# Part III — BUSINESS MATHEMATICS

### ( English Version )

Time Allowed : 3 Hours ]                    [ Maximum Marks : 200

## SECTION – A

N. B. :   i)   Answer *all* the 40 questions.

ii)   Each question carries *one* mark.

iii)  Choose and write the correct answer from the four choices given.                                                        40 × 1 = 40

1.  If the minor of $a_{23}$ equals the cofactor of $a_{23}$ in $|a_{ij}|$ then the minor of $a_{23}$ is

    a)   1                                        b)   2

    c)   0                                        d)   3.

2.  For what value of $k$ the matrix, $A = \begin{pmatrix} 2 & k \\ 3 & 5 \end{pmatrix}$ has no inverse ?

    a)   $\frac{3}{10}$                            b)   $\frac{10}{3}$

    c)   3                                        d)   10.

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3. The rank of $n \times n$ matrix each of whose elements is 2 is

    a)  1                                      b)  2

    c)  $n$                                    d)  $n^2$.

4. The inverse of the relation $\begin{matrix} & x & y \\ a & \begin{pmatrix} 0 & 1 \\ b & 1 & 0 \end{pmatrix} \end{matrix}$ is

    a)  $\begin{matrix} & a & b \\ x & \begin{pmatrix} 0 & 1 \\ y & 1 & 0 \end{pmatrix} \end{matrix}$

    b)  $\begin{matrix} & x & y \\ a & \begin{pmatrix} 0 & -1 \\ b & -1 & 0 \end{pmatrix} \end{matrix}$

    c)  $\begin{matrix} & a & b \\ x & \begin{pmatrix} 0 & 0 \\ y & 1 & 1 \end{pmatrix} \end{matrix}$

    d)  $\begin{matrix} & x & y \\ a & \begin{pmatrix} 0 & 0 \\ b & 0 & 0 \end{pmatrix} \end{matrix}$.

5. The number of Hawkins-Simon conditions for the viability of an input-output model is

    a)  1                                      b)  3

    c)  4                                      d)  2.

6.  The length of the latus rectum of the parabola $3x^2 + 8y = 0$, is

    a) $\dfrac{8}{3}$                          b) $\dfrac{2}{3}$

    c) $8$                                     d) $\dfrac{3}{8}$.

7.  Eccentricity of the hyperbola $\dfrac{x^2}{4} - \dfrac{y^2}{5} = 1$ is

    a) $\dfrac{3}{2}$                          b) $\dfrac{9}{4}$

    c) $\dfrac{5}{4}$                          d) $4$.

8.  Asymptotes of a hyperbola pass through

    a)  one of the foci

    b)  one of the vertices

    c)  centre of the hyperbola

    d)  one end of its latus rectum.

9.  Equation of the directrix of the parabola $y^2 = -8x$ is

    a)  $x + 2 = 0$                            b)  $x - 2 = 0$

    c)  $y + 2 = 0$                            d)  $y - 2 = 0$.

10. If 60 units of some product cost Rs. 1,400 and 40 units cost Rs. 1,200 to manufacture, then the variable cost per unit is

    a)  Rs. 100                                b)  Rs. 2,600

    c)  Rs. 10                                 d)  Rs. 5.

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11. For the cost function $C = \frac{1}{10} e^{2x}$, the marginal cost is

a) $\frac{1}{10}$

b) $\frac{1}{5} e^{2x}$

c) $\frac{1}{10} e^{2x}$

d) $\frac{1}{10} e^{x}$

12. If 20 units of some product cost Rs. 2,500 and 50 units cost Rs. 3,400 to produce, the linear cost function is

a) $y = 30x + 1900$

b) $y = 20x + 5900$

c) $y = 50x + 3400$

d) $y = 10x + 900.$

13. For the function $y = 3x + 2$ the average rate of change of $y$ when $x$ increases from 1·5 to 1·6 is

a) 1

b) 0·5

c) 0·6

d) 3.

14. The slope of the normal to the curve $\sqrt{x} + \sqrt{y} = 5$ at ( 9, 4 ) is

a) $\frac{2}{3}$

b) $-\frac{2}{3}$

c) $\frac{3}{2}$

d) $-\frac{3}{2}.$

15. $f(x, y) = \dfrac{x^{\frac{1}{2}} + y^{\frac{1}{2}}}{x^{\frac{1}{3}} + y^{\frac{1}{3}}}$ is a homogeneous function of degree

  a)   $\dfrac{1}{2}$       b)   $\dfrac{1}{3}$

  c)   $\dfrac{1}{6}$       d)   $\dfrac{1}{5}$.

16. If $u = \log\left(e^x + e^y\right)$ then $\dfrac{\partial u}{\partial x} + \dfrac{\partial u}{\partial y}$ is equal to

  a)   $\dfrac{1}{e^x + e^y}$       b)   $\dfrac{e^x}{e^x + e^y}$

  c)   $1$       d)   $e^x + e^y$.

17. The marginal revenue is Rs. 40 and the average revenue is Rs. 60. The elasticity of demand with respect to price is

  a)   1       b)   0

  c)   2       d)   3.

