Date : 11-04-2010 **Duration : 3 Hours** **Max. Marks : 237**

PAPER - 2

Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.

INSTRUCTIONS

A. General :

1. This Question Paper contains 57 questions.
2. The question paper CODE is printed on the right hand top corner of this sheet and also on the back page of this booklet.
3. No additional sheets will be provided for rough work.
4. Blank papers, clipboard, log tables, slide rules, calculators, cellular phones, pagers and electronic gadgets in any form are not allowed.
5. **Log and Antilog tables** are given
6. The answer sheet, a machine-gradable **Objective Response Sheet (ORS)**, is provided separately.
7. Do not Tamper / mutilate the **ORS** or this Booklet.
8. Do not break the seals of the question-paper booklet before being instructed to do so by the invigilators.

B. Filling the bottom-half of the ORS :

9. The ORS has **CODE** printed on its lower and upper parts.
10. Make sure the **CODE** on the ORS is the same as that on this booklet. **If these Codes do not Match, ask for a change of the Booklet.**
11. Write your Registration No., Name and Name of Centre and Sign with pen in appropriate boxes. **Do not write these anywhere else.**
12. Darken the appropriate bubbles **under** each digit of your Registration No. with **HB Pencil**.

C. Question paper format and Marking scheme :

13. The question paper consists of 3 parts (Chemistry, Mathematics and Physics) and each part consists of **four** Sections.
14. For each question in **Section–I :** you will be **awarded 5 marks** if you have darkened **only** the bubble corresponding to the correct answer and **zero mark** if no bubbles are darkened. In all other cases, **minus two (–2) mark** will be awarded.
15. For each question in **Section–II :** you will be **awarded 3 marks** if you have darken the bubble corresponding to the correct answer and **zero mark** if no bubble is darkened. No negative marks will be awarded for incorrect answers in this Section.
16. For each question in **Section–III :** you will be **awarded 3 marks** if you darken **only** the bubble corresponding to the correct answer and **zero mark** if no bubbles are darkened. In all other cases, **minus one (–1) mark** will be awarded.
17. For each question in **Section–IV :** you will be **awarded 2 marks** for each row in which you have darkened the bubble(s) corresponding to the correct answer. Thus, each question in this section carries a maximum of **8 marks.** There is **no negative marks** awarded for incorrect answer(s) in this Section.

Useful Data :

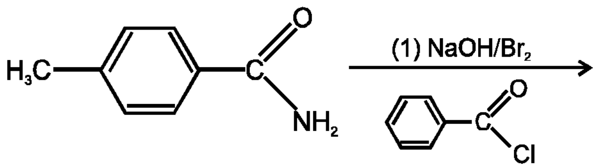
Atomic Numbers : Be 4; N 7; O 8; Al 13 ; Si 14; Cr 24 ; Fe 26; Fe 26; Zn 30; Br 35.

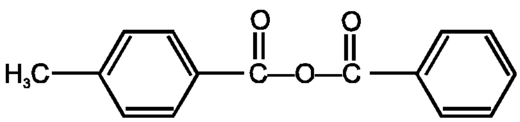
1 amu = 1.66×10^{-27} kg	R = 0.082 L-atm K^{-1} mol^{-1}
h = 6.626×10^{-34} J s	$N_A = 6.022 \times 10^{23}$
$m_e = 9.1 \times 10^{-31}$ kg	e = 1.6×10^{-19} C
c = 3.0×10^{8} m s^{-1}	F = 96500 C mol^{-1}
$R_H = 2.18 \times 10^{-18}$ J	$4\pi\varepsilon_0 = 1.11 \times 10^{-10}$ J^{-1} C^2 m^{-1}

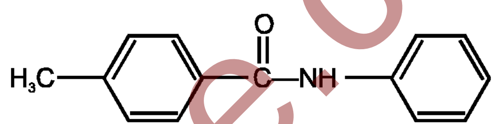
CHEMISTRY

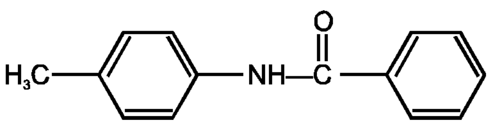
PART-I
SECTION - I
(*Single Correct Choice Type)*

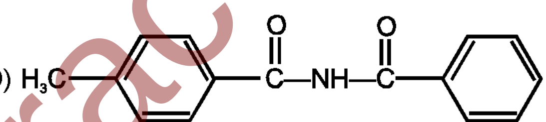
This section contains 6 multiple choice questions. Each question has four choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

1. In the reaction  T, the strucutre of the Product T is :

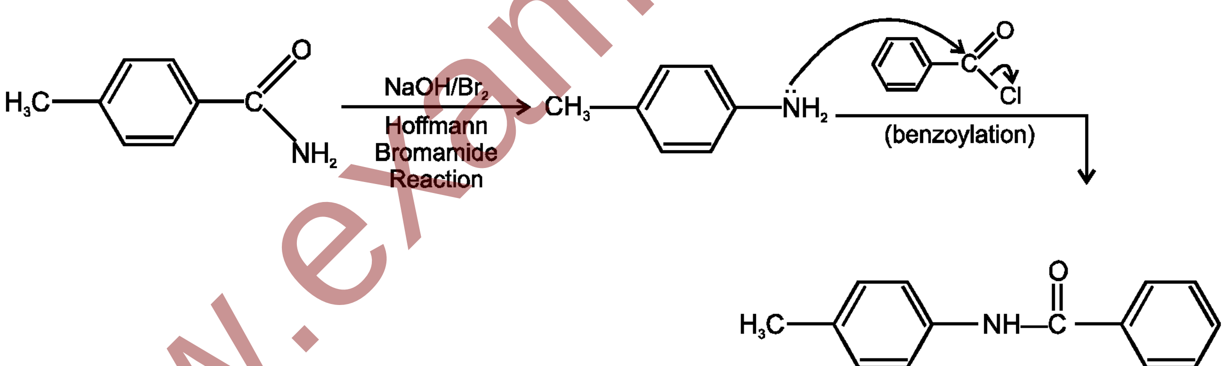
(A) 

(B) 

(C) 

(D) 

Ans. (C)

Sol. 

(Anilide)

2. Assuming that Hund's rule is violated, the bond order and magnetic nature of the diatomic molecule B_2 is :

(A) 1 and diamagnetic
(B) 0 and diamagnetic
(C) 1 and paramagnetic
(D) 0 and paramagnetic

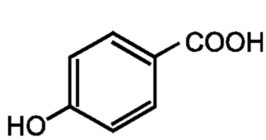
Ans. (A)

Sol. B_2 (total number of electrons = 10)

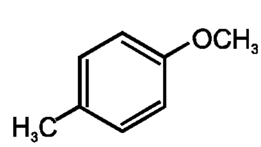
$$\sigma_{1s}^{2}\ \sigma^{*}{}_{1s}^{2}\ \sigma_{2s}^{2}\ \sigma^{*}{}_{2s}^{2}\ \pi_{2p_x}^{2} = \pi_{2p_y}^{0}\ \sigma_{2p_z}^{0}$$

So, bond order = $\frac{6-4}{2}$ = 1 and molecule will be diamagnetic.

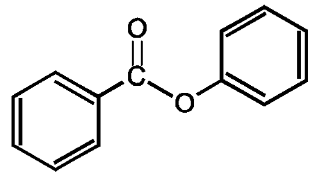
3. The compounds P, Q and S



P



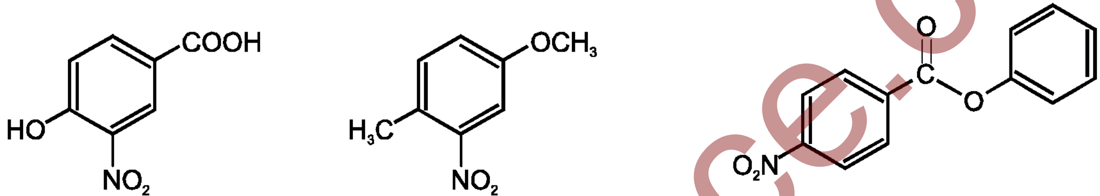
Q



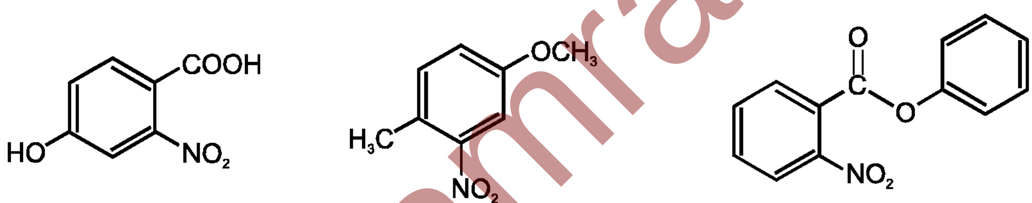
S

were separately subjected to nitration using HNO_3 / H_2SO_4 mixture. The major product formed in each case respectively, is :

