1. If $f(x) = f(a - x)$ and $g(x) + g(a - x) = 2$, then the value of $\int_0^a f(x)\, g(x)\, dx$ is

(a) $\int_0^a f(x)\, dx$ (b) $\int_0^a g(x)\, dx$

(c) $\int_0^a [g(x) - f(x)]\, dx$ (d) $\int_0^a [g(x) + f(x)]\, dx$

Sol: Ans [a] Let $I = \int_0^a f(x)\, g(x)\, dx$

$$I = \int_0^a f(a-x)\, g(a-x)\, dx = \int_0^a f(x)\,.\,[2 - g(x)]\, dx = 2\int_0^a f(x)\, dx - \text{I}$$

$$\Rightarrow I = \int_0^a f(x)\, dx$$

2. The differential equation of the family of the curves $x^2 + y^2 - 2ax = 0$ is

(a) $x^2 - y^2 - 2xyy'' = 0$ (b) $y^2 - x^2 = 2xyy'$ (c) $x^2 + y^2 + 2y'' = 0$ (d) none of these

Sol: Ans [b] Differentiating the given equation we get

$$2x + 2y\frac{dy}{dx} - 2a = 0 \Rightarrow a = x + y\frac{dy}{dx}$$

$$\Rightarrow \quad x^2 + y^2 - 2x\left(x + y\frac{dy}{dx}\right) = 0$$

$$\Rightarrow \quad y^2 - x^2 - 2xy\frac{dy}{dx} = 0$$

3. If $y = \cos^{-1}\left(\frac{1 - \ln x}{1 + \ln x}\right)$, then $\frac{dy}{dx}$ at $x = e$ is

(a) $-\frac{1}{e}$ (b) $-\frac{1}{2e}$ (c) $\frac{1}{2e}$ (d) $\frac{1}{e}$

Sol: Ans [b] $y = \cos^{-1}\left(\frac{1 - \ln x}{1 + \ln x}\right)$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{-1}{\sqrt{1-\left(\frac{1-\ln x}{1+\ln x}\right)^2}} \cdot \frac{(1+\ln x)\left(-\frac{1}{x}\right)-(1-\ln x)\frac{1}{x}}{(1+\ln x)^2}$$

$$\Rightarrow \quad \left(\frac{dy}{dx}\right)_{x=e} = -\frac{1}{2e}$$

4. The sum of the series $\frac{1}{2}+\frac{3}{4}+\frac{7}{8}+\frac{15}{16}+\dots$ upto n-terms is

(a) $n-1+\frac{1}{2^n}$ (b) $n+\frac{1}{2^n}$ (c) $2n+\frac{1}{2^n}$ (d) $n+1+\frac{1}{2^n}$

Sol: Ans [a] $T_n = 1-\frac{1}{2^n}$

$$S_n = n-\left(\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\dots\frac{1}{2^n}\right) = n-\frac{\left(\frac{1}{2}\right)\left(1-\frac{1}{2^n}\right)}{\left(1-\frac{1}{2}\right)} = n-1+\frac{1}{2^n}$$

5. The equation of the plane passing through the mid point of the line of join of the points (1, 2, 3) and (3, 4, 5) and perpendicular to it is

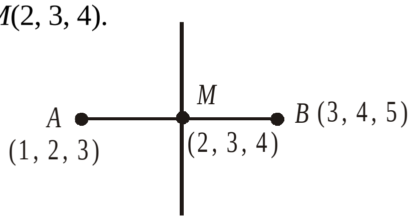
(a) $x + y + z = 9$ (b) $x + y + z = -9$ (c) $2x + 3y + 4z = 9$ (d) $2x + 3y + 4z = -9$

Sol: Ans [a] The mid point of the line of join of the points is $M(2, 3, 4)$.

