role of queuing theory in decision making and discuss its application.

Ans. The queuing theory, also called the waiting line theory, owes its development to A K Erlang's efforts to analyse telephone traffic congestion with a view to satisfying the randomly arising demand for the services of the Copenhagen automatic telephone system, in the year 1909. The theory is used in situations where the 'customers' arrive at some 'service station(s)' for some service; wait (occasionally not); and then leave the system after getting the service. In such 'arrival and departure' problems, the customers might be people waiting to deposit their electricity bills at a cash counter, machines waiting to be repaired in a factory's repair shop, aeroplanes waiting to land at an airport, patients in a hospital who need treatment ... and so on. The service stations in such problems are the cash counters in the electricity office, repairmen in the shop, runways at the airport and doctors attending the patients, respectively. Some more examples of queuing situations are given in Table 9.1.

The waiting lines develop because the service to a customer may not be rendered immediately as the customer reaches the service facility. Thus, lack of adequate service facility would cause waiting lines of customers to be formed. The only way that the service demand can be met with ease is to increase the service capacity (and raising the efficiency of the existing capacity if possible) to the existing level. The capacity might be built to such high level as can always meet the peak demand with no queues. But adding to capacity may be a costly affair and uneconomic after a stage because then it shall remain idle to varying degrees when there are no or few customers. A manager, therefore, has to decide on an appropriate level of service which is neither too low nor too high. Providing too low service would cause excessive waiting which has a cost in terms of customer frustration, loss of goodwill in the long run, direct cost of idle employees (where, for example, the employees have to wait near the store to obtain the supplies of materials, parts or tools needed for their work) or loss associated with poor employee morale resulting from being idle. On the other hand, too high a service level would result in very high set up cost and idle time for the service station(s). Thus, the goal of queuing modelling is the achievement of an economic balance between the cost of providing service and the cost associated with the wait required for that service.

General structure of a queuing system.

The general structure of a queuing system is depicted in figure



Situation	Arriving Customers	Service Facility
(a) Passage of customers through a supermarket checkout	Shoppers	Checkout counters
(b) Flow of automobile traffic through a road network	Automobiles	Road network
(c) Transfer of electronic messages	Electronic messages	Transmission lines
(d) Banking transactions	Bank patrons	Bank tellers
(e) Flow of computer programme through a computer system	Computer programmes	Central processing unit
(f) Sale of theatre tickets	Theatre goers	Ticket booking windows
(g) Arrival of trucks to carry fruits and vegetables from a central market	Trucks	Loading crews and facilities
(h) Registration of unemployed at employment exchange	Unemployed personnel	Registration assistants

(i) Occurrences of fires	Fires equipment	Firemen and
(j) Flow of ships to the seashore	Ships	Harbour and docking facilities
(k) Calls at police control room	Service calls	Policeman

Q. 6. (b) The manager of a bank observes that, on an average, 18 customers are served by a cashier in an hour. Assuming that the service time has an exponential distribution, what is the probability that?

(i) A customer shall be free within 3 minutes

(ii) a customer shall be serviced in more than 12 minutes.

Ans. (a) Here

$$\mu = 18 \text{ customers/hour}$$

$$T = 3 \text{ minutes} = 0.05 \text{ hour}$$

$$\therefore P(\text{less than 3 minutes}) = 1 - 2.7183^{-(18)(0.05)} = 0.593$$

(b) With

$$\mu = 18 \text{ customers/hour,}$$

$$T = 12 \text{ minutes} = 0.20 \text{ hour,}$$

$$\begin{aligned} P(\text{more than 12 minutes}) &= e^{-\mu T} \\ &= 2.7183^{-(18)(0.20)} \\ &= 0.027. \end{aligned}$$

Q. 7. (a) What is simulation? Describe the simulation process.

Ans. Using simulation, an analyst can introduce the constants and variables related to the problem, set up the possible courses of action and establish criteria which act as measures of effectiveness. The benefit of simulation from the viewpoint of the analyst stems from the fact that the results of taking a particular course of action can be estimated prior to its implementation in the real world. Instead of using hunches and intuition to determine what may happen, the analyst using simulation can test and evaluate various alternatives and select the one that gives the best results.

Process of simulation :

Broadly, there are four phases of the simulation process. They are :

(a) Definition of the problem and statement of objectives.

- (b) Construction of an appropriate model,
- (c) Experimentation with the model constructed; and
- (d) evaluation of the results of simulation.