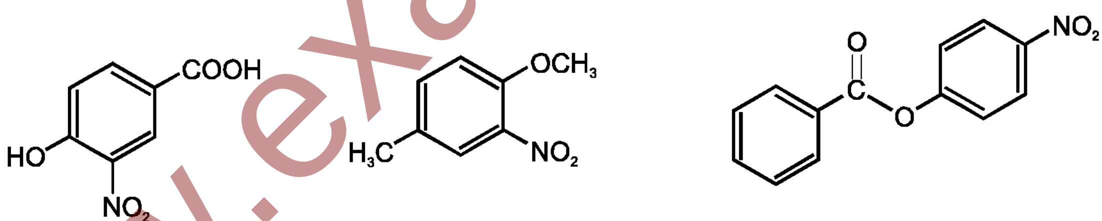
(A)



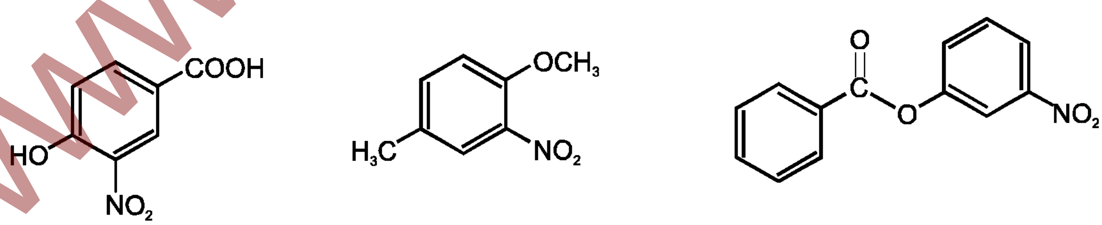
(B)



(C)

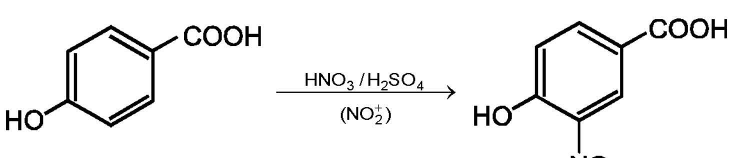


(D)



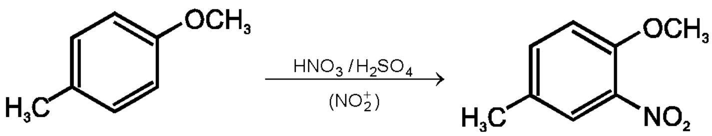
Ans. **(C)**

Sol.



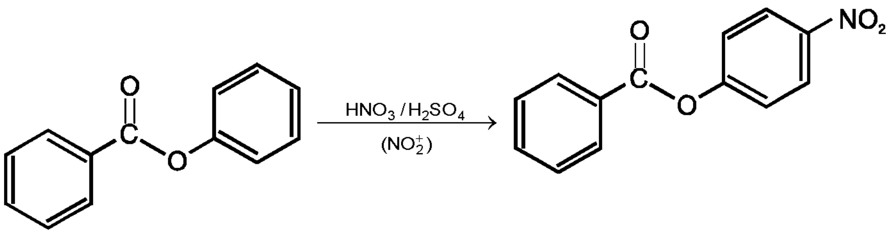
P

(–OH is o/p director)



Q

(–OCH_3 is stronger activator)



S

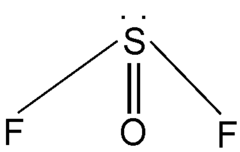
(Substitution takes place in activated ring at least crowded p-position)

4. The species having pyramidal shape is :

(A) SO_3 (B) BrF_3 (C) SiO_3^{2-} (D) OSF_2

Ans. (D)

Sol.



pyramidal shape

5. The complex showing a spin-only magnetic moment of 2.82 B.M. is :

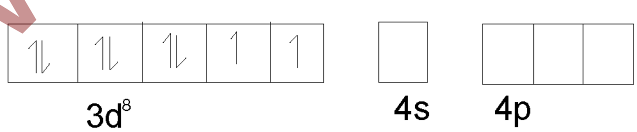
(A) $Ni(CO)_4$ (B) $[NiCl_4]^{2-}$ (C) $Ni(PPh_3)_4$ (D) $[Ni(CN)_4]^{2-}$

Ans. (B)

Sol. Ni : $3d^8$ $4s^2$

Ni^{2+}, $3d^8$

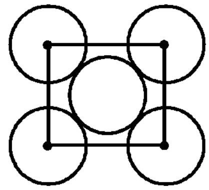
since Cl is a weak field ligand, so it will not cause a paring of electron.



N = 2

$\mu = \sqrt{N(N+2)} = \sqrt{2(2+2)}$ B.M. $= \sqrt{8}$ B.M. = 2.82 B.M.

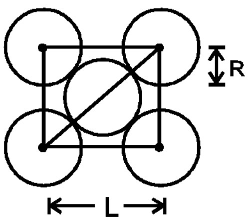
6. The packing efficiency of the two dimensional square unit cell shown below is :



(A) 39.27% (B) 68.02% (C) 74.05% (D) 78.54%

Ans. **(D)**

Sol.



$4R = L\sqrt{2}$

so, $L = 2\sqrt{2}R$

Area of square unit cell = $(2\sqrt{2}R)^2 = 8R^2$

Area of atoms present in one unit cell = $\pi R^2 + 4\left(\frac{\pi R^2}{4}\right) = 2\pi R^2$

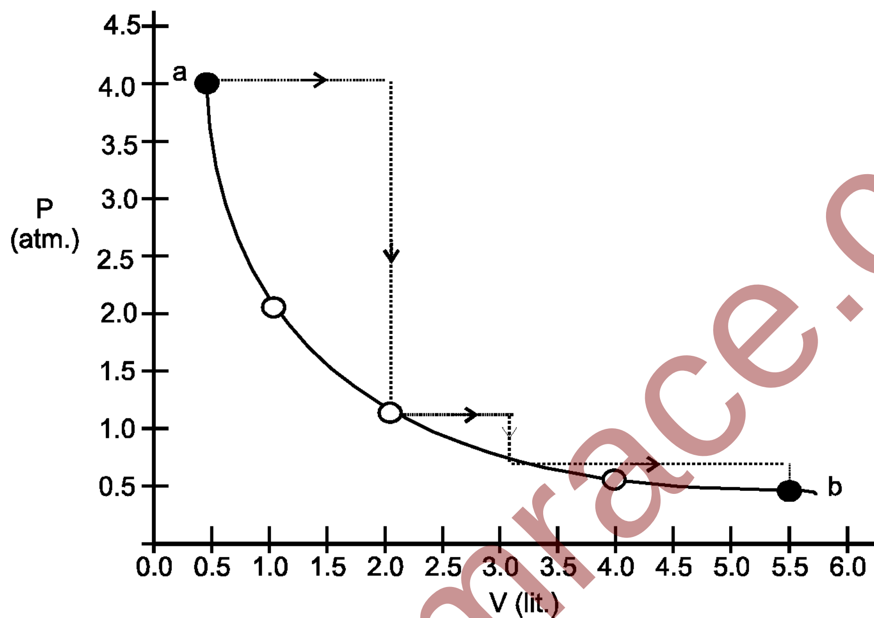
so, packing efficiency = $\frac{2\pi R^2}{8R^2} \times 100$

$= \frac{\pi}{4} \times 100 = 78.54\%$

SECTION - II
(Integer Type)

This section contains 5 questions. The answer to each question is a single-digit integer, ranging from 0 to 9. The correct digit below the question number in the ORS is to be bubbled.

7. One mole of an ideal gas is taken from **a** and **b** along two paths denoted by the solid and the dashed lines as shown in the graph below. If the work done along the solid line path is w_s and that along the dotted line path is w_d, then the integer closest to the ratio w_d / w_s is :



Ans. 2

Sol. Process shown by solid line is reversible isothermal

So, work $W_s = -4 \times 0.5 \ln (5.5/0.5)$

$= -2 \ln 11$ L atm.

