M-08

Department of Mathematics and Statistics
University of Hyderabad
Entrance examination - M.Sc. Statistics, 2008

Hall Ticket Number:

Time: 2 hours
Max. Marks: 75

Part A: 25 marks
Part B: 50 marks

Instructions

1. Write your Booklet Code and Hall Ticket Number in the OMR Answer Sheet given to you. Also write the Hall Ticket Number in the space provided above.

2. There is negative marking. Do not gamble.

3. Answers are to be marked on the OMR answer sheet following the instructions provided there upon.

4. Hand over the question paper booklet and the OMR answer sheet at the end of the examination.

5. No additional sheets will be provided. Rough work can be done in the question paper itself/ space provided at the end of the booklet.

6. Calculators are not allowed.

7. There are a total of 50 questions in part A and B together.

8. The appropriate answer should be coloured in by either a blue or black ballpoint or sketch pen. DO NOT USE A PENCIL.

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SECTION A

Each question carries 1 mark for a correct answer and 0.33 marks for a wrong answer.

1. Let A and B be mutually exclusive (disjoint) events with strictly positive probabilities of occurrences. Then
(A) A^c and B^c are independent.
(B) A^c and B^c are mutually exclusive.
(C) $P(A \cup B) = 1$.
(D) $P(A|B) = P(B|A)$.

2. In a school there are 200 children born between 1st January 1999 and 30th June 1999 (end dates are included). The probability that no two children have the same birthday is
(A) 0

(B) 1

(C) $\frac{363}{365}$

(D) $\frac{2}{365}$

3. If an event A occurs whenever event B occurs then it is true that
(A) $P(A^c) \leq P(B^c)$
(B) $P(A^c) \geq P(B^c)$
(C) $P(A) = P(B)$
(D) $P(A) = P(B^c)$

4. For the two events A and B, the probability of each event is at least 0.8. $P(A \cap B)$ is in the interval
(A) $[0, 0.3]$
(B) $(0.3, 0.5]$
(C) $[0.5, 0.6)$
(D) $[0.6, 0.8]$

5. Suppose M is the mode and S^2 the variance of n observations. If a negative constant a is added to all the observations, the mode and variance of these observations will be respectively
(A) $M + \mathrm{a},\ S^2$
(B) $M - \mathrm{a},\ S^2 - \mathrm{a}^2$
(C) $M - \mathrm{a},\ S^2$
(D) $M + \mathrm{a},\ S^2 + \mathrm{a}^2$

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6. Consider the data: -212, -0.3, -0.28, -1.4, -0.6, -0.2, 0, 0.1, 0.2, 0.2, 1.2, -0.6, 4008. The appropriate measure of central tendancy among the following is

(A) Arithmetic Mean

(B) Geometric Mean

(C) Median

(D) Mode

7. For a set of observations, the mean is 58, the median is 62 and the mode is 63. The data are

(A) skewed to the right

(B) skewed to the left

(C) symmetric about the median

(D) bimodal

8. Out of 100 students in a class, 80 students got 40 marks in Mathematics and 20 got 50 marks. The mean, median, and mode of the marks in Mathematics for this class are respectively

(A) 40, 40, 40

(B) 40, 40, 70

(C) 42, 40, 40

(D) 45, 40, 40

9. $X \sim U(-\beta, \beta)$ that is, the random variable X has uniform (or rectangular) distribution in the interval $(-\beta, \beta)$. For $0 < \alpha < \beta$, $P(-\alpha < X < \alpha)$ is

(A) $\frac{\alpha}{2\beta}$

(B) $\frac{2\alpha}{\beta}$

(C) $\frac{\alpha}{4\beta}$

(D) $\frac{\alpha}{\beta}$

10. Let $X_1 \sim B(n_1, p_1)$, $X_2 \sim B(n_2, p_2)$. Suppose $p_1 > p_2$ and $E(X_1) = E(X_2)$, then

(A) $E(X_1^2) > E(X_2^2)$

(B) $E(X_1^2) = E(X_2^2)$

(C) $E(X_1^2) < E(X_2^2)$

(D) $E(X_1^2)$ and $E(X_2^2)$ can not be compared.

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11. X_1 and X_2 are Poisson random variables with parameters λ_1 and λ_2 respectively. If $P(X_1 > 0) > P(X_2 > 0)$, then
(A) $E(X_1) > E(X_2)$
(B) $E(X_1) < E(X_2)$
(C) $E(X_1) = E(X_2)$
(D) It is not possible to say any thing definitely.