Hence equation of the plane is

$2(x - 2) + 2(y - 3) + 2(z - 4) = 0$

$x + y + z = 9$



6. The equation of the circle concentric to the circle $2x^2 + 2y^2 - 3x + 6y + 2 = 0$ and having area double the area of this circle is

(a) $8x^2 + 8y^2 - 24x + 48y - 13 = 0$ (b) $16x^2 + 16y^2 + 24x - 48y - 13 = 0$

(c) $16x^2 + 16y^2 - 24x + 48y - 13 = 0$ (d) $8x^2 + 8y^2 + 24x - 48y - 13 = 0$

Sol: Ans [c] The given circle is

$$x^2 + y^2 - \frac{3}{2}x + 3y + 1 = 0$$

Its centre is $\left(\frac{3}{4}, -\frac{3}{2}\right)$ and radius = $\sqrt{\frac{9}{16}+\frac{9}{4}-1} = \sqrt{\frac{9+36-16}{16}}$ = $\sqrt{\frac{29}{16}}$

Area of required circle = $\pi r^2 = 2\pi\times\frac{29}{16} \Rightarrow r^2 = \frac{29}{8} \Rightarrow r = \sqrt{\frac{29}{8}}$

Equation of required circle = $\left(x-\frac{3}{4}\right)^2+\left(y\ \frac{3}{2}\right)^2 = \frac{29}{8}$

$\Rightarrow \quad x^2-\frac{3}{2}x+y^2+3y+\frac{9}{16}+\frac{9}{4}-\frac{29}{8}=0$

$\Rightarrow \quad x^2-\frac{3}{2}x+y^2+3y-\frac{13}{16}=0$

$\Rightarrow \quad 16x^2 + 16y^2 - 24x + 48y - 13 = 0$

7. The domain of the function $f(x)=\frac{\cos^{-1}x}{[x]}$ is

(a) $[-1, 0)\cup\{1\}$ (b) $[-1, 1]$ (c) $[-1, 1)$ (d) none of these

Sol: Ans [a] For given function : $-1\le x\le 1$ but

$[x]\neq 0 \Rightarrow x\notin[0,1) \Rightarrow x\in[-1,0)\cup\{1\}$

8. Let $f(x)=\begin{cases}\dfrac{\tan x-\cot x}{x-\frac{\pi}{4}}, & x\neq\frac{\pi}{4}\\ a & x=\frac{\pi}{4}\end{cases}$

the value of *a* so that *f*(*x*) is continuous at $x=\frac{\pi}{4}$ is

(a) 2 (b) 4 (c) 3 (d) 1

Sol: Ans [b] $a = \underset{x\to\frac{\pi}{4}}{Lt}\ \frac{\tan x-\cot x}{x-\frac{\pi}{4}} = \frac{\sec^2 x+\text{cosec}^2 x}{1} = 2+2=4$

9. If e and e' are the eccentricities of hyperbolas $\frac{x^2}{z^2} - \frac{y^2}{b^2} = 1$ and its conjugate hyperbola, then the value of $\frac{1}{e^2} + \frac{1}{e'^2}$ is

(a) 0 (b) 1 (c) 2 (d) None of these

Sol: Ans [b] $e^2 = 1 + \frac{b^2}{a^2}$, $e'^2 = 1 + \frac{a^2}{b^2}$

$$\frac{1}{e^2} = \frac{a^2}{a^2 + b^2}, \quad \frac{1}{e'^2} = \frac{b^2}{a^2 + b^2}$$

$$\therefore \quad \frac{1}{e^2} + \frac{1}{e'^2} = 1$$

10. The value of the $\int \frac{\sin x + \cos x}{3 + \sin 2x} dx$ is

(a) $\frac{1}{4} \ln\left(\frac{2 - \sin x - \cos x}{2 + \sin x + \cos x}\right) + c$ (b) $\frac{1}{2} \ln\left(\frac{2 + \sin x}{2 - \sin x}\right) + c$

(c) $\frac{1}{4} \ln\left(\frac{1 + \sin x}{1 - \sin x}\right) + c$ (d) none of these

Sol: Ans [a] $I = \int \frac{\sin x + \cos x}{3 + \sin x} dx = \int \frac{\sin x + \cos x}{4 - (\sin x - \cos x)^2} dx$