For dotted process (three step irreversible) work done will be

$$W_d = -\{4 \times 1.5 + 1 \times 1 + \frac{2}{3} \times 2.5\} \text{ L atm.}$$

$$= -\{6 + 1 + \frac{5}{3}\} \text{ L atm.} \quad = -\frac{26}{3} \text{ L atm.}$$

$$\text{so, } \frac{W_d}{W_s} = \frac{26}{3 \times 2 \ln 11} \approx 2.$$

8. Among the following, the number of elements showing only one non-zero oxidation state is :

O, Cl, F, N, P, Sn, Tl, Na, Ti

Ans. 2

Sol. Only Na & F will show one non-zero oxidation state. These are Na^+ & F^-.

9. Silver (atomic weight = 108 g mol^{-1}) has a density of 10.5 g cm^{-3}. The number of silver atoms on a surface of area 10^{-12} m^2 can be expressed in scientific notation as $y \times 10^x$. The value of x is :

Ans. 7

Sol. Volume of one mole of silver atoms

$$= \frac{108}{10.5} \ cm^3/mole$$

$$\text{volume of one silver atom} = \frac{108}{10.5} \times \frac{1}{6.022 \times 10^{23}} cm^3$$

$$\text{so, } \frac{4}{3}\pi R^3 = \frac{108}{10.5} \times \frac{1}{6.022 \times 10^{23}} = 1.708 \times 10^{-23}$$ [neglecting the void space]

$R^3 = 0.407 \times 10^{-23}$ cm^3

$R^3 = 0.407 \times 10^{-29}$ m^3

Area of each silver atom

$\pi R^2 = \pi \times (0.407 \times 10^{-29} \ m^3)^{2/3}$

so, number of silver atoms in given area.

$$= \frac{10^{-12}}{(0.407 \times 10^{-29} m^3)^{2/3}} = \frac{10^8}{(\pi \times 2)}$$

$= 1.6 \times 10^7 = y \times 10^x$

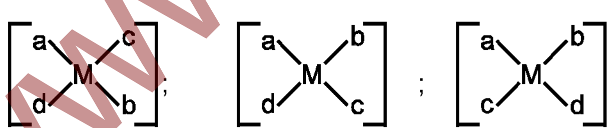
$x = 7$

10. Total number of geometrical isomers for the complex $[RhCl(CO)(PPh_3)(NH_3)]$ is :

Ans. 3

Sol. [M(abcd)] complex is square planar, so will have 3 geometrical isomers.

(i) (a T b) (c T d) ; (ii) (a T c) (b T d) ; (iii) (a T d) (b T c)



11. The total number of diprotic acids among the following is :

H_3PO_4	H_2SO_4	H_3PO_3	H_2CO_3	$H_2S_2O_7$
H_3BO_3	H_3PO_2	H_2CrO_4	H_2SO_3	

Ans. 6

Sol. H_2SO_4 , H_3PO_3 , H_2CO_3 , $H_2S_2O_7$, H_2CrO_4 , H_2SO_3

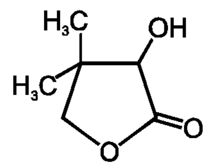
All are diprotic acids

SECTION - III
(Comprehension Type)

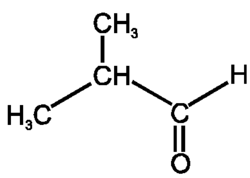
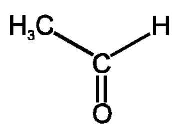
This section contains 2 Paragraphs. Based upon the first paragraph 3 multiple choice questions have to be answered. Each of these question has four choice (A), (B), (C) and (D) out of which **ONLY ONE is correct.**

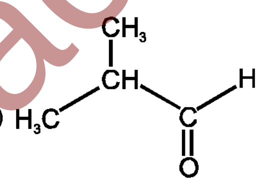
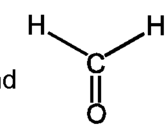
Paragraph for Question Nos. 12 to 14

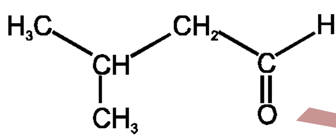
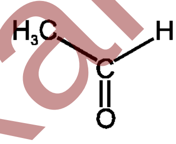
Two aliphatic aldehydes P and Q react in the presence of aqueous K_2CO_3 to give compound R, which upon treatment with HCN provides compound S. On acidification and heating, S gives the product shown below :

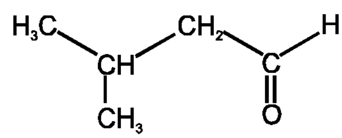
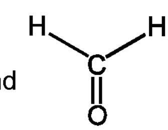


12. The compounds P and Q respectively are :

(A)  and 

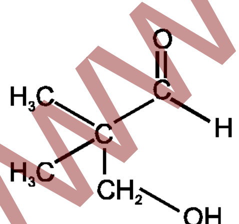
(B)  and 

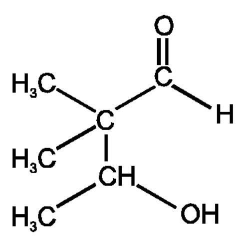
(C)  and 

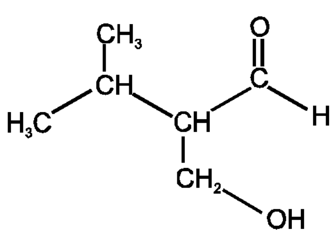
(D)  and 

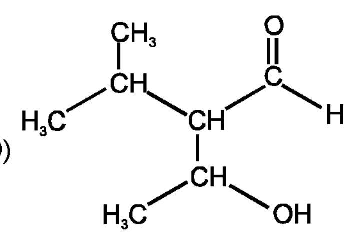
Ans. (B)

13. The compound R is :

(A) 

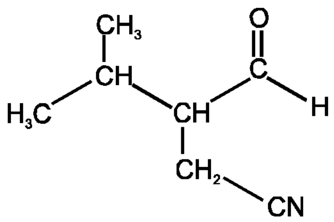
(B) 

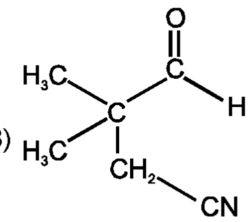
(C) 

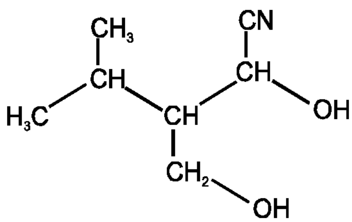
(D) 

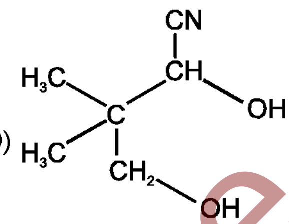
Ans. (A)

14. The compound S is :

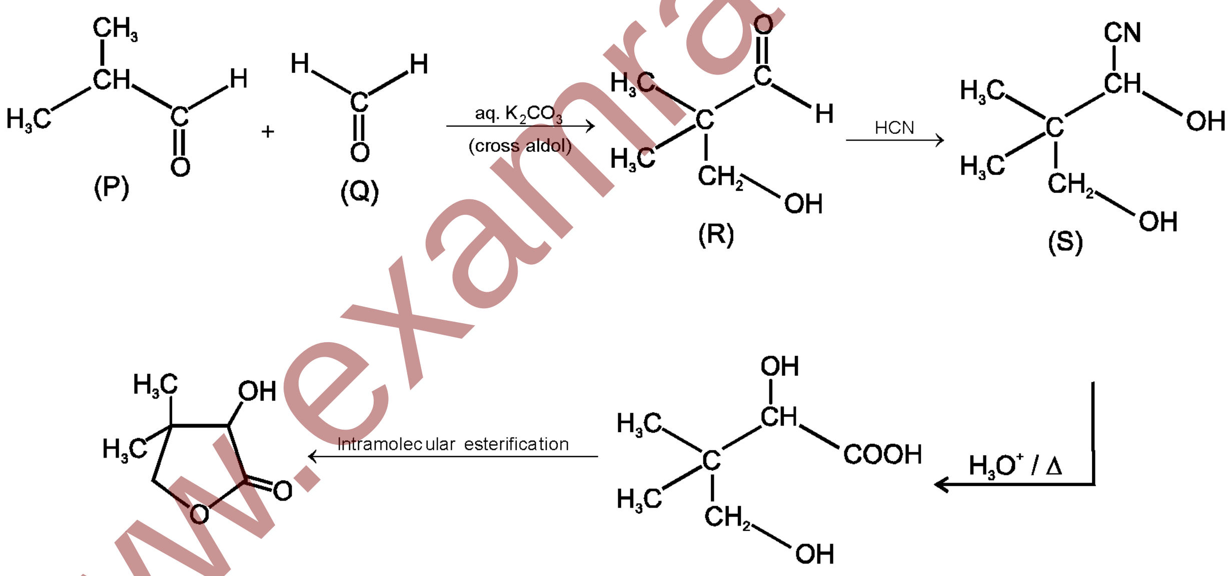
(A) 

(B) 

(C) 

(D) 

Ans. **(D)**

Sol. 