12. The heights of adult men are normally distributed with mean 1.65 meters and standard deviation σ meters. 30% of this population is of height less than 1.45 meters. The percentage of adult men taller than 1.85 m is
(A) 25%
(B) 30%
(C) 40%
(D) 70%

13. X is a random variable with probability density function (pdf):

$$f_X(x) = \frac{1}{\sqrt{\pi}} \exp\{-x^2\}, \quad -\infty < x < \infty.$$

The mean and standard deviation of X are respectively
(A) 0 and $\frac{1}{2}$

(B) 0 and $\frac{1}{\sqrt{2}}$

(C) 0 and $\sqrt{2}$

(D) 1 and $\frac{1}{\sqrt{2}}$.

14. The random variables X and Y are such that $X + Y$ and $X - Y$ are positively correlated. Then
(A) $\text{Var}(X + Y) > \text{Var}(X - Y)$
(B) $\text{Var}(X + Y) < \text{Var}(X - Y)$
(C) $\text{Var}(X) < \text{Var}(Y)$
(D) $\text{Var}(X) > \text{Var}(Y)$

15. The correlation coefficient $\rho(X, Y)$ between X and Y is $-\frac{3}{4}$. Then $\rho(U, V)$, where $U = -2X + 7$ and $V = 3Y + 8$
(A) can not be determined from the given information

(B) is 0

(C) is $\frac{3}{4}$

(D) is -$\frac{3}{4}$

16. X is a random variable with $P(X = 1) = \frac{1}{100}$ and $P(X = 0) = \frac{99}{100}$. Suppose Y is another random variable for which $E(Y|X = 0) = -1000$ and $E(Y|X = 1) = 200,000$ then $E(Y)$
(A) can not be determined from the given information
(B) is 99500
(C) is 0
(D) is 1010

17. Two random variables X and Y are uncorrelated if
(A) $\text{Var}(X + Y) = \text{Var}(X - Y)$
(B) $\text{Var}(XY) = \text{Var}(X)\text{Var}(Y)$
(C) $\text{Var}(X - Y) = \text{Var}(X) - \text{Var}(Y)$
(D) $\text{Var}(X) = \text{Var}(Y)$

18. The area of a circle inscribed in a square (that is all the sides of the square are tangents to the circle) whose side is 2π cm is
(A) π^2 sq cm

(B) $\frac{\pi^2}{2}$ sq cm

(C) π^3 sq cm

(D) $\frac{\pi^3}{2}$ sq cm

19. A and B two $m \times n$ matrices, then
(A) Rank(A+B)=Rank(A)+Rank(B)
(B) Rank(A+B)< min{Rank(A), Rank(B)}
(C) Rank(A+B)> max{Rank(A), Rank(B)}
(D) None of the above.

20. A three digit number is obtained by drawing 3 numbers denoted by a, b, c from a purse containing 9 marbles numbered 1,2,...,9 with replacement. The set of all possible outcomes is
(A) $\{abc \mid \{a,b,c\} \subset \{1,2,\ldots,9\}\}$
(B) $\{\{a,b,c\} \mid \{a,b,c\} \subset \{1,2,\ldots,9\}\}$
(C) $\{abc \mid a \in \{1,2,\ldots,9\}, b \in \{1,2,\ldots,9\}, c \in \{1,2,\ldots,9\}\}$
(D) $\{abc \mid a < b < c, a \in \{1,2,\ldots,9\}, b \in \{1,2,\ldots,9\}, c \in \{1,2,\ldots,9\}\}$.

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21. The number of elements in the set $S = \{(0, x_1, \ldots, x_{n-1}, 1) : x_i \in \{0, 1\}, i = 1, \ldots, n-1\}$ is
(A) n
(B) 2^{n-1}
(C) 2^n
(D) $n2^{n-1}$.

22. For testing a null hypothesis, the level α critical region is C, if the level of significance of the test is reduced to α_1, the corresponding critical region C_1
(A) contains C
(B) is contained in C
(C) is equal to C
(D) none of the above can be said with certainity.

23. Suppose you are told that there is a direct relationship between the price of a certain fruit and the amount of rainfall during the growing season. It can be concluded that
(A) Price of the fruit tends to be high when the rainfall is high.
(B) Price of the fruit tends to be low when the rainfall is high.
(C) A large amount of rain causes price of the fruit to rise.
(D) A lack of rainfall causes price of the fruit to rise.

24. The limit of the sequence $\{a_n\}_{n\geq 1}$ as $n \to \infty$ where $a_n = \left(1 + \frac{1}{n^{\frac{n-1}{n}}}\right)^n$ is
(a) 0
(b) 1
(c) $\frac{1}{2}$
(d) none of the above

25. The negation of the statement 'Ashok has read every book in the library' is
(A) Ashok has not read any book in the library.
(B) Ashok has not read at least one book in the library.
(C) Ashok has read at most one book in the library.
(D) Ashok has read only one book in the library.

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SECTION-B

Each question carries 2 marks for a correct answer and 0.66 marks for a wrong answer.