Each of the phases calls for the performance of a number of preliminary tasks. Of these, the two major tasks are collection of data and selection of means by which the simulation activity would replicate the random behaviour of the real world.

The first step in problem solving of any situation is to identify and clearly define the problem and list the objective(s) that the solution is intended to achieve. This is true of simulation as well. A clear statement, not only facilitates the development of an appropriate model but also provides a basis for evaluation of the simulation results. In general, simulation aims to determine how the system under consideration would behave under certain conditions. Naturally, the more specific the analyst is about what he is looking for, the greater the chances that the simulation model will be designed to accomplish that. Thus, the scope and the level of detail of the simulation should be decided upon carefully.

The next step in simulation is the development of a suitable model. During the course of a simulation, the model mimics the important elements of what is being simulated. A simulation model may be a physical or mathematical model, a mental conception or a combination. Many simulations involve physical models. Examples include a scaled down model of an aeroplane or ship constructed out of wood or other material. Since physical models are relatively expensive to build, mathematical models are often preferred. In such a model, mathematical symbols or equations are used to represent the relationships in the system.

Collection of data is a significant aspect of model development, and the quantum and type of data needed are directly governed by the scope and extent of the detail of the simulation. The data are needed both for model development and evaluation. Obviously, the model for simulation must be so designed that it would enable evaluation of the key decision alternatives. An ancillary step here is of designing experiments. The experiments help answer the 'what if ...' types of questions in simulation studies. By going through this process the analyst is able to learn about the system behaviour.

Q. 7. (b) Distinguish between deterministic and stochastic simulation models.

Ans. Once the simulation model is developed, the next step is to run it. If the model is deterministic, with all its parameters known and constant, then only a single run would suffice. On the other hand, if the simulation is stochastic in nature, with the parameters subject to random variation, then a number of runs would be needed to get a clear picture of the model performance. The probabilistic simulation is akin to the random sampling where each run represents one observation. Thus, statistical theory can be used to determine the optimal sample sizes. Evidently, the greater the variability inherent in the simulation results, the larger would be the simulation runs needed to obtain a reasonable degree of confidence that the results are truly indicative of the system behaviour.

The last step in the process of simulation is to analyse and interpret the results of the runs. The interpretation of results is, in a large measure, dependent on the extent to which the simulation model portrays the reality.

Obviously, closer the approximation of the real system by the simulation model, lesser will be the need for adjusting the results and also lesser will be the risk inherent in applying the results.

Monte Carlo simulation :

Although simulation can be of many types, our discussion will focus on the probabilistic simulation using the Monte Carlo method. Also called computer simulation, it can be described as a numerical technique that involves modelling a stochastic system with the objective of predicting the system's behaviour. The chance element is a very significant feature of Monte Carlo simulation and this approach can be used when the given process has a random, or chance, component.

In using the Monte Carlo method, a given problem is solved by simulating the original data with random number generators. Basically, its use requires two things. First, as mentioned earlier, we must have a model, that represents an image of the reality of the situation. Here the model refers to the probability distribution of the variable in question. What is significant here is that the variable may not be known to explicitly follow any of the theoretical distributions like Poisson, Normal, etc.. The distribution may be obtained by direct observation or from past records. To illustrate, suppose that a bakery keeps a record of the sale of the number of cakes of a certain type. Information relating to 200 days' sales is,

Demand (No. of cakes)	5	6	7	8	9	10	11	12	Total
(No. of cakes)									
No. of days :	4	10	16	50	62	38	12	8	200

Assuming that this is an adequate representation of the distribution of demand for the cake, we can derive the probability distribution of demand by expressing each of the frequencies in terms of proportions. This is done by dividing each one of the values by 200-the total frequency. The resultant distribution follows :

Demand :	5	6	7	8	9	10	11	12	
(No. of cakes)									
Probability :	.02	.05	.08	.25	.31	.19	.06	.04	

Q. 8. Define 'decision.' Explain the illustrate the following principles of decision making :

- (i) Laplace
- (ii) Maximin
- (iii) Maximax
- (iv) Hurwicz

Ans. The decision theory, also called the decision analysis, is used to determine optimal strategies where a decision maker is faced with several decision alternatives and an uncertain, or risky, pattern of future events. To recapitulate, all decision making situations are characterized by the fact that two or more alternate courses of action are available to the decision maker to choose from. Further, a decision may be defined as the selection by the decision maker of an act, considered to be best according to some predestinated standard, from among the available options. The decision making process, thus, involves the following steps :

- (a) Identification of the various possible outcomes, called states of nature or events, E_i 's, for the decision problem. The events are beyond the control of the decision maker.
- (b) Identification of all the courses of action, A_j 's, or the strategies that are available to the decision maker. The decision maker has control over choice of these.
- (c) Determination of the pay-off function which describes the consequences resulting from the different combinations of the acts and events. The pay-offs may be designated as V_{ij} 's—the pay-off resulting from i th event and j th strategy.
- (d) Choosing from among the various alternatives on the basis of some criterion, which may involve the information given in step (c) only or which may require and incorporate some additional information.