Paragraph for Question Nos. 15 to 17

The hydrogen-like species Li^{2+} is in a spherically symmetric state S_1 with one radial node. Upon absorbing light the ion undergoes transition to a state S_2. The state S_2 has one radial node and its energy is equal to the ground state energy of the hydrogen atom.

15. The state S_1 is :

(A) 1s (B) 2s (C) 2p (D) 3s

Ans. **(B)**

Sol. For lower state (S_1)

No. of radial node = 1 = n – ℓ – 1

Put n = 2 and ℓ = 0 (as higher state S_2 has n = 3)

So, it would be 2s (for S_1 state)

16. Energy of the state S_1 in units of the hydrogen atom ground state energy is :

(A) 0.75 (B) 1.50 (C) 2.25 (D) 4.50

Ans. (C)

Sol. Energy of state S_1

$$= -13.6\left(\frac{3^2}{2^2}\right) \text{ eV/atom}$$

$$= \frac{9}{4}\text{(energy of H-atom in ground state)}$$

= 2.25 (energy of H-atom in ground state).

17. The orbital angular momentum quantum number of the state S_2 is :

(A) 0 (B) 1 (C) 2 (D) 3

Ans. **(B)**

Sol. For state S_2

No. of radial node = 1 = n – ℓ – 1 .... (eq.-1)

Energy of S_2 state = energy of e^- in lowest state of H-atom

= – 13.6 eV/atom

$$= -13.6\left(\frac{3^2}{n^2}\right) \text{ eV/atom}$$

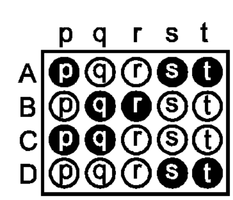
n = 3.

put in equation (1) ℓ = 1

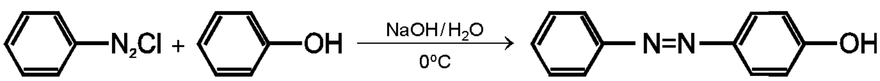
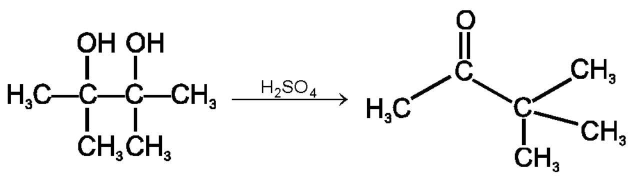
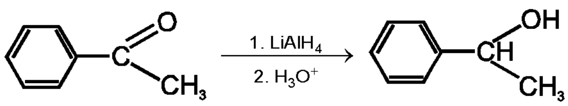
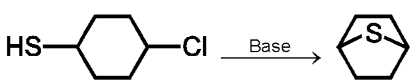
so, orbital $\Rightarrow$ 3p (for S_2 state).

SECTION - IV
(Matrix - Type)

This section contains 2 questions. Each question has four statements (A, B, C and D) given in **Column-I** and five statements (p,q,r,s and t) in **Column-II**. Any given statement in **Column-I** can have correct matching with **ONE OR MORE** statement(s) in **Column-II**. For example, if for a given question, statement B matches with the statements given in q and r, then for that particular question against statement B, darken the bubbles corresponding to q and r in the ORS.

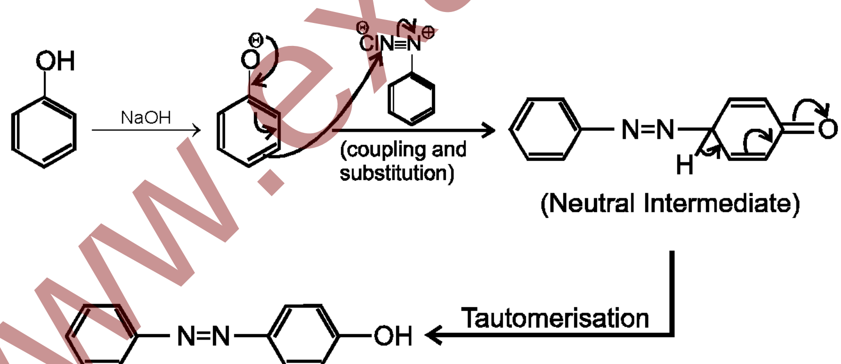


18. Match the reactions in **Column I** with appropriate options in **Column II**.

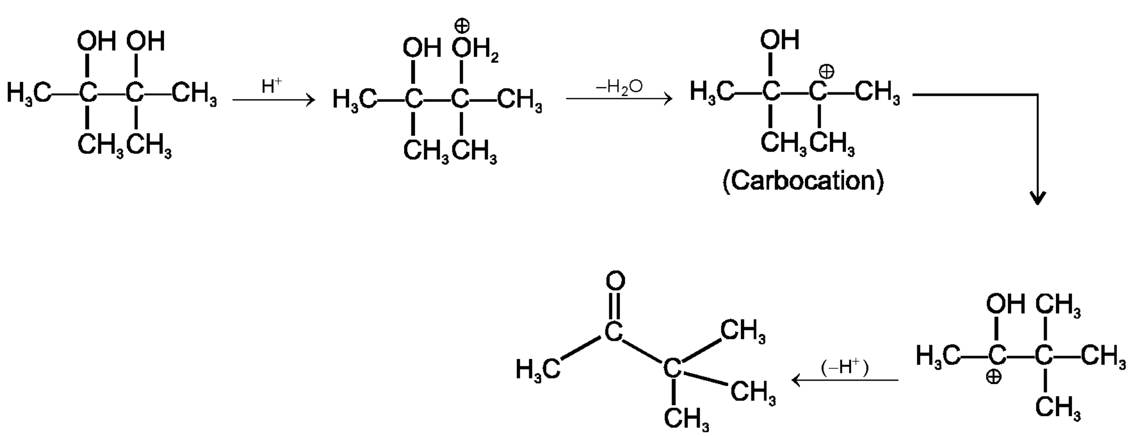
Column I	**Column II**
(A) 	(p) Racemic mixture
(B) 	(q) Addition reaction
(C) 	(r) Substitution reaction
(D) 	(s) Coupling reaction
	(t) Carbocation intermediate

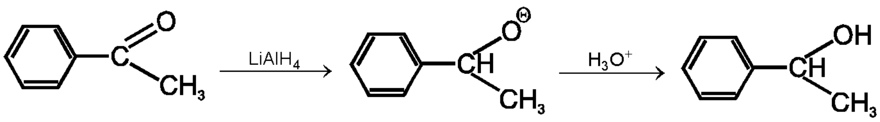
Ans. **(A) - r,s ; (B) - t ; (C) - p, q ; (D) - r**

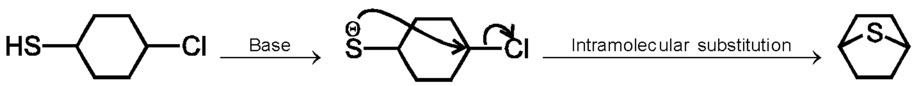
Sol. (A)



(B)



(C)  (racemic mixture)

(D) 

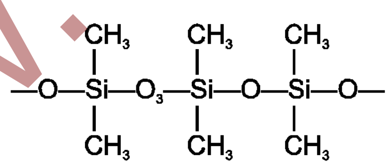
19. All the compounds listed in **Column I** react with water. Match the result of the respective reactions with the appropriate options listed in **Column II**.

