Register Number | | | | | | |

Part III — BUSINESS MATHEMATICS

(New Syllabus)

(English Version)

Time Allowed : 3 Hours] [Maximum Marks : 200

SECTION – A

N. B. : i) Answer *all* the 40 questions.

ii) Each question carries *one* mark.

iii) Choose and write the correct answer from the four choices given. $40 \times 1 = 40$

1. If $AB = BA = |A| I$, then the matrix B is

a) inverse of A

b) the transpose of A

c) the adjoint of A

d) $2A$.

2. The inverse of $\begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix}$ is

a) $\begin{pmatrix} 0 & \frac{1}{2} \\ 2 & 0 \end{pmatrix}$

b) $\begin{pmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & 0 \end{pmatrix}$

c) $\begin{pmatrix} 0 & -\frac{1}{2} \\ \frac{1}{2} & 1 \end{pmatrix}$

d) $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$.

3. The rank of a zero matrix is

a) 0 b) 1

c) -1 d) ∞.

4. The relation $R = \begin{array}{c} \\ a \\ b \end{array} \begin{array}{c} \begin{array}{cc} a & b \end{array} \\ \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \end{array}$ is

a) Reflexive

b) Symmetric

c) Transitive

d) Reflexive and Symmetric.

5. If $T = \begin{array}{c} \\ A \\ B \end{array} \begin{array}{c} \begin{array}{cc} A & B \end{array} \\ \begin{pmatrix} 0{\cdot}7 & 0{\cdot}3 \\ x & 0{\cdot}8 \end{pmatrix} \end{array}$ is a transition probability matrix, then the value of x is

a) 0·3 b) 0·2

c) 0·8 d) 0·7.

6. Equation of the directrix of $x^2 = 4ay$ is

a) $x + a = 0$ b) $x - a = 0$

c) $y + a = 0$ d) $y - a = 0$.

7. The length of the latus rectum of $4x^2 + 9y^2 = 36$ is

a) $\frac{4}{3}$ b) $\frac{8}{3}$

c) $\frac{4}{9}$ d) $\frac{8}{9}$.

8. The difference between the focal distances of any point on the hyperbola is equal to length of its

a) transverse axis b) semi-transverse axis

c) conjugate axis d) semi-conjugate axis.

9. Eccentricity of the rectangular hyperbola is

a) 2 b) $\frac{1}{2}$

c) $\sqrt{2}$ d) $\frac{1}{\sqrt{2}}$.

10. If the variable cost per unit is Rs. 40, fixed cost is Rs. 900 and unit selling price is Rs. 70, then the profit equation is

a) $P = 30x - 900$ b) $P = 15x - 70$

c) $P = 40x - 900$ d) $P = 70x + 3600$.

11. For the cost function $C = \frac{1}{10} e^{2x}$, the marginal cost is

a) $\frac{1}{10}$ b) $\frac{1}{5} e^{2x}$

c) $\frac{1}{10} e^{2x}$ d) $\frac{1}{10} e^{x}$

12. For the function $y = 3x + 2$, the average rate of change of y when x increases from 1·5 to 1·6 is

a) 1 b) 0·5

c) 0·6 d) 3.

13. The slope of the tangent at (2, 8) on the curve $y = x^3$ is

a) 3 b) 12

c) 6 d) 8.

14. The slope of the normal to the curve $\sqrt{x} + \sqrt{y} = 5$ at (9, 4) is

a) $\frac{2}{3}$ b) $-\frac{2}{3}$

c) $\frac{3}{2}$ d) $-\frac{3}{2}$.

15. The maximum value of $f(x) = \cos x$ is

a) 0 b) $\frac{\sqrt{3}}{2}$

c) $\frac{1}{2}$ d) 1.

16. $y = x^3$ is always

a) an increasing function of x b) a decreasing function of x

c) a constant function d) none of these.

17. If $u = x^y$ ($x > 0$) then $\frac{\partial u}{\partial y}$ is equal to

a) $x^y \log x$ b) $\log x$

c) $y^x \log x$ d) $\log y^x$.

18. The elasticity of demand when the marginal revenue is zero, is

a) 1 b) 2

c) -5 d) 0.

19. If $f(x)$ is an even function then $\int_{-a}^{a} f(x)\,dx$ is

a) $2\int_{0}^{a} f(x)\,dx$ b) $\int_{0}^{a} f(x)\,dx$

c) $-2a$ d) $2a$.

20. The area bounded by $y = x$, y-axis and $y = 1$ is

a) 1 b) $\frac{1}{2}$

c) log 2 d) 2.