26. 20% of the individuals in a group can speak English and 52% of the group are females. The percentage of individuals who are either English speaking or male is at most

(A) 48%

(B) 52%

(C) 68%

(D) 72%

27. All arrangements of the 9 numbers 1, ..., 9 in a row are equally likely. The probability that no two odd numbers are next to each other is

(A) $\frac{1}{9!}$

(B) $\frac{1}{\binom{9}{5}}$

(C) $\frac{4!}{9!}$

(D) $\frac{5!}{9!}$

28. A and B are independent events with equal probabilities. If $P(A \cup B) = 0.75$, then

(A) $P(A) = P(B) = 0.25$

(B) $P(A) = P(B) = 0.5$

(C) $P(A) = P(B) = 0.6$

(D) $P(A) = P(B) = 0.75$

29. A packet of 5 shirts contains no defective or 1 defective shirt with probabilities 0.5 each. A dealer picks up a shirt from a packet at random and rejects the packet if the selected shirt is defective, otherwise he accepts the packet. The probability that there is a defective shirt in an accepted packet is

(A) $\frac{4}{9}$

(B) $\frac{1}{2}$

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(C) $\frac{5}{9}$

(D) $\frac{2}{3}$

30. There are 20 balls numbered 1, 2, ..., 20, two balls each have to be placed in 10 boxes. If every arrangement is equally likely, the probability that each box gets both balls with even numbers or both balls with odd numbers is

(A) $\dfrac{\binom{10}{5}\binom{10}{5}}{\binom{20}{10}}$

(B) $\dfrac{2^{10}}{\binom{20}{10}}$

(C) $\dfrac{2^{10}5!}{\binom{20}{10}}$

(D) $\dfrac{\binom{10}{5}}{\binom{20}{10}}$

31. $X \sim B(7, \frac{1}{3})$, let $Y = 7 - X$, then
(A) $P(X - Y = -7) > P(X - Y = 7)$
(B) $P(X - Y = -5) = P(X - Y = 5)$
(C) $P(X - Y = -3) = P(X - Y = 1)$
(D) $P(X - Y = 0) = \frac{1}{2}$

32. Consider the quadratic equation $ax^2 + bx + c = 0$, obtained by selecting a, b and c from the set $\{1, 2, 3\}$ (a, b and c need not be distinct). The probability that the quadratic equation will have only real roots is

(A) $\frac{1}{27}$

(B) $\frac{2}{27}$

(C) $\frac{4}{27}$

(D) $\frac{8}{27}$

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33. $P(A) = 0.8$, $P(B|A) = 0.9$, $P(C|A \cap B) = 0.9$, then $P(A^c \cup B^c \cup C^c)$
(A) is 0.352
(B) is 0.648
(C) is 0.81
(D) can not be determined from the information given.

34. Given below are data on Y, X_1 and X_2

Y:	0.6	0.2	0.2	0.1	0.1	0.05	0.01	0
X_1:	2.01	2	2	2	2	2	2	2
X_2:	50	2	2	2	2	2	2	2

Let ρ_i be the correlation coefficient between Y and X_i, $i = 1, 2$, then
(A) $\rho_1 = \rho_2$
(B) $\rho_1 > \rho_2$
(C) $\rho_1 < \rho_2$
(D) $\rho_1 = 0$

35. Let $x_1 < x_2 < x_3$ be 3 real numbers and suppose we are given the values of $x_3 - x_1$ and $x_3 - x_2$. With this information we
(A) can find the mean and the variance of x_1, x_2, x_3
(B) can not find either the mean or the variance of x_1, x_2, x_3
(C) can find the mean but not the variance of x_1, x_2, x_3
(D) can find the variance but not the mean of x_1, x_2, x_3

36. If X is a Poisson random variable with mean 2, then $E[X(X-1)(X-2)]$ is
(A) 2
(B) 4
(C) 8
(D) 16

37. A random variable X follows Binomial(n, $\frac{1}{4}$) distribution. If $P(X = 4) = P(X = 5)$, then n is equal to
(A) 9
(B) 10
(C) 19
(D) 25

38. A random variable X has mean 5. If $P(|X - 5| > 5) = 0.1$, then the variance of X is
(A) at most 1.5
(B) equal to 1

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(C) equal to 2

(D) at least 2.5

39. The median of the random variable X whose pdf is given by

$$f_X(x) = \begin{cases} 12x^2(1-x), & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

is in

(A) $(0, \frac{1}{4}]$

(B) $(\frac{1}{4}, \frac{1}{2}]$

(C) $(\frac{1}{2}, \frac{3}{4}]$

(D) $(\frac{3}{4}, \frac{4}{5}]$

40. The joint pdf of two random variables X and Y is

$$f(x,y) = \begin{cases} \exp\{-y\}, & 0 < x < y < \infty \\ 0, & \text{otherwise} \end{cases}$$