18. The production function for a firm is $P = 3L^2 - 5KL + 2K^2$. The marginal productivity of capital ($K$) when $L = 2$ and $K = 3$ is

  a)   5       b)   3

  c)   6       d)   2.

19. $\displaystyle\int_{-2}^{2} x^4 \, dx$ is

  a)   $\dfrac{32}{5}$       b)   $\dfrac{64}{5}$

  c)   $\dfrac{16}{5}$       d)   $\dfrac{8}{5}$.

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20. The area bounded by the curve $y = e^x$, the x-axis and the lines $x = 0$ and $x = 2$

    is

    a)  $e^2 - 1$

    b)  $e^2 + 1$

    c)  $e^2$

    d)  $e^2 - 2.$

21. If the marginal cost function $MC = 3e^{3x}$, then the cost function is

    a)  $\dfrac{e^{3x}}{3}$

    b)  $e^{3x} + k$

    c)  $9e^{3x}$

    d)  $3e^{3x}.$

22. The order and degree of the differential equation

    $$\left(\frac{dy}{dx}\right)^2 - 3\frac{d^3y}{dx^3} + 7\frac{d^2y}{dx^2} + \frac{dy}{dx} = x + \log x \text{ are}$$

    a)  1 and 3

    b)  3 and 1

    c)  2 and 3

    d)  3 and 2.

23. The solution of $x\,dx + y\,dy = 0$ is

    a)  $x^2 + y^2 = c$

    b)  $\dfrac{x}{y} = c$

    c)  $x^2 - y^2 = c$

    d)  $xy = c.$

24. The integrating factor of $\dfrac{dy}{dx} + \dfrac{2y}{x} = x^3$ is

    a)  $2 \log x$

    b)  $e^{x^2}$

    c)  $3 \log(x^2)$

    d)  $x^2.$

25. The complementary function of the differential equation $\left( D^2 - D \right) y = e^x$ is

   a)   $A + Be^x$                          b)   $( Ax + B ) e^x$

   c)   $A + Be^{-x}$                        d)   $( A + Bx ) e^{-x}$.

26. $E =$

   a)   $1 + \Delta$                         b)   $1 - \Delta$

   c)   $\nabla + 1$                         d)   $\nabla - 1$.

27. $\Delta f( x ) =$

   a)   $f( x + h )$                         b)   $f( x ) - f( x + h )$

   c)   $f( x + h ) - f( x )$               d)   $f( x ) - f( x - h )$.

28. If the probability density function of a variable $X$ is defined as

   $f( x ) = C x ( 2 - x ),\ 0 < x < 2,$   then the value of $C$ is

   a)   $\dfrac{4}{3}$                       b)   $\dfrac{3}{2}$

   c)   $\dfrac{3}{4}$                       d)   $\dfrac{3}{5}$.

29. The normal distribution curve is

   a)   bimodal                             b)   unimodal

   c)   skewed                              d)   none of these.

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39. A time series consists of

     a)    two components

     b)    three components

     c)    four components

     d)    none of these.

40. The component of a time series attached to long term variation is termed as

     a)    cyclic variations              b)    secular trend

     c)    irregular variation            d)    all of these.

## SECTION – B

*N. B. :*    i)    Answer any *ten* out of *fifteen* questions given.

         ii)    Each question carries *six* marks.          $10 \times 6 = 60$

41. Find the value of $k$, if the equations $x + 2y - 3z = -2$, $3x - y - 2z = 1$,

     $2x + 3y - 5z = k$ are consistent.

42. Given $A = \begin{pmatrix} 1 & -1 & 1 \\ 2 & 1 & 1 \\ 3 & 1 & -1 \end{pmatrix}$, verify that $|\ adj\ A\ | = |\ A\ |^2$.

43. The supply of a commodity is related to the price by the relation $x = 5\sqrt{2p - 10}$. Show that the supply curve is a parabola. Find its vertex and the price below which supply is zero.

44. Find the equilibrium price and equilibrium quantity for the following demand and supply functions :

$$q_d = 4 - 0.05\,p \quad \text{and} \quad q_s = 0.8 + 0.11\,p.$$

45. Find the equation of the tangent and normal to the curve $xy = 9$ at $x = 4$.

46. A firm produces $x$ tons of a valuable metal per month at a total cost $C$ given by $C = \text{Rs.}\left(\dfrac{1}{3}x^3 - 5x^2 + 75x + 10\right)$. Find at what level of output, the marginal cost attains its minimum.

47. The marginal cost at a production level of $x$ units is given by

$C'(x) = 85 + \dfrac{375}{x^2}$. Find the cost of producing 10 incremental units after 15 units have been produced.

