[3861]-151

F. E. (Semester - I) Examination - 2010

ENGINEERING MATHEMATICS - I

(2008 Pattern)

Time: 3 Hours

Mx. Marks: 100

Instructions:

- (1) Answers to the two sections should be written in separate answer books.
- (2) Black figures to the right indicate full marks.
- (3) Neat diagrams must be drawn wherever necessary.
- (4) Assume suitable data if necessary.
- (5) Use of electronic pocket calculator is allowed.

SECTION - I

Q.1) (A) Reduce the following matrix A to its normal form and hence find its rank, where [05]

$$A = \begin{bmatrix} 2 & -1 & 1 & 3 \\ 2 & 4 & -1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$$

(B) Examine the consistency of the system of the following equations. If consistent, solve system of the equations: [06]

1

$$x + y - z + t = 2$$

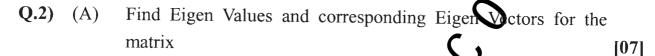
$$2x + 3y + 4t = 9$$

$$y - 2z + 3t = 2$$



$$\mathbf{A} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 2 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

OR



$$A = \begin{bmatrix} 2 & -1 & 1 \\ 1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

Examine whether the following ectors are linearly dependent. (B)

If so, find the relation between them:

$$X_1 = (2, -2, 4), X_2 = (-1, 3, -3), X_3 = (1, 1, 1)$$
 [05]

Find values of a, b, c so that the matrix (C)

$$A = \begin{bmatrix} 0 & 2b & c \\ a & b & c \\ a & & c \end{bmatrix}$$

becomes an orthogonal matrix.

Q.3) (A) If $\frac{Z-1}{i}$ is a purely imaginary number, then show that the locus of Z is a circle.

(B) Show that the continued product of all values of $(1 + i\sqrt{3})\frac{1}{4}$ is $2\left[\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}\right]$

is
$$2\left[\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}\right]$$
 [05]

[06]

[06]

(C)	If α , β are roots of an equation,	
	$\sin^2\theta z^2 - (2\sin\theta\cos\theta) z + 1 = 0$, prove that	
	$\alpha^n + \beta^n = 2\cos\theta \csc^n\theta$, where n is an integer.	[05]

OR

Q.4) (A) Find
$$\tanh x$$
 if $5 \sinh x - \cosh x = 5$ [05]

- (B) If $u + iv = \sin (x + iy)$, prove that:
 - (a) $u^2 \operatorname{cosec}^2 x v^2 \operatorname{sec}^2 x = 1$

(b)
$$u^2 \operatorname{sech}^2 x + v^2 \operatorname{cosech}^2 x = 1$$
 [05]

(C) A square lies above real axis it Argand's diagram and has two of its vertices at origin and the point 3 + 2i. Find the rest two vertices of the square. [06]

Q.5) (A) If
$$y = \frac{x^3}{x^2 - 1}$$
,

then find nth order differential coefficient of y w.r. to x. [05]

(B) If
$$y = \sin^{-1} [3x - 4x^3]$$
,
prove that $(X - x^2) y_{n+2} - (2n + 1) xy_{n+1} - n^2y_n = 0$. [05]

(C) Test convergence of the series: (Any One) [06]

(a)
$$1 + \frac{1}{2} + \frac{1 \cdot 3}{2 \cdot 4} + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6} + \dots$$

(b) $\sum_{n=1}^{\infty} \frac{(n+1) x^n}{n^2}$

OR

3

Q.6) (A) If
$$y = (2x + 1) \log (4x + 3)$$
,
then find y_{20} . [05]

(B) If
$$y = \left[x + \sqrt{x^2 - 1}\right]^m$$
,
prove that $(x^2 - 1) y_{n+2} + (2n + 1) xy_{n+1} + (n^2 - n^2) y_n = 0$. [05]

(C) Test convergence of the series: (Any One) [06]