The conditional pdf of X given $Y = y$, $f(x|y)$ is

(A)

$$f(x|y) = \begin{cases} \frac{1}{y}, & 0 < x < y \\ 0, & \text{otherwise} \end{cases}$$

(B)

$$f(x|y) = \begin{cases} 1, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

(C)

$$f(x|y) = \begin{cases} \exp\{-x\}, & x > 0 \\ 0, & \text{otherwise} \end{cases}$$

(D)

$$f(x|y) = \begin{cases} x\exp\{-x\}, & x > 0 \\ 0, & \text{otherwise} \end{cases}$$

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41. The random variable X follows exponential distribution with mean $\frac{1}{2}$. Then

(A) $P(X < 2) > P(X > 2)$

(B) $P(X < 2) < P(X > 2)$

(C) $P(X < 2) = P(X > 2)$

(D) $P(X < 2) > P(X > 0)$.

42. X_1, X_2 is a random sample from the normal population with mean μ and variance μ^2. Then unbiased estimators for μ and μ^2 respectively are

(A) $\frac{X_1+X_2}{2}$ and $(\frac{X_1-X_2}{2})^2$

(B) $\frac{X_1+X_2}{4}$ and $\frac{X_1^2-X_2^2}{2}$

(C) $\frac{X_1+X_2}{2}$ and $\frac{X_1^2+X_2^2}{4}$

(D) $\frac{2X_1-X_2}{2}$ and $\frac{2X_1^2-X_2^2}{5}$.

43. X has pdf

$$f(x) = \begin{cases} \frac{1}{\theta^2}, & 0 < x < \theta^2 \\ 0, & \text{otherwise} \end{cases}$$

where $\theta \neq 0$. Let $X_1, X_2, \ldots, X_{20}$ be a random sample from X. The maximum likelihood estimator of θ^2 is

(A) $\frac{1}{20}\sum_{i=1}^{20} X_i$

(B) $\frac{1}{19}\sum_{i=1}^{19} X_i^2$

(C) $\max\{X_1^2, X_2^2, \ldots, X_{20}^2\}$

(D) $\max\{X_1, X_2, \ldots, X_{20}\}$.

44. $X_1, X_2, \ldots, X_n$, $n > 3$ is a random sample from the normal distribution with mean 0 and variance σ^2. The best unbiased estimator for σ^2 among the four given below is

(A) $\frac{1}{n}\sum_{i=1}^{n} X_i^2$

(B) $\frac{1}{n}(\sum_{i=1}^{n} X_i)^2$

(C) $\frac{X_1^2+X_2^2+X_3^2}{3}$

(D) $\frac{2X_1^2+X_2^2}{3}$

45. Let $X_1, X_2, \ldots, X_n$ be a random sample from the normal distribution with mean μ and variance 1. The most powerful test for $H_0 : \mu = \mu_0$ against $H_1 : \mu > \mu_0$ is reject H_0 if for a suitable constant k

(A) $\overline{X} < k$

(B) $\overline{X} \geq k$

(C) $\overline{X} \neq k$

(D) None of the above.

46. Let X is a Poisson distributed random variable with mean 2, then $E(2^X)$
(A) does not exist

(B) is $\exp\{2\}$

(C) is $\exp\{4\}$

(D) is $\exp\{6\}$.

47. There are 20 slips in a bag numbered $1, 2, \ldots, 20$. Three slips are taken without replacement, the probability that the smallest of these three is at most 7, the next smallest equal to 8 and the largest is at least 12 is
(A) strictly less than $\frac{1}{20}$

(B) at least $\frac{1}{20}$ but less than $\frac{1}{15}$

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(C) at least $\frac{1}{15}$ but less than $\frac{1}{10}$

(D) at least $\frac{1}{10}$.

48. Given the Linear Programming Problem maximize $8x_1 + 4x_2$ subject to

$$\begin{aligned} 3x_1 + 4x_2 &\geq 6 \\ 5x_1 + 2x_2 &\leq 10 \\ x_1 + 4x_2 &\leq 4, \\ \\ x_1 \geq 0, \ x_2 &\geq 0. \end{aligned}$$

From the graphical representation of these conditions the optimal solution is
(A) $(2, 0)$

(B) $(1, 3)$

(C) $(2, 1/2)$

(D) none of the above.

49. The value of $\sum_{x=0}^{n} 2^{2x} \binom{2n}{2x}$ is
(A) 3^{2n}

(B) 3^{2n-1}

(C) $\frac{3^{2n}-1}{2}$

(D) $\frac{3^{2n}+1}{2}$.

50. Let $A = \{x \in \Re : |x - 5| < 4\}$ and $B = \{x \in \Re : (x^2 - 3x + 2) > 0\}$.
Then, $A \cap B$ is
(A) empty

(B) an open interval

(C) a closed interval

(D) a set consisting of 2 points.