Column I	**Column II**
(A) $(CH_3)_2SiCl_2$	(p) Hydrogen halide formation
(B) XeF_4	(q) Redox reaction
(C) Cl_2	(r) Reacts with glass
(D) VCl_5	(s) Polymerization
	(t) O_2 formation

Ans. (A – p, s) ; (B – p, q, r, t); (C–p, q) ; (D–p)

Sol. (A) $(CH_3)_2SiCl_2 + H_2O \longrightarrow (CH_3)_2\ Si(OH)_2 + HCl$

$\downarrow$

 (polymer)

(B) $3XeF_4 + 6H_2O \longrightarrow XeO_3 + 2Xe + \frac{3}{2}O_2 + 12HF$

(C) $Cl_2 + H_2O \longrightarrow HCl + HOCl$

(D) $VCl_5 + H_2O \longrightarrow VOCl_3 + 2HCl$ (First step of hydrolysis)

$VCl_5 + 2H_2O \longrightarrow VO_2Cl + 4HCl$ (Complete hydrolysis)

PART-II
SECTION - I
Single Correct Choice Type

This section contains 6 multiple choice questions. Each question has four choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

20. Let f be a real-valued function defined on the interval (–1, 1) such that $e^{-x} f(x) = 2 + \int_0^x \sqrt{t^4+1}\,dt$, for all $x \in (-1, 1)$ and let f^{-1} be the inverse function of f. Then $(f^{-1})'(2)$ is equal to

(A) 1 (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{1}{e}$

Ans. **(B)**

Sol. $f(x) = e^x\left(2+\int_0^x \sqrt{t^4+1}\,dt\right)$

Let $g(x) = f^{-1}(x)$ $\Rightarrow$ $g(f(x)) = x$

$\Rightarrow$ $g'(f(x))\, f'(x) = 1$

$\Rightarrow$ $g'(2) = \frac{1}{f'(0)}$ $(\because f(0) = 2)$

Now $f'(x) = e^x\left(2+\int_0^x \sqrt{t^4+1}\,dt\right) + e^x\sqrt{x^4+1}$ (Applying Leibinitz Rule)

$\Rightarrow$ $f'(0) = 2 + 1 = 3$

$\Rightarrow$ $g'(2) = \frac{1}{3}$

$\Rightarrow$ $(f^{-1})'(2) = \frac{1}{3}$

21. A signal which can be green or red with probability $\frac{4}{5}$ and $\frac{1}{5}$ respectively, is received by station A and then transmitted to station B. The probability of each station receiving the signal correctly is $\frac{3}{4}$. If the signal received at station B is green, then the probability that the original signal was green is

(A) $\frac{3}{5}$ (B) $\frac{6}{7}$ (C) $\frac{20}{23}$ (D) $\frac{9}{20}$

Ans. **(C)**

Probability (P) = $\dfrac{P(GGG)+P(GRG)}{P(GGG)+P(GRG)+P(RGG)+P(RRG)}$

$\Rightarrow$ P = $\dfrac{\frac{4}{5}\times\frac{3}{4}\times\frac{3}{4}+\frac{4}{5}\times\frac{1}{4}\times\frac{1}{4}}{\frac{4}{5}\times\frac{3}{4}\times\frac{3}{4}+\frac{4}{5}\times\frac{1}{4}\times\frac{1}{4}+\frac{1}{5}\times\frac{1}{4}\times\frac{3}{4}+\frac{1}{5}\times\frac{3}{4}\times\frac{1}{4}}$

$\Rightarrow$ P = $\dfrac{36+4}{36+4+3+3} = \dfrac{40}{46} = \dfrac{20}{23}$

22. If the distance of the point P(1, –2, 1) from the plane x + 2y – 2z = α, where α > 0, is 5, then the foot of the perpendicular from P to the plane is

(A) $\left(\frac{8}{3}, \frac{4}{3}, -\frac{7}{3}\right)$ (B) $\left(\frac{4}{3}, -\frac{4}{3}, \frac{1}{3}\right)$ (C) $\left(\frac{1}{3}, \frac{2}{3}, \frac{10}{3}\right)$ (D) $\left(\frac{2}{3}, -\frac{1}{3}, \frac{5}{2}\right)$

Ans. (A)

Sol. D = $\left|\dfrac{1-4-2-\alpha}{3}\right|$ = 5

α + 5 = 15 ($\because$ α > 0)

$\Rightarrow$ α = 10

$\Rightarrow$ plane is x + 2y – 2z – 10 = 0

for positive be (α, β, γ)

$\dfrac{\alpha-1}{1} = \dfrac{\beta+2}{2} = \dfrac{\gamma-1}{-2} = -\left(\dfrac{1-4-2-10}{9}\right) = \dfrac{5}{3}$ $\Rightarrow$ a = $\dfrac{8}{3}$, $\beta = \dfrac{4}{3}$, $\gamma = -\dfrac{7}{3}$

23. Let S = {1, 2, 3, 4}. The total number of unordered pairs of disjoint subsets of S is equal to

(A) 25 (B) 34 (C) 42 (D) 41

Ans. (D)

Sol.

S = {1, 2, 3, 4}

Each element can be put in 3 ways either in subsets or we don't put in any subset.

So total number of unordered pairs = $\dfrac{3\times3\times3\times3-1}{2}$ + 1 = 41. [Both subsets can be empty also]

24. For r = 0, 1, ..., 10, let A_r, B_r and C_r denote, respectively, the coefficient of x^r in the expansions of $(1 + x)^{10}$, $(1 + x)^{20}$ and $(1 + x)^{30}$. Then $\sum_{r=1}^{10} A_r(B_{10}B_r - C_{10}A_r)$ is equal to

(A) $B_{10} - C_{10}$ (B) $A_{10}(B^2_{10} - C_{10}A_{10})$ (C) 0 (D) $C_{10} - B_{10}$

Ans. (D)

Sol. $B_{10}\sum_{r=1}^{10} A_rB_r - C_{10}\sum_{r=1}^{10}(A_r)^2 = {}^{20}B_{10}\,({}^{30}C_{20} - 1) - {}^{30}C_{10}\,({}^{20}C_{10} - 1) = {}^{30}C_{10} - {}^{20}C_{10} = C_{10} - B_{10}$

[By sum of series of product of two binomial coefficients]

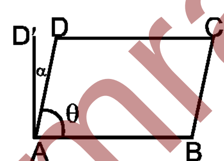
25. Two adjacent sides of a parallelogram ABCD are given by

$\overrightarrow{AB} = 2\hat{i} + 10\hat{j} + 11\hat{k}$ and $\overrightarrow{AD} = -\hat{i} + 2\hat{j} + 2\hat{k}$. The side AD is rotated by an acute angle α in the plane of the parallelogram so that AD becomes AD′. If AD′ makes a right angle with the side AB, then the cosine of the angle α is given by

(A) $\frac{8}{9}$ (B) $\frac{\sqrt{17}}{9}$ (C) $\frac{1}{9}$ (D) $\frac{4\sqrt{5}}{9}$

Ans. **(B)**

Sol. $\cos\theta = \frac{-2+20+22}{15\times 3} = \frac{8}{9}$ [Using dot product]



$\theta + \alpha = 90°$

$\alpha = 90° - \theta$

$\cos\alpha = \sin\theta = \frac{\sqrt{17}}{9}$

SECTION - II
(Integer Type)

This section contains 5 questions. The answer to each question is a single-digit integer, ranging from 0 to 9. The correct digit below the question number in the ORS is to be bubbled.

26. Let k be a positive real number and let

$A = \begin{bmatrix} 2k-1 & 2\sqrt{k} & 2\sqrt{k} \\ 2\sqrt{k} & 1 & -2k \\ -2\sqrt{k} & 2k & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 2k-1 & \sqrt{k} \\ 1-2k & 0 & 2\sqrt{k} \\ -\sqrt{k} & -2\sqrt{k} & 0 \end{bmatrix}$. If det (adj A) + det (adj B) = 10^6, then [k] is equal to

(**Note** : adj M denotes the adjoint of a square matrix M and [k] denotes the largest integer less than or equal to k].

Ans. 4

Sol. $\det(A) = \begin{vmatrix} 2k-1 & 2\sqrt{k} & 2\sqrt{k} \\ 2\sqrt{k} & 1 & -2k \\ -2\sqrt{k} & 2k & -1 \end{vmatrix}$ $\quad C_2 \to C_2 - C_3$

$= \begin{vmatrix} 2k-1 & 0 & 2\sqrt{k} \\ 2\sqrt{k} & 1+2k & -2k \\ -2\sqrt{k} & 2k+1 & -1 \end{vmatrix}$ $\quad R_2 \to R_2 - R_3$

$= \begin{vmatrix} 2k-1 & 0 & 2\sqrt{k} \\ 4\sqrt{k} & 0 & 1-2k \\ 2\sqrt{k} & 2k+1 & -1 \end{vmatrix} = (2k+1)^3$

$\because$ B is a skew-symmetric matrix of odd order therefore det(B) = 0

Now det (adj A) + det (adj B) = 10^6

$\Rightarrow$ $\{(2k+1)^3\}^2 + 0 = 10^6$

$\Rightarrow$ $2k + 1 = 10$, as $k > 0$

$\Rightarrow$ $k = 4.5$

$\Rightarrow$ $[k] = 4$

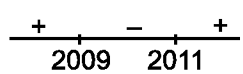
27. Let f be a function defined on **R** (the set of all real numbers) such that
$f'(x) = 2010\,(x-2009)\,(x-2010)^2\,(x-2011)^3\,(x-2012)^4$, for all $x \in$ **R**.
If g is a function defined on **R** with values in the interval $(0, \infty)$ such that $f(x) = \ell n\,(g(x))$, for all $x \in$ **R**, then the number of points in **R** at which g has a local maximum is

Ans. 1

Sol. $f'(x) = 2010\,(x-2009)\,(x-2010)^2\,(x-2011)^3\,(x-2012)^4$

$f(x) = \ell n\,(g(x))$

$\Rightarrow$ $g(x) = e^{f(x)}$



$\Rightarrow$ $g'(x) = e^{f(x)} \cdot f'(x)$

only point of maxima [Applying first derivative test]

28. Let $a_1, a_2, a_3, \ldots, a_{11}$ be real numbers satisfying $a_1 = 15$, $27 - 2a_2 > 0$ and $a_k = 2a_{k-1} - a_{k-2}$ for $k = 3, 4, \ldots, 11$.

