This question paper contains 4 printed pages.]

Your Roll No . .

6176

B.Sc. (Hons.) Computer Science / I Sem. J

Paper – 102 : Discrete Structures

(Admissions of 2001 and onwards)

Time : 3 Hours **Maximum Marks : 75**

(Write your Roll No on the top immediately on receipt of this question paper)

Note : Attempt **all** questions
Parts of a question must be answered together

1 (a) A palindrome is a word that reads the same forward or backward How many seven-letter palindromes can be made out of the English alphabet ? 2

(b) Give the first six numbers when we list the four digit permutations of $x = \{1, 3, 5, 7, 8\}$ in lexicographic ordering 3

2 (a) Use Huffman Method to construct an optimal binary prefix code for the following symbols with given frequencies · 4
a 20 b 10 c . 15 d 25 e 30
What is the average number of bits required to encode ?

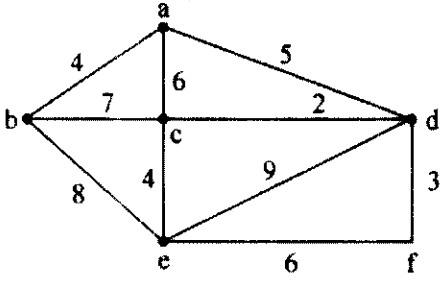
(b) Show that a connected graph with n vertices and (n – 1) edges, does not have any cycles 3

(c) What is the out degree of the root and leaves of a regular m-ary tree ? Derive the minimum height of an n-node regular m-ary tree 3

3 (a) How many edges does an undirected complete graph, K_n have ? Consider an undirected graph G = (V, E) with n vertices and m edges Lets $G' = (V, E')$ be the complement of G with respect to K_n How many edges does G' has ? 2

(b) Show that in a linear connected planar graph with 16 edges and 10 vertices, each of the regions is bounded by 4 edges 3

(c) Find the shortest path between 'a' and 'f' in the following graph : 4



Show all the steps

4 (a) Consider the following numeric function, a

where $a_r = \begin{cases} 1 & 0 \le r \le 12 \\ 2 & r \ge 13 \end{cases}$ 3

Compute $b = S^5a$ and $c = S^{-5}a$.

(b) Solve the following recurrence relation 4

$a_r - 7a_{r-1} + 10\, a_{r-2} = 3^r$

given $a_0 = 0$ and $a_1 = 1$

(c) Let a, b and c be numeric functions such that $a * b = c$ Given that 3

$$a_r = \begin{cases} 1 & r = 0 \\ 2 & r = 1 \\ 0 & r \geq 2 \end{cases} \qquad c_r = \begin{cases} 1 & r = 0 \\ 0 & r \geq 1 \end{cases}$$

Determine b

5 (a) Test the validity of the following arguments 4

"If I go to my office tomorrow, then I must get up before 7 a.m. and if I attend the dinner party at the club, I will return home late If I return home late and get up before 7 a m , I will not sleep well I want to sleep well Therefore, either I will not go to office or I will not attend the dinner party "

(b) Obtain the principle disjunctive normal form of the following 3

$P \rightarrow (P \wedge (Q \rightarrow P))$

(c) Given the truth values of P and Q as T (true) and those of R & S as F (false), find the truth values of the following 2

(i) $(\neg(P \wedge Q) \vee \neg R) \vee (((\neg P \wedge Q) \vee \neg R) \wedge S)$

(ii) $\neg((P \rightarrow Q) \wedge (R \rightarrow S))$

(d) Convert the following prefix formula into completely parenthesized infix form 2

$\rightarrow \rightarrow PQ \rightarrow \rightarrow QR \rightarrow PR$

6. (a) Prove tautology 4

$(p \rightarrow q) \rightarrow (\neg q \rightarrow \neg p)$

(b) Express the statement "If a person is female and is a parent, then this person is someone's mother " as a logical expression involving predicates, quantifiers with universe of discourse consisting of all people. 2

(c) Prove that 3

$(x)\,(P(x) \rightarrow Q(x)) \wedge (x)\,(Q(x) \rightarrow R(x))$
$\Rightarrow (x)\,(P(x) \rightarrow R(x))$

7 (a) Give the loop invariant to prove correctness of algorithm 3

Sequential Search (A, n, x)

index = 1,
while (index $\leq$ n) and A[index] $\neq x$ do
index ++,
if index > n then
index = 0,

(b) Illustrate the operation of MERGE-SORT on the following array A = <8, 5, 4, 2, 10> Give the recurrence relation for MERGE-SORT 3

8 (a) Prove that $\frac{1}{2}n^2 - 2n = ②(n^2)$ 4

Compute the constants c_1, c_2 and n_0.

(b) Use big– 0 (0) notation to estimate the sum of the squares of first n numbers 3

9 (a) Solve the following using Master's Theorem if applicable Give reasons if Master's Theorem is not applicable 3

$T(n) = 16(n/4) + n^2$

(b) Use substitution method to prove that the recurrence $T(n) = T(n-1) + ②(n)$ has the solution $T(n) = ②(n^2)$ 3

(c) Find a simple formula for $\sum_{k=1}^{n} (2k+1)$ 2