Put $\sin x - \cos x = t$ and $(\sin x + \cos x) = dt$

$$\Rightarrow \quad I = \int \frac{dt}{4 - t^2} = \frac{1}{4} \ln \frac{2 - t}{2 + t} = \frac{1}{4} \ln\left(\frac{2 - \sin x - \cos x}{2 + \sin x + \cos x}\right) + c$$

11. The solution of the differential equation

$$\frac{dy}{dx} - \frac{\tan y}{x} = \frac{\tan y \sin y}{x^2} \text{ is}$$

(a) $\frac{x}{\sin y} + \ln x = c$ (b) $\frac{y}{\sin x} + \ln = c$ (c) $\ln y + x = c$ (d) $\ln x + y = c$

Sol: Ans [a] $\frac{dy}{dx} - \frac{\tan y}{x} = \frac{\tan y \sin y}{x^2}$

$$\cot y \operatorname{cosec} y \frac{dy}{dx} - \frac{\operatorname{cosec} y}{x} = \frac{1}{x^2}$$

Put $-\operatorname{cosec} y = t$

$$\Rightarrow \quad \operatorname{cosec} y \cot y \frac{dy}{dx} = \frac{dt}{dx}$$

$$\Rightarrow \quad \frac{dt}{dx} + \frac{t}{x} = \frac{1}{x^2}$$

I.F. = $e^{\int \frac{1}{x} dx} = x$

$$\Rightarrow \quad tx = \int \frac{1}{x} dx + c \quad \Rightarrow \quad tx = \ln x + c \quad \Rightarrow \quad \frac{x}{\sin y} + \ln x = c$$

12. For a party 8 guests are invited by a husband and his wife. They sit around a circular table for dinner. The probability that the husband and his wife sit together is

(a) $\frac{2}{7}$ (b) $\frac{2}{9}$ (c) $\frac{1}{9}$ (d) $\frac{4}{9}$

Sol: Ans [b] The favorable ways of sitting are = 2 × 8 !

Total number of ways of sitting are = 9 !

$$\text{Probability} = \frac{2 \times 8!}{9!} = \frac{2}{9}$$

13. If $I_m\left(\frac{z-1}{2z+1}\right) = -4,$ then locus of z is

(a) ellipse (b) parabola (c) straight line (d) circle

Sol: Ans [d] $I_m\left(\frac{(x-1)+iy}{(2x+1)+2iy}\right) = -4$

$\Rightarrow$ $y(2x + 1) - 2y(x - 1) = -4[(2x + 1)^2 + 4y^2]$

$\Rightarrow$ It is a circle

14. The equation $(x - b)(x - c) + (x - a)(x - b) + (x - a)(x - c) = 0$ has all its roots.

(a) positive (b) real (c) imaginary (d) negative

Sol: Ans [b] The equation is

$3x^2 - 2(a + b + c)x + ab + bc + ca = 0$

$\Delta = 4(a + b + c)^2 - 12(ab + bc + ca)$

$= 4[a^2 + b^2 + c^2 - ab - bc - ca]$

$= 2[(a - b)^2 + (b - c)^2 + (c - a)^2] \quad \Rightarrow \quad \Delta > 0$

15. The sum of coefficients of the expansion $\left(\frac{1}{x}+2x\right)^n$ is 6561. The coefficient of term independent of x is

(a) 16.8_{C_4} (b) 8_{C_4} (c) 8_{C_5} (d) none of these

Sol: Ans [a] Put $x = 1$ we get $3^n = 6561 = 3^8 \Rightarrow n = 8$

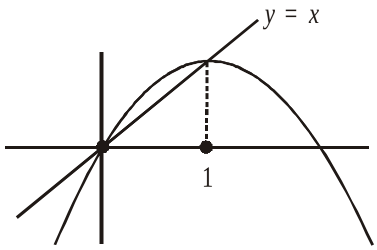
$$T_{r+1} = 8_{C_r}\left(\frac{1}{x}\right)^r (2x)^{8-r} = 8_{C_r}\ 2^{8-r}\ x^{8-2r} \Rightarrow r = 4$$