21. The marginal revenue of a firm is $MR = 15 - 8x$. Then the revenue function is

a) $15x - 4x^2 + k$ b) $\frac{15}{x} - 8$

c) -8 d) $15x - 8$.

22. The degree and order of the differential equation $\frac{d^2y}{dx^2} - 6\sqrt{\frac{dy}{dx}} = 0$ are

a) 2 and 1 b) 1 and 2

c) 2 and 2 d) 1 and 1.

23. The solution of $x\,dy + y\,dx = 0$ is

a) $x + y = c$ b) $x^2 + y^2 = c$

c) $xy = c$ d) $y = cx$.

24. The particular integral of the differential equation

$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = e^{3x}$ is

a) $\frac{e^{3x}}{2!}$ b) $\frac{x^2 e^{3x}}{2!}$

c) $\frac{xe^{3x}}{2!}$ d) $9e^{3x}$.

25. The integrating factor of $\frac{x\,dy}{dx} - y = e^x$ is

a) $\log x$ b) $e^{-\frac{1}{x}}$

c) $\frac{1}{x}$ d) $-\frac{1}{x}$.

26. When $h = 1$, $\Delta\left(x^2\right) =$

a) $2x$ b) $2x - 1$

c) $2x + 1$ d) 1.

27. The normal equations of fitting a straight line $y = ax + b$ are $10a + 5b = 15$ and $30a + 10b = 43$. The slope of the line of best fit is

a) 1·2 b) 1·3

c) 13 d) 12.

28. If a discrete random variable has the probability mass function as

x	0	1	2	3
$p(x)$	k	$2k$	$3k$	$5k$

then the value of k is

a) $\frac{1}{11}$ b) $\frac{2}{11}$

c) $\frac{3}{11}$ d) $\frac{4}{11}$.

29. The normal distribution curve is

a) bimodal b) unimodal

c) skewed d) none of these.

30. If X is a Poisson variate with $P(X = 1) = P(X = 2)$, the mean of the Poisson variate is equal to

a) 1 b) 2

c) -2 d) 3.

31. The mean and variance of the Binomial distribution are

a) np, npq b) pq, npq

c) $np, \sqrt{npq}$ d) np, nq.

32. The standard error of the sample mean is

a) Type I error

b) Type II error

c) standard deviation of the sampling distribution of the mean

d) variance of the sampling distribution of the mean.

33. The central limit theorem states that the sampling distribution of the mean will approach normal distribution

a) as the size of the population increases

b) as the sample size increases and becomes larger

c) as the number of samples gets larger

d) as the sample size decreases.

34. Probability of rejecting null hypothesis when it is true is

a) Type I error b) Type II error

c) Sampling error d) Standard error.

35. The number of ways in which one can select 2 customers out of 10 customers is

a) 90 b) 60

c) 45 d) 50.

36. A time series consists of

a) two components
b) three components
c) four components
d) none of these.

37. Laspeyre's index formula uses the weights of

a) the base year quantities
b) the current year prices
c) average of the weights of number of years
d) none of these.

38. Control charts in statistical quality consist of

a) three control lines
b) upper and lower limits
c) the level of process
d) all of these.

39. The range of correlation coefficient is

a) 0 to ∞
b) $-\infty$ to ∞
c) -1 to 1
d) none of these.

40. If X and Y are two variates, there can be atmost

a) one regression line
b) two regression lines
c) three regression lines
d) none of these.

SECTION – B

N. B. : i) Answer any *ten* out of *fifteen* questions given.

ii) Each question carries *six* marks. $10 \times 6 = 60$

41. Verify $(AB)^{-1} = B^{-1}A^{-1}$ when $A = \begin{pmatrix} 3 & 1 \\ 2 & -1 \end{pmatrix}$ and $B = \begin{pmatrix} -6 & 0 \\ 0 & 9 \end{pmatrix}$.

42. Find k if the equations $2x + 3y - z = 5$, $3x - y + 4z = 2$ and $x + 7y - 6z = k$ are consistent.

43. Find the equation of the hyperbola whose eccentricity is $\sqrt{3}$, focus is $(1, 2)$ and the corresponding directrix is $2x + y = 1$.

44. The demand curve for a monopolist is given by $x = 100 - 4p$.

i) Find the total revenue, average revenue and marginal revenue.

ii) At what value of x, the marginal revenue is equal to zero ?

45. Find the equations of the tangent and normal to the curve

$x = a \cos\theta$, $y = b \sin\theta$ at $\theta = \frac{\pi}{4}$.