48. Solve : $(x + y)\,dy + (x - y)\,dx = 0$.

49. Solve : $\left(1 + x^2\right)\dfrac{dy}{dx} + y = 1$.

50. If $f(0) = 5$, $f(1) = 6$, $f(3) = 50$, $f(4) = 105$, find $f(2)$ by using Lagrange's formula.

51. Find $y$ when $x = 0.2$ given that

| $x$ : | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $y$ : | 176 | 185 | 194 | 202 | 212 |

( Use Gregory-Newton's forward formula )

52. Ten coins are thrown simultaneously. Find the probability of getting at least seven heads.

53. A random sample of size 5 is drawn without replacement from a finite population consisting of 41 units. If the population S.D. is 6.25, find the S.E. of the sample mean.

54. Calculate the regression equation of $X$ on $Y$ from the following data :

| $X$ : | 10 | 12 | 13 | 12 | 16 | 15 |
|---|---|---|---|---|---|---|
| $Y$ : | 40 | 38 | 43 | 45 | 37 | 43 |

55. Solve the following, using graphical method :

Maximize $Z = 3x_1 + 4x_2$

subject to the constraints

$2x_1 + x_2 \leq 40$

$2x_1 + 5x_2 \leq 180$

$x_1, x_2 \geq 0$.

## SECTION – C

N. B. :   i)    Answer any *ten* questions out of *fifteen* questions given.

ii)   Each question carries *ten* marks.                          $10 \times 10 = 100$

56. Solve by Cramer's rule :

    $x + y = 2, \quad y + z = 6, \quad z + x = 4.$

57. Two products A and B currently share the market with shares 60% and 40%

    each respectively. Each week some brand switching takes place. Of those who

    bought A the previous week, 70% buy it again whereas 30% switch over to B.

    Of those who bought B the previous week, 80% buy it again whereas 20%

    switch over to A. Find their shares after one week and after two weeks. If the

    price war continues, when is the equilibrium reached ?

58. Find the centre, vertices, eccentricity, foci and latus rectum of the ellipse

    $7x^2 + 4y^2 - 14x + 40y + 79 = 0.$

59. Prove that for the cost function $C = 100 + x + 2x^2$, where $x$ is the output, the

    slope of AC curve = $\frac{1}{x}$ ( MC – AC ). ( MC is the marginal cost and AC is the

    average cost )

60. The demand for a quantity $A$ is $q_1 = 16 - 3p_1 - 2p_2^2$. Find

 (i) the partial elasticities $\dfrac{Eq_1}{Ep_1}$, $\dfrac{Eq_1}{Ep_2}$

 (ii) the partial elasticities for $p_1 = 2$ and $p_2 = 1$.

61. A manufacturer has to supply his customer with 600 units of his products per year. Shortages are not allowed and storage cost amounts to 60 paise per unit per year. When the set up cost is Rs. 80, find

 (i) the economic order quantity

 (ii) the minimum average yearly cost.

62. Evaluate : $\displaystyle\int_0^{\pi/2} \dfrac{a \sin x + b \cos x}{\sin x + \cos x}\, dx.$

63. Find the consumers' surplus and the producers' surplus under market equilibrium if the demand function is $p_d = 20 - 3x - x^2$ and the supply function is $p_s = x - 1$.

64. Solve : $\left(D^2 - 14D + 49\right) y = 3 + e^{7x}.$

65. Fit a straight line to the following data :

| $x$ : | 4 | 8 | 12 | 16 | 20 | 24 |
|---|---|---|---|---|---|---|
| $y$ : | 7 | 9 | 13 | 17 | 21 | 25 |

66. The diameter of shafts produced in a factory conforms to normal distribution. 31% of the shafts have a diameter less than 45 mm and 8% have more than 64 mm. Find the mean and standard deviation of the diameter of shafts.

Given :

| Z : | 0·5 | 1·41 |
|------|------|------|
| Area : | 0·19 | 0·42 |

67. For the following probability density function $f(x)$ compute the mean and variance :

$$f(x) = \begin{cases} \dfrac{1}{2\sqrt{x}} & , \quad \text{if } 0 < x < 1 \\ 0 & , \quad \text{otherwise} \end{cases}$$

68. Out of 1000 TV viewers, 320 watched a particular programme. Find 95% confidence limits for TV viewers who watched this programme.

69. Calculate Fisher's ideal index from the following data :

| Commodity | Price | | Quantity | |
|-----------|-------|------|----------|------|
|  | 1985 | 1986 | 1985 | 1986 |
| A | 8 | 20 | 50 | 60 |
| B | 2 | 6 | 15 | 10 |
| C | 1 | 2 | 20 | 25 |
| D | 2 | 5 | 10 | 8 |
| E | 1 | 5 | 40 | 30 |

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70. The following data shows the value of sample mean $\overline{X}$ and the range R for ten samples of size 5 each. Calculate the values for central line and control limits for mean chart and range chart and determine whether the process is in control.

| Sample No. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Mean $\overline{X}$ | 11·2 | 11·8 | 10·8 | 11·6 | 11·0 | 9·6 | 10·4 | 9·6 | 10·6 | 10·0 |
| Range R | 7 | 4 | 8 | 5 | 7 | 4 | 8 | 4 | 7 | 9 |

( Given for $n = 5$, $A_2 = 0·577$, $D_3 = 0$, $D_4 = 2·115$ )