(i) Laplace principle :

The Laplace Principle is based on the simple philosophy that if we are uncertain about the various events then we may treat them as equally probable. Under this assumption, the expected (mean) value of pay-off for each strategy is determined and the strategy with highest mean value is adopted. Of course, if the pay-offs are in terms of costs, we choose the strategy with the lowest average cost.

Act	Mean (Expected) Pay-off
A_1	$(360 + 360 + 360 + 360 + 360 + 360)/6 = \text{Rs } 360.0$
A_2	$(310 + 380 + 380 + 380 + 380 + 380)/6 = \text{Rs } 368.3$
A_3	$(260 + 330 + 400 + 400 + 400 + 400)/6 = \text{Rs } 365.0$
A_4	$(210 + 280 + 350 + 420 + 420 + 420)/6 = \text{Rs } 350.0$
A_5	$(160 + 230 + 300 + 370 + 440 + 440)/6 = \text{Rs } 323.3$
A_6	$(110 + 180 + 250 + 320 + 390 + 460)/6 = \text{Rs } 285.0$

Since the expected pay-off for A_2 is the maximum, it would be adopted. Thus, the bookstore manager would buy 19 copies of the book if he chooses to adopt the Laplace rule for taking decision.

(ii) Maximin or minimax principle :

This principle is adopted by pessimistic decision makers who are conservative in their approach. Using this approach the minimum pay-offs resulting from adoption of various strategies are considered and among

these values the maximum one is selected. It involves therefore, choosing the best (the maximum) profit from the set of worst (the minimum) profits.

When dealing with the costs, the maximum cost associated with each alternative is considered and the alternative which minimizes this maximum cost is chosen. In this context, therefore, the principle used is minimax-the best (the minimum cost) of the worst (the maximum cost).

For our example, the minimum profit associated with various strategies is as follows :

A_1 : Rs. 360	A_3 : Rs. 260	A_5 : 160
A_2 : Rs 310	A_4 : Rs 210	A_6 : Rs 110

Since the maximum of these is Rs 360, the strategy A_1 is selected corresponding to the maximin principle of choice.

(iii) Maximax of minimin principle :

The maximax principle is optimists' principle of choice. It suggests that for each strategy, the maximum profit should be considered and the strategy with which the highest of these values is associated should be chosen. The optimist obviously desires a chance for the maximum pay-off in the decision matrix.

For example the maximum pay-off associated with the different strategies is as follows :

A_1 : Rs. 360	A_3 : Rs. 400	A_5 : Rs. 440
A_2 : Rs. 380	A_4 : Rs. 420	A_6 : Rs. 460

The highest profit being Rs 460, strategy A_6 of ordering 23 copies of the book is the decision according to the maximax principle.

In decision problems dealing with costs, the minimum cost for each alternative is considered and then the alternative which minimizes the minimum cost is selected. The principle in this case is obviously termed minimin.

(iv) Hurwicz principle :

The Hurwicz principle of decision making stipulates that a decision maker's view may fall somewhere between the extreme pessimism of the maximin principle and the extreme optimism of the maximax principle. This principle provides a mechanism by which different levels of optimism and pessimism may be shown. For this an index of optimism, α , is defined on scale ranging from 0 to 1. An $\alpha = 0$ indicates extreme pessimism while $\alpha = 1$ represents extreme optimism.

For taking a decision using this principle, first the decision maker's degree of optimism is indicated on the scale. Assuming that the decision maker is able to reflect a degree of optimism by assigning a particular value of α , we multiply the maximum profit for each strategy A_j by α , and the minimum profit for it by $1 - \alpha$. The sum of these products, called the Hurwicz Criterion, is obtained for each strategy and we select the alternative which maximizes this quantity. Obviously, when $\alpha = 0$, only the minimum of the profits for each strategy would be considered and the decision would in effect be according to the maximin criterion. Similarly, when $\alpha = 1$, the decision would be identical to that arrived through the maximax principle.