If $\frac{a_1^2 + a_2^2 + \ldots + a_{11}^2}{11} = 90$, then the value of $\frac{a_1 + a_2 + \ldots + a_{11}}{11}$ is equal to

Ans. 0

Sol. $a_1 = 15$

$\frac{a_k + a_{k-2}}{2} = a_{k-1}$ for $k = 3, 4, \ldots, 11$

$\Rightarrow$ $a_1, a_2, \ldots\ldots, a_{11}$ are in AP

$a_1 = a = 15$

$\frac{a_1^2 + a_2^2 + \ldots + a_n^2}{11} = 90 \quad \Rightarrow \quad \frac{(15)^2 + (15+d)^2 + \ldots + (15+10d)^2}{11} = 90$

$\Rightarrow$ $9d^2 + 30d + 27 = 0$ $\Rightarrow$ $d = -3$ or $-\frac{9}{7}$

Since $27 - 2a_2 > 0$ $\Rightarrow$ $a_2 < \frac{27}{2}$ $\Rightarrow$ $d = -3$

$$\frac{a_1 + a_2 + ... + a_{11}}{11} = \frac{11}{2}\ \frac{[30 + 10(-3)]}{11} = 0$$

29. Consider a triangle ABC and let a, b and c denote the lengths of the sides opposite to vertices A, B and C respectively. Suppose a = 6, b = 10 and the area of the triangle is $15\sqrt{3}$. If $\angle ACB$ is obtuse and if r denotes the radius of the incircle of the triangle, then r^2 is equal to

Ans. 3

Sol. Area of triangle = $\frac{1}{2}$ ab sin C = $15\sqrt{3}$

$\Rightarrow$ $\frac{1}{2}$. 6 . 10 sin C = $15\sqrt{3}$

$\Rightarrow$ sin C = $\frac{\sqrt{3}}{2}$

$\Rightarrow$ C = $\frac{2\pi}{3}$ (C is obtuse angle)

Now cos C = $\frac{a^2 + b^2 - c^2}{2ab}$

$\Rightarrow$ $-\frac{1}{2} = \frac{36 + 100 - c^2}{2.6.10}$ $\Rightarrow$ c = 14

$\therefore$ $r = \frac{\Delta}{s} = \frac{15\sqrt{3}}{\frac{6 + 10 + 14}{2}} = \sqrt{3}$

$\Rightarrow$ $r^2 = 3$

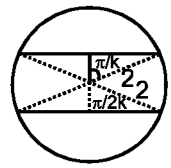
30. Two parallel chords of a circle of radius 2 are at a distance $\sqrt{3} + 1$ apart. If the chords subtend at the center, angles of $\frac{\pi}{k}$ and $\frac{2\pi}{k}$, where k > 0, then the value of [k] is

[**Note** : [k] denotes the largest integer less than or equal to k]

Ans. 3

Sol. Since distance between parallel chords is greater than radius, therefore both chords lie on opposite side of centre.

$$2\cos\frac{\pi}{2k} + 2\cos\frac{\pi}{k} = \sqrt{3} + 1$$



Let $\frac{\pi}{2k} = \theta$

$\therefore \quad 2\cos\theta + 2\cos 2\theta = \sqrt{3} + 1$

$\Rightarrow \quad 2\cos\theta + 2(2\cos^2\theta - 1) = \sqrt{3} + 1$

$\Rightarrow \quad 4\cos^2\theta + 2\cos\theta - (3 + \sqrt{3}) = 0$

$$\therefore \quad \cos\theta = \frac{-2 \pm \sqrt{4 + 16(3+\sqrt{3})}}{2(4)} = \frac{-2 \pm 2\sqrt{1 + 12 + 4\sqrt{3}}}{2(4)} = \frac{-1 \pm \sqrt{(\sqrt{12}+1)^2}}{4} = \frac{-1 \pm (2\sqrt{3}+1)}{4}$$

$\Rightarrow \quad \cos\frac{\pi}{2k} = \frac{\sqrt{3}}{2}, \frac{-(\sqrt{3}+1)}{2}$ Rejected

$\Rightarrow \quad \frac{\pi}{2k} = \frac{\pi}{6} \quad \Rightarrow \quad k = 3 \quad \Rightarrow \quad [k] = 3$

SECTION - III
Paragraph Type

This section contains 2 Paragraphs. Based upon the first paragraph 3 multiple choice questions have to be answered. Each of these question has four choice (A), (B), (C) and (D) out of which ONLY ONE is correct.

Paragraph for Question Nos. 31 to 33

Consider the polynomial

$f(x) = 1 + 2x + 3x^2 + 4x^3$

Let s be the sum of all distinct real roots of f(x) and let t = |s|

31. The real number s lies in the interval.

(A) $\left(-\frac{1}{4}, 0\right)$ (B) $\left(-11, \frac{3}{4}\right)$ (C) $\left(-\frac{3}{4}, -\frac{1}{2}\right)$ (D) $\left(0, \frac{1}{4}\right)$

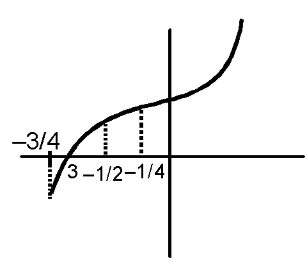
Ans. (C)

Sol. $f(x) = 1 + 2x + 3x^2 + 4x^3$

$f'(x) = 2 + 6x + 12x^2 > 0$ [as a > 0, D < 0]

f(x) is increasing function so it can atmost one real root.

Using inter mediate value theorem



$f\left(-\frac{3}{4}\right) . f\left(-\frac{1}{2}\right) < 0$

$\therefore$ (C) is correct

32. The area bounded by the curve y = f(x) and the lines x = 0, y = 0 and x = t, lies in the interval

(A) $\left(\frac{3}{4}, 3\right)$ (B) $\left(\frac{21}{64}, \frac{11}{16}\right)$ (C) (9, 10) (D) $\left(0, \frac{21}{64}\right)$

Ans. (A)

Sol. By estimation of integration

$$\int_0^{1/2} f(x)\,dx < \int_0^{t} f(x)dx < \int_0^{3/4} f(x)\,dx$$

$$\Rightarrow \frac{15}{16} < \int_0^{t} f(x)dx < \frac{525}{256}$$

Hence option (A) is correct

33. The function f′(x) is

(A) increasing in $\left(-t, \frac{1}{4}\right)$ and decreasing in $\left(-\frac{1}{4}, t\right)$

(B) decreasing in $\left(-t, -\frac{1}{4}\right)$ and increasing in $\left(-\frac{1}{4}, t\right)$

(C) increasing in (–t, t)

(D) decreasing in (–t, t)

Ans. (B)

Sol. $f'(x) = 2 + 6x + 12x^2$

$\Rightarrow f''(x) = 6 + 24x$

$\Rightarrow f''(x) = 6\,(4x + 1) > 0 \Rightarrow x > -\frac{1}{4}$

Paragraph for Question Nos. 34 to 36

Tangents are drawn from the point P(3, 4) to the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ touching the ellipse at point A and B.