(a)
$$\frac{1}{1^2 + m} + \frac{2}{2^2 + m} + \frac{3}{3^2 + m} + \dots$$

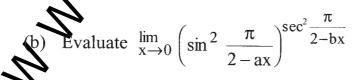
(b)
$$\sum_{n=1}^{\infty} \frac{4.7.10 \dots (3n+1)}{1.2.3.4 \dots n}$$

SECTION-JI

Q.7) (A) Expand
$$\frac{x}{e^x - 1}$$
 upto x^4 . [05]

- (B) Use Taylor's Theorem to obtain approximate value of $\sqrt{10}$ to four decimal places. [05]
- (C) Solve: (Any One) [06]
 - (a) Find a and b, if

$$\lim_{x \to 0} \frac{ash^2x + blogcosx}{x^4} = -\frac{1}{2}$$



OR

- Expand $\sin^{-1}x$ in ascending powers of x. Q.8) (A) [05]
 - Expland $3x^2 2x^2 + x 4$ in powers of (x + 2). (B) [05]
 - (C) Solve: (Any One) [06]
 - (a) Evaluate $\lim_{x\to 0} \frac{\tanh x 2\sin x + x}{x^5}$
 - (b) Evaluate $\lim_{x\to 0} \frac{e^x \sin x x x^2}{x^2 + x \log(1-x)}$
- Solve: (Any Two)

Find value of n so that
$$u = r^n(3\cos^2 \theta - 1)$$

satisfies $\frac{\partial}{\partial r} \left(r^2 \frac{\partial u}{\partial r} \right) + \frac{1}{\sin \theta} \left(\sin \theta \frac{\partial u}{\partial \theta} \right)$.

(B) If ux + vy = 0; $\frac{u}{x} = 1$,

show that
$$: \left(\frac{\partial v}{\partial v}\right)_x - \left(\frac{\partial u}{\partial x}\right)_y = \frac{x^2 + y^2}{x^2 - y^2}$$

If $x = e^u$ cesecv; $y = e^u$ cotv, then

show that :
$$\left(\frac{\partial z}{\partial x}\right)^2 - \left(\frac{\partial z}{\partial y}\right)^2 = e^{-2u} \left[\left(\frac{\partial z}{\partial u}\right)^2 - \sin^2 v \left(\frac{\partial z}{\partial v}\right)^2 \right]$$

OR

[16]

Q.10) Solve: (Any Two)

[16]

(A) If
$$u = (2x + 3y)^n + \frac{1}{(x - y)^n}$$
,

show that : $x^2 u_{xx} + 2xy u_{xy} + y^2 u_{yy} + xu_x + u_y = n^2 u$.

(B) If
$$x^3 + y^3 - 3axy = 0$$
,

show that : $\frac{d^2y}{dx^2} + \frac{2a^3 xy}{(y^2 - ax)^3} = 0.$

(C) If
$$f(xy^2, z - 2x) = 0$$
,

prove that : $x \frac{\partial z}{\partial x} - \frac{y}{2} \frac{\partial z}{\partial y} = 2x$.

Q.11) (A) The area of Δ ABC is calculated using the formula $\Delta = \frac{1}{2} \text{ absinC. Errors of 2\%, 3\%, 4\% are made in measuring a, b, C respectively. If the correct value of C is 30°, find % error in the calculated value of <math>\Delta$. [06]

(B) If
$$x = u + y$$
; $y = v^2 + w^2$; $z = w^3 + u^3$,

show that : $\frac{\partial u}{\partial x} = \frac{vw}{vw + u^2}$. [06]

(C) Find Stationary values of u = x + y + z if $xy + yz + zx = 3a^2$. [06]

OR

- Q.12) (A) Verify JJ' = 1 for $x = e^{u}tanv$ and $y = e^{u}secv$. [06]
 - (B) Examine for functional dependence/independence. If dependent, find relation between them:

$$u = \frac{x - y}{x + a} \qquad ; \qquad v = \frac{x + a}{y + a}$$
 [06]

(C) The sum of three positive numbers is 'a'. Determine maximum value of their product. [06]