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B.Sc.(Hons.) II Sem./Computer Science

Paper 203–Calculus II

(Admissions of 2001 and onwards)

Time   3 Hours   *Maximum Marks   75*

(Write your Roll No on the top immediately

on receipt of this question paper)

All *questions are compulsory*

All *questions carry equal marks*

Use of calculator is permitted

1   State min-max inequality for integrals By dividing the interval [0, 1] into two subintervals, use min-max inequality to prove that

$$\frac{7}{12} \leq \int_0^{\cdot} \frac{1}{1+x}\,dx \leq \frac{5}{6}$$

2   Graph the function

$$f(x) = 2 - |x|, \ -1 \leq x \leq 1$$

using area, evaluate the integral $\int^1 (2 - |x|)\,dx$

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3   Solve the following initial value problem for $\vec{r}$ as a function of t.

$$\frac{d\vec{r}}{dt} = \frac{3}{2}(t+1)^{1/2}\,\hat{i} + e^{-t}\,\hat{j} + \frac{1}{t+1}\,\hat{k}$$

$$\vec{r}(0) = \hat{k}$$

4   A solid lies between planes perpendicular to the x-axis at $x = -1$ and $x = 1$. The cross sections perpendicular to the x-axis between these planes are squares whose diagonals run from the parabola $y = -\sqrt{x}$ to the parabola $y = \sqrt{x}$. Find the volume of the solid generated.

5   Find the average value of the function $f(x, y) = xy$ over the region D, where $D = \{(x, y)\ \ x^2 + y^2 \le 1,\ x \ge 0,\ y \ge 0\}$. Also find two points in D where $f(x, y)$ takes its average value.

6   Evaluate by changing to polar co-ordinates, the integral

$$\int_{1}^{1}\int_{0}^{\sqrt{x}} e^{-(x^2+y^2)}\,dy\,dx$$

7   Find the centre of mass of a thin plate of constant density δ, bounded by the lines $x = 0$, $y = x$ and the parabola $y = 2 - x^2$ in the first quadrant.

8   Evaluate the integral

$$\int \int_{0}^{-x} \int_{0}^{4-x^2-y} x\,dz.dy.dx$$

9   Find the surface area of the solid generated by revolving

the curve $y=\sqrt{x+1}$, $1 \leq x \leq 5$ about X-axis

10   Find the Fourier Series of the function $f(x) = |x|$,

$-\pi \leq x \leq \pi$, what is the sum of the series at $x = 0, \dfrac{3\pi}{2}, 3\pi$?

11   Verify that $u = x^2 - y^2 - y$ is harmonic in the whole
complex plane and find a conjugate harmonic function
V of u

12   Find

(i) $\ln (4 + 3i)$

(ii) z satisfying $\ln z = 4 - 3i$

(iii)   Principal value of i^i

13.   Evaluate $\int R e z \, dz$

where C is vertical from $1 + i$ to $1 + 2i$, then horizontal
to $3 + 2i$

14.   Find Laurent series of expansion of ·

$f(z) = \dfrac{-2z+3}{z^2 - 3z + 2}$ with centre O

15.   For counterclockwise circle $C . |z| = \dfrac{3}{2}$, evaluate

$\int \dfrac{\tan z}{z^2 - 1} dz$ using residue theorem