34. The coordinates of A and B are

(A) (3, 0) and (0, 2) (B) $\left(-\frac{8}{5}, \frac{2\sqrt{161}}{15}\right)$ and $\left(-\frac{9}{5}, \frac{8}{5}\right)$

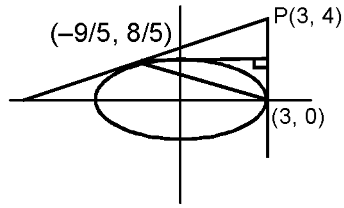
(C) $\left(-\frac{8}{5}, \frac{2\sqrt{161}}{15}\right)$ and (0, 2) (D) (3, 0) and $\left(-\frac{9}{5}, \frac{8}{5}\right)$

Ans. (D)

Sol. Equation of chord of contact

$\frac{x}{3} + y = 1$

$x = 3(1 - y)$



Solving with ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$

$(1 - y)^2 + \frac{y^2}{4} = 1$

$4(y^2 + 1 - 2y) + y = 4$

$4y^2 - 8y = 0$

$y = 0 \;\&\; \frac{8}{5}$

$\Rightarrow\; x = 2 \;\&\; 3\left(1 - \frac{8}{5}\right) \quad \Rightarrow\; x = 3,\; -\frac{9}{5}$

$\Rightarrow$ Points are $(3, 0)$ and $\left(-\frac{9}{5}, \frac{8}{5}\right)$

35. The orthocentre of the triangle PAB is

(A) $\left(5, \frac{8}{7}\right)$ (B) $\left(\frac{7}{5}, \frac{25}{8}\right)$ (C) $\left(\frac{11}{5}, \frac{8}{5}\right)$ (D) $\left(\frac{8}{25}, \frac{7}{5}\right)$

Ans. (C)

Sol. y coordinate of the orthocentre must be $\frac{8}{5}$

36. The equation of the locus of the point whose distances from the point P and the line AB are equal, is

(A) $9x^2 + y^2 - 6xy - 54x - 62y + 241 = 0$ (B) $x^2 + 9y^2 + 6xy - 54x + 62y - 241 = 0$

(C) $9x^2 + 9y^2 - 6xy - 54x - 62y - 241 = 0$ (D) $x^2 + y^2 - 2xy + 27x + 31y - 120 = 0$

Ans. (A)

Sol. $\sqrt{(x-3)^2 + (y-4)^2} = \frac{|x + 3y - 3|}{\sqrt{1+9}}$

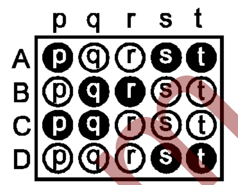
$\Rightarrow\; 10\,\{(x^2 + 9 - 6x) + [y^2 + 16 - 8y]\} = (x + 3y - 3)^2$

$= x^2 + 9y^2 + 9 + 6xy - 6xy - 6x - 18y$

$\Rightarrow\; 9x^2 + y^2 - 6xy - 54x - 62y + 241 = 0$

SECTION - IV (Matrix - Type)

This section contains 2 questions. Each question has four statements (A, B, C and D) given in **Column-I** and five statements (p,q,r,s and t) in **Column-II**. Any given statement in **Column-I** can have correct matching with **ONE OR MORE** statement(s) in **Column-II**. For example, if for a given question, statement B matches with the statements given in q and r, then for that particular question against statement B, darken the bubbles corresponding to q and r in the ORS.



37. Match the statements in **Column-I** with those in **Column-II.**
[Note : Here z takes values in the complex plane and Im z and Re z denote, respectively, the imaginary part and the real part of z.]

Column-I

(A) The set of points z satisfying $|z - i|z|| = |z + i|z||$ is contained in or equal to

(B) The set of points z satisfying $|z + 4| + |z - 4| = 10$ is contained in or equal to

(C) If $|w| = 2$, then the set of points $z = w - \frac{1}{w}$ is contained in or equal to

(D) If $|w| = 1$, then the set of points $z = w + \frac{1}{w}$ is contained in or equal to

Column-II

(p) an ellipse with eccentricity $\frac{4}{5}$

(q) the set of points z satisfying Im z = 0

(r) the set of point z satisfying $|\text{Im } z| \le 1$

(s) the set of points z satisfying $|\text{Re } z| \le 2$

(t) the set of points z satisfying $|z| \le 3$

Ans. (A) - (q,r), (B)-(p), (C) - (p,s,t), (D) - (q,r,s,t)

Sol. (A) $|z - i|z|| = |z + i|z||$

$\Rightarrow |x + iy - i\sqrt{x^2 + y^2}| = |x + iy + i\sqrt{x^2 + y^2}|$

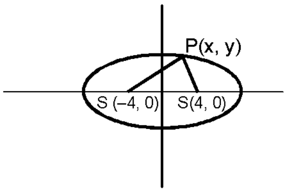
$\Rightarrow x^2 + (y - \sqrt{x^2 + y^2})^2 = x^2 + (y + \sqrt{x^2 + y^2})^2$

$\Rightarrow 4y\sqrt{x^2 + y^2} = 0 \quad \Rightarrow y = 0 \Rightarrow \text{Im } z = 0$

(B) $|z + 4| + |z - 4| = 10$

Ellipse with $2a = 10 \Rightarrow a = 5$

$ae = 4 \Rightarrow e = \frac{4}{5}$



(C) Let $w = 2(\cos\theta + i\sin\theta)$

$$z = 2(\cos\theta + i\sin\theta) - \frac{(\cos\theta - i\sin\theta)}{2}$$

$$= \frac{3\cos\theta + 5i\sin\theta}{2} \Rightarrow \frac{3\cos\theta}{2},\ y = \frac{5\sin\theta}{2}$$

$$= \frac{x^2}{9/4} + \frac{y^2}{25/4} = 1 \qquad e = \frac{4}{5}$$

$$|z| = \sqrt{\frac{9\cos^2\theta}{4} + \frac{25\sin^2\theta}{4}} = \sqrt{\frac{9 + 16\sin^2\theta}{4}} = \sqrt{\frac{9}{4} + 4\sin^2\theta} \le \frac{5}{2}$$

$$|\text{Re } z| = \left|\frac{3}{2}\cos\theta\right| \le \frac{3}{2}$$

(D) $z = \cos\theta + i\sin\theta + \cos\theta - i\sin\theta = 2\cos\theta$

$\therefore\ |z| \le 2$

$\therefore\ \text{Im}(z) = 0$

$(\text{Re } z) \Rightarrow |2\cos\theta| \le 2$

$|z| \le 2$

38. Match the statements in **Column-I** with those in **Column-II.**

Column-I	Column-II
(A) A line from the origin meets the lines $\frac{x-2}{1}=\frac{y-1}{-2}=\frac{z+1}{1}$ and $\frac{x-\frac{8}{3}}{2}=\frac{y+3}{-1}=\frac{z-1}{1}$ at P and Q respectively. If length PQ = d, then d^2 is	(p) -4
(B) The values of x satisfying $\tan^{-1}(x+3)-\tan^{-1}(x-3)=\sin^{-1}\left(\frac{3}{5}\right)$ are	(q) 0
(C) Non-zero vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ satisfy $\vec{a}\cdot\vec{b}=0$, $(\vec{b}-\vec{a}).(\vec{b}+\vec{c})=0$ and $2\lvert\vec{b}+\vec{c}\rvert=\lvert\vec{b}-\vec{a}\rvert$. If $\vec{a}=\mu\vec{b}+4\vec{c}$ then possible value of μ are	(r) 4
(D) Let f be the function on $[-\pi, \pi]$ given by $f(0) = 9$ and $f(x)=\frac{\sin\left(\frac{9x}{2}\right)}{\sin\left(\frac{x}{2}\right)}$ for $x \neq 0$. The value of $\frac{2}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$ is	(s) 5
	(t) 6

Ans. **(A) → (t), (B) → (p, r), (C) → (q,s), (D) → (r)**

Sol. **(A)** Let the line through origin is $\frac{x}{\lambda}=\frac{y}{\mu}=\frac{z}{1}$

$\Rightarrow$ $x=\lambda z$, $y=\mu z$ ....(1)

To find point of intersection of line (1) and line $\frac{x-2}{1}=\frac{y-1}{-2}=\frac{z+1}{1}$ ....(2)

we have $\frac{\lambda z-2}{1}=\frac{\mu z-1}{-2}=z+1$

$\Rightarrow$ $z=\frac{3}{\lambda-1}=\frac{-1}{\mu+2}$

$\Rightarrow$ $\lambda+3\mu+5=0$ ....(3)

$$\Rightarrow \quad \frac{\lambda_1^2(4-\mu)-\lambda_2^2}{4} = 0$$

$$\Rightarrow \quad \lambda_2^2 = \lambda_1^2(4-\mu) \quad \ldots\ldots\ldots\ldots.(2)$$

from (1) & (2)