$\Rightarrow$ Coefficient = $8_{C_4}\ 2^4 = 16.8_{C_4}$

16. The area enclosed between the curves $y = x$ and $y = 2x - x^2$ is

(a) $\frac{1}{2}$ (b) $\frac{1}{6}$ (c) $\frac{1}{3}$ (d) $\frac{1}{4}$

Sol: Ans [b] Solving the curves,



$x = 2x - x^2 \Rightarrow x = 0$ and $x = 1$.

$$\text{Area} = \int_0^1 \left[(2x-x^2)-x\right]dx = \int_0^1 (x-x^2)\,dx = \left(\frac{x^2}{2}-\frac{x^3}{3}\right)_0^1 = \frac{1}{2}-\frac{1}{3}=\frac{1}{6}$$

17. There are 12 white and 12 red balls in a bag. Balls are drawn one by one with replacement from the bag. The probability that 7th drawn ball is 4th white is

(a) $\frac{1}{4}$ (b) $\frac{1}{8}$ (c) $\frac{1}{2}$ (d) $\frac{1}{3}$

Sol: Ans [c] The probability that 7th drawn ball is 4th white = $\frac{1}{2}$

Since balls are replaced.

18. In an ellipse the angle between the lines joining the foci with the positive end of minor axis is a right angle, the eccentricity of the ellipse is

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\sqrt{2}$ (d) $\sqrt{3}$

Sol: Ans [a] Given that $b = ac \Rightarrow b^2 = a^2 e^2$

Again $a^2 e^2 = a^2 (1 \quad e^2)$

$$2e^2 = 1 \Rightarrow e = \frac{1}{\sqrt{2}}$$

19. If $|\bar{a}| = 3, |\bar{b}| = 5$ and $|\bar{c}| = 4$ and $\bar{a} + \bar{b} + \bar{c} = 0$, then the value of $\bar{a}.\bar{b} + \bar{b}.\bar{c}$ is equal to

(a) 0 (b) –25 (c) 25 (d) none of these

Sol: Ans [b] $\bar{a} + \bar{b} + \bar{c} = 0 \Rightarrow \bar{a}.\bar{b} + |\bar{b}|^2 + \bar{b}.\bar{c} = 0 \Rightarrow \bar{a}.\bar{b} + \bar{b}.\bar{c} = -|\bar{b}|^2 = -25$

20. The equation of a line is $6x – 2 = 3y – 1 = 2z – 2$ the direction ratios of the line are

(a) 1, 2, 3 (b) 1, 1, 1 (c) $\frac{1}{3}, \frac{1}{3}, \frac{1}{3}$ (d) $\frac{1}{3}, \frac{-1}{3}, \frac{1}{3}$

Sol: Ans [a] $6x – 2 = 3y – 1 = 2z – 2 \Rightarrow \frac{x - \frac{1}{3}}{1} = \frac{y - \frac{1}{3}}{2} = \frac{z-1}{3}$

$\Rightarrow$ (1, 2, 3) are direction ratios.

21. The value of $\int_0^{\frac{\pi}{2}} \frac{\sin^2 x - \cos^2 x}{\sin^3 x + \cos^3 x} dx$ is

(a) 0 (b) (c) (d)

Sol: Ans [a] $\int_0^{\frac{\pi}{2}} \frac{\sin^2 x - \cos^2 x}{\sin^3 x + \cos^3 x} dx = 0$

Since $f\left(\frac{\pi}{2} - x\right) = -f(x)$

22. The value of $\int \frac{dx}{x + \sqrt{x-1}}$ is

(a) $\log\left(x + \sqrt{x-1}\right) + \sin^{-1}\left(\sqrt{\frac{x-1}{x}}\right) + c$ (b) $\log\left(x + \sqrt{x-1}\right) + c$

(c) $\ln\left(x + \sqrt{x-1}\right) - \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{2\sqrt{x-1}+1}{\sqrt{3}}\right) + c$ (d) none of these