46. A manufacturer can sell x items per week at a price of $p = 600 - 4x$ rupees. Production cost of x items works out to Rs. C where $C = 40x + 2000$. How much production will yield maximum profit ?

47. The elasticity of demand with respect to price for a commodity is a constant and is equal to 2. Find the demand function and hence the total revenue function given that when the price is 1, the demand is 4.

48. Solve : $\frac{dy}{dx} + y \cos x = \frac{1}{2} \sin 2x$.

49. Solve : $\left(D^2 + 10D + 25 \right) y = 5e^x$.

50. From the following data, find $f(3)$.

x :	1	2	3	4	5
$f(x)$:	2	5	—	14	32

51. Using Lagrange's formula find $y(11)$ from the following table :

x :	6	7	10	12
y :	13	14	15	17

52. Find the mean, variance and standard deviation of the following probability distribution :

X :	– 3	– 2	– 1	0	1	2	3
$P(x)$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$

53. 1% of the outgoing + 2 students in a school have joined I.I.T. Madras. What is the probability that in a group of 500 such students 2% or more will be joining I.I.T. Madras ?

Given :

Z	2·1	2·2	2·3
Area	0·4821	0·4826	0·4830

54. Calculate the correlation coefficient from the following data :

X :	12	9	8	10	11	13	7
Y :	14	8	6	9	11	12	3

55. From the following data calculate the price index number by Laspeyre's method.

Commodity	***Base Year***		***Current Year***	
	Price	***Quantity***	***Price***	***Quantity***
A	5	25	6	30
B	10	5	15	4
C	3	40	2	50
D	6	30	8	35

SECTION - C

N. B. : i) Answer any *ten* questions out of *fifteen* questions given.

ii) Each question carries *ten* marks. $10 \times 10 = 100$

56. By matrix method solve the equations $x - 2y + 3z = 1$; $3x - y + 4z = 3$; $2x + y - 2z = -1$.

57. Two products P and Q share the market currently with shares 70% and 30% each respectively. Each week some brand switching takes place. Of those who bought P in the previous week, 80% buy it again whereas 20% switch over to Q. Of those who bought Q in the previous week, 40% buy it again whereas 60% switch over to P. Find their shares after two weeks. If the price war continues, when is the equilibrium reached ?

58. Find the centre, eccentricity, foci and directrices of the ellipse

$$3x^2 + 4y^2 - 6x + 8y - 5 = 0.$$

59. Find the equation of the tangent and normal to the curve

$y(x-2)(x-3) - x + 7 = 0$ at the point where it cuts the x-axis.

60. Find the maximum and minimum values of the function

$f(x) = 2x^3 - 15x^2 + 24x - 15.$

61. The demand for a commodity A is $q_1 = 240 - p_1^2 + 6p_2 - p_1 p_2$. Find the partial elasticities $\frac{Eq_1}{Ep_1}$ and $\frac{Eq_1}{Ep_2}$ when $p_1 = 5$ and $p_2 = 4$.

62. Evaluate $\int_0^{\pi/2} \frac{\sqrt{\sin^3 x}}{\sqrt{\sin^3 x} + \sqrt{\cos^3 x}}\, dx$.

63. Find the consumer's surplus and producer's surplus under market equilibrium if the demand function $p_d = 20 - 3x - x^2$ and supply function $p_s = x - 1$.

64. The net profit p and quantity x satisfy the differential equation $\frac{dp}{dx} = \frac{2p^3 - x^3}{3xp^2}$.

Find the relationship between net profit and demand given that $p = 20$ when $x = 10$.

65. Fit a straight line to the following data :

x :	100	200	300	400	500	600
y :	90·2	92·3	94·2	96·3	98·2	100·3

66. Find the mean and variance for the following probability distribution :

$$f(x) = \begin{cases} 2e^{-2x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

67. Ten coins are thrown simultaneously. Find the probability of getting at least 7 heads.

68. A sample of 400 students is found to have a mean height of 171·38 cm. Can it reasonably be regarded as a sample from a large population with mean height of 171·17 cm and standard deviation of 3·3 cm ? (Test at 5% level).

69. Solve the following using graphical method :

Maximize $Z = 5x_1 + 6x_2$

subject to the constraints,

$$3x_1 + 2x_2 \leq 120$$

$$4x_1 + 6x_2 \leq 260$$

$$x_1, x_2 \geq 0.$$

70. Calculate the seasonal indices for the following data using average mehod :

Year	Quarters			
	I	II	III	IV
1982	72	68	80	70
1983	76	70	82	74
1984	74	66	84	80
1985	76	74	84	78
1986	78	74	86	82