Sol: Ans [c] $\int \frac{dx}{x + \sqrt{x-1}}$ Put $x – 1 = t^2$

$dx = 2t\, dt$

$$= \int \frac{2t\, dt}{t^2 + 1 + t} = \int \frac{2t+1}{t^2+t+1} dt - \int \frac{1}{t^2+t+1} dt$$

$$= \ln\left(t^2 + t + 1\right) - \int \frac{dt}{\left(t + \frac{1}{2}\right)^2 + \frac{3}{4}} = \ln\left(t^2 + t + 1\right) - \frac{2}{\sqrt{3}} \tan^{-1} \frac{t + \frac{1}{2}}{\frac{\sqrt{3}}{2}} + c$$

$$= \ln\left(x + \sqrt{x-1}\right) - \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{2\sqrt{x-1}+1}{\sqrt{3}}\right) + c$$

23. $y = \sin^{-1}\frac{x}{2} + \cos^{-1}\frac{x}{2}$, then the value of $\frac{dy}{dx}$ is

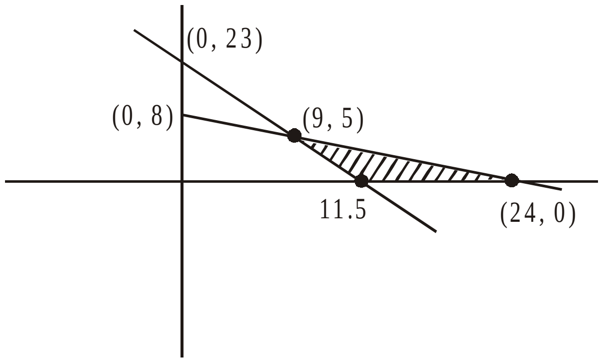
(a) 1 (b) –1 (c) 0 (d) 2

Sol: Ans [c] $y = \frac{\pi}{2} \Rightarrow \frac{dy}{dx} = 0$

24. $z = 4x + 2y$, $4x + 2y \geq 46$, $x + 3y \leq 24$ and x and y are greater than or equal to zero, then the maximum value of z is

(a) 46 (b) 96 (c) 52 (d) none of these

Sol: Ans [b] $z(9, 5) = 36 + 10 = 46$



$z(11.5, 0) = 46$

$z(24, 0) = 96$

$y^2 = 4x$ $x^2 = 4y - 12$ $\frac{y^4}{16} = 4y - 12$

25. On one bank of river there is a tree. On another bank, an observer makes an angle of elevation of 60º at the top of the tree. The angle of elevation of the top of the tree at a distance 20 m away from the bank is 30º. The width of the river is

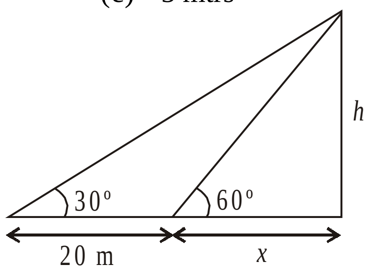
(a) 20 mtrs (b) 10 mtrs (c) 5 mtrs (d) 1 m

Sol: Ans [b] $h = x\sqrt{3}$

and $h = \frac{(x+20)}{\sqrt{3}}$

$\Rightarrow 3x = x + 20$

$\Rightarrow x = 10$ mtrs



26. The magnitude of cross product of two vectors is $\sqrt{3}$ times the dot product the angle between the vectors is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

Sol: Ans [b] $|\bar{a} \times \bar{b}| = \sqrt{3}\,|\bar{a} . \bar{b}|$

$\tan\theta = \sqrt{3}$

$\theta = \frac{\pi}{3}$

27. If $Dr = \begin{vmatrix} r & 1 & \frac{n(n+1)}{2} \\ 2r-1 & 4 & n^2 \\ 2^{r-1} & 5 & 2^n - 1 \end{vmatrix}$, then the value of $\sum_{r=0}^{n} Dr$ is

(a) 0 (b) 1 (c) $\frac{n(n+1)(2n+1)}{6}$ (d) none of these

Sol: Ans [a] $\sum_{r=0}^{n} Dr = \begin{vmatrix} \sum r & 1 & \frac{n(n+1)}{2} \\ 2\sum r - 1 & 4 & n^2 \\ 2^{\sum r - \sum 1} & 5 & 2^n - 1 \end{vmatrix} = 0$

28. If $\begin{vmatrix} -12 & 0 & \lambda \\ 0 & 2 & -1 \\ 2 & 1 & 15 \end{vmatrix} = 546$ then the value of λ is

(a) –1 (b) –2 (c) –3 (d) 4

Sol: Ans [c] Solving we get $\lambda = -3$

29. If $A = \begin{bmatrix} 1 & x \\ x^2 & 4y \end{bmatrix}$ and $B = \begin{bmatrix} -3 & 1 \\ 1 & 0 \end{bmatrix}$

adj. $A + B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then values of x and y are

(a) 1, 1 (b) $(\pm 1, 1)$ (c) 1, 0 (d) none of these

Sol: Ans [a] $\begin{bmatrix} 4y & -x^2 \\ -x & 1 \end{bmatrix} + \begin{bmatrix} -3 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow$ $4y - 3 = 1$ $\Rightarrow$ $y = 1$

$-x + 1 = 0$ $\Rightarrow$ $x = 1$

30. If $\tan^{-1}\frac{1-x}{1+x} = \frac{1}{2}\tan^{-1} x,$ then value of x is

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\sqrt{3}$ (d) 2

Sol: Ans [b] Let $x = \tan\theta$

$$\tan^{-1}\tan\left(\frac{\pi}{4}-\theta\right)=\frac{1}{2}\tan^{-1}(\tan\theta)$$

$$\frac{\pi}{4}-\theta=\frac{\theta}{2} \quad \Rightarrow \quad \frac{3\theta}{2}=\frac{\pi}{4} \quad \Rightarrow \quad \theta=\frac{\pi}{6}$$

$$x=\tan\frac{\pi}{6}=\frac{1}{\sqrt{3}}$$

31. The number of values of k for which $(\log x)^2 - \log x - \log k = 0$ (is/are)

(a) 1 (b) 2 (c) 3 (d) 4

Sol: Ans [b] Let $\log x = t$ then

$t^2 - t = \log k$

$\Rightarrow$ k will have two values

32. The value of $\underset{\alpha\to 0}{Lt}\dfrac{\operatorname{cosec}^{-1}(\sec\alpha)+\cot^{-1}(\tan\alpha)+\cot^{-1}\cos(\sin^{-1}\alpha)}{\alpha}$ is

(a) 0 (b) –1 (c) –2 (d) 1

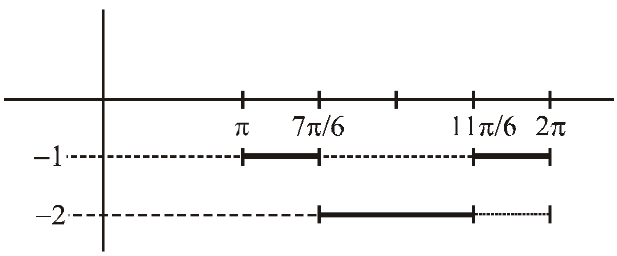
Sol: Ans [c] $\underset{\alpha\to 0}{Lt}\dfrac{\frac{\pi}{2}-\alpha+\frac{\pi}{2}-\alpha+\cot^{-1}\sqrt{1-\alpha^2}}{\alpha}$

$$-2-\frac{1}{2}\times 0=-2$$

33. The value of $\displaystyle\int_{\pi}^{2\pi}[2\sin x]\,dx$ is

(a) $\frac{\pi}{3}$ (b) $-\frac{4\pi}{3}$ (c) $\frac{4\pi}{3}$ (d) $-\frac{\pi}{3}$

Sol: Ans [b]



$$\int_{\pi}^{2\pi}[2\sin x]\,dx=-\frac{\pi}{6}-\pi-\frac{\pi}{6}=-\frac{4\pi}{3}$$