$12 + \mu^2 - 8\mu = 12 - 3\mu$

$\Rightarrow \quad \mu^2 - 5\mu = 0 \quad \Rightarrow \quad \mu = 0, 5$

(D) $$I = \frac{2}{\pi}\int_{-\pi}^{\pi} \frac{\sin\frac{9x}{2}}{\sin\frac{x}{2}}\,dx = \frac{4}{\pi}\int_0^{\pi} \frac{\sin\frac{9x}{2}\cos\frac{x}{2}}{\sin\frac{x}{2}\cos\frac{x}{2}}\,dx = \frac{8}{\pi}\int_0^{\pi} \frac{\sin\frac{9x}{2}\cos\frac{x}{2}}{\sin x}\,dx$$

$$I = \frac{4}{\pi}\int_0^{\pi} \frac{\sin 5x + \sin 4x}{\sin x}\,dx \quad \ldots\ldots(i)$$

(using $\int_0^b f(x)dx = \int_0^b f(a+b-x)dx$)

$$= \frac{4}{\pi}\int_0^{\pi} \frac{\sin 5x - \sin 4x}{\sin x}\,dx \quad \ldots\ldots(ii)$$

Add (i) and (ii)

$$I = \frac{4}{\pi}\int_0^{\pi} \frac{\sin 5x}{\sin x}\,dx$$

Consider

$$I_k - I_{k-2} = \frac{4}{\pi}\int_0^{\pi} \frac{\sin kx - \sin(k-2)x}{\sin x} = \frac{8}{\pi}\int_0^{\pi} \frac{\cos(k-1)x\sin x}{\sin x}$$

$I_k = I_{k-2}$

so $I_5 = I_3 \quad \Rightarrow \quad I_5 = I_1 = \frac{4}{\pi}\int_0^{\pi} dx = 4$

Aliter

Let $$I = \frac{2}{\pi}\int_{-\pi}^{\pi} \frac{\sin(9x/2)}{\sin(x/2)}\,dx$$

$$I = \frac{4}{\pi}\int_0^{\pi} \frac{\sin(9x/2)}{\sin(x/2)}\,dx \quad \ldots\ldots.(1)$$ ($\because$ f(x) is even function)

$$I = \frac{4}{\pi}\int_0^{\pi}\frac{\cos(9x/2)}{\cos(x/2)}dx \quad ....(2)$$

(using $\int_0^b f(x)dx = \int_0^b f(a+b-x)dx$)

Add (1) & (2)

$$I = \frac{4}{\pi}\int_0^{\pi}\frac{\sin 5x}{2\sin(x/2)\cos(x/2)}dx = \frac{4}{\pi}\int_0^{\pi}\frac{\sin 5x}{\sin x}dx$$

$$\Rightarrow I = \frac{8}{\pi}\int_0^{\pi/2}\frac{\sin 5x}{\sin x}dx$$

$$\Rightarrow I = \frac{8}{\pi}\int_0^{\pi/2}\left(\frac{16\sin^5 x - 20\sin^3 x + 5\sin x}{\sin x}\right)dx$$

$$\Rightarrow I = \frac{8}{\pi}\int_0^{\pi/2}(16\sin^4 x - 20\sin^2 x + 5)dx$$

$$\Rightarrow I = \frac{8}{\pi}\left[16x\frac{3\times 1\times \pi}{4\times 2\times 2} - 20\times\frac{1\times \pi}{2\times 2} + \frac{5\pi}{2}\right]$$

$$\Rightarrow I = \frac{8}{\pi}\left[3\pi - 5\pi - \frac{5\pi}{2}\right]$$

$$\Rightarrow I = 4$$

PHYSICS

PART-III

SECTION - I

Single Correct Choice Type

This section contains 6 multiple choice questions. Each question has four choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

39. A biconvex lens of focal length 15 cm is in front of a plane mirror. The distance between the lens and the mirror is 10 cm. A small object is kept at a distance of 30 cm from the lens. The final image is

(A) Virtual and at a distance of 16 cm from mirror

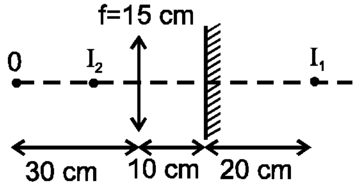
(B) Real and at distance of 16 cm from the mirror

(C) Virtual and at a distance of 20 cm form the mirror

(D) Real and at a distance of 20 cm from the mirror

Ans. **(B)**

Sol.



First image,

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

$$\frac{1}{v}-\frac{1}{-30}=\frac{1}{15}$$

v = 30, image in formed 20 cm behind the mirror.

Second image, by plane mirror will be at 20 cm infront of plane mirror.

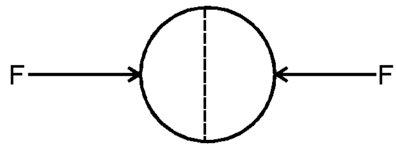
For third image, $\frac{1}{v}-\frac{1}{10}=\frac{1}{15}$

$$\frac{1}{v}=\frac{1}{10}+\frac{1}{15}=\frac{3+2}{30}=\frac{5}{30}$$

v = 6 cm

Ans. Final image is real & formed at a distance of 16 cm from mirror.

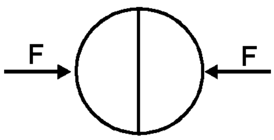
40. A uniformly charged thin spherical shell of radius R carries uniform surface charge density of σ per unit area. It is made of two hemispherical shells, held together by pressing them with force F (see figure). F is proportional to



(A) $\frac{1}{\varepsilon_0}\sigma^2R^2$ (B) $\frac{1}{\varepsilon_0}\sigma^2R$ (C) $\frac{1}{\varepsilon_0}\frac{\sigma^2}{R}$ (D) $\frac{1}{\varepsilon_0}\frac{\sigma^2}{R^2}$

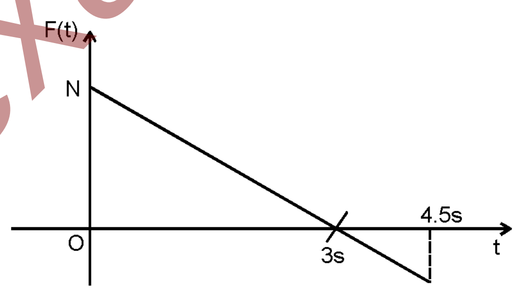
Ans. **(A)**

Sol.



Electrostatics repulsive force ; $F_{ele} = \left(\frac{\sigma^2}{2\varepsilon_0}\right)\pi R^2$; $F = F_{ele} = \frac{\sigma^2 \pi R^2}{2\varepsilon_0}$

41. A block of mass 2 kg is free to move along the x-axis. It is at rest and from t = 0 onwards it is subjected to a time-dependent force F (t) in the x direction. The force F (t) varies with t as shown in the figure. The kinetic energy of the block after 4.5 seconds is :



(A) 4.50 J (B) 7.50 J (C) 5.06 J (D) 14.06 J

Ans. **(C)**

Sol. $\int F dt = \Delta p$

$\Rightarrow \quad \frac{1}{2} \times 4 \times 3 - \frac{1}{2} \times 1.5 \times 2 = p_f - 0$

$\Rightarrow \quad p_f = 6 - 1.5 = \frac{9}{2}$

K.E. $= \frac{p^2}{2m} = \frac{81}{4 \times 2 \times 2}$;K.E. = 5.06 J **Ans.**

42. A hollow pipe of length 0.8 m is closed at one end. At its open end a 0.5 m long uniform string is vibrating in its second harmonic and it resonates with the fundamental frequency of the pipe. If the tension in the wire is 50 N and the speed of sound is 320 ms^{-1}, the mass of the string is :

(A) 5 grams (B) 10 grams (C) 20 grams (D) 40 grams

Ans. (B)

Sol. Fundamental frequency of close organ pipe = $\frac{V_1}{4\ell_1}$

Second harmonic frequency of string = $\frac{2V_2}{2\ell_2}$

So, $\frac{V_1}{4\ell_1} = \frac{V_2}{\ell_2}$

$= \frac{320}{4\times 0.8} = \frac{1}{0.5}\sqrt{\frac{50}{\mu}}$

$2500 = \frac{50}{\mu}$

$\mu = \frac{1}{50} = \frac{m}{0.5}$

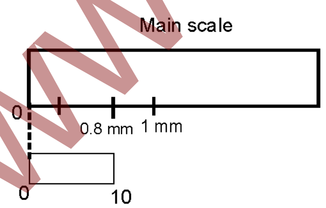
m = 10 gm.

43. A vernier calipers has 1 mm marks on the main scale. It has 20 equal division on the Vernier scale which match with 16 main scale divisions. For this Vernier calipers, the least count is :

(A) 0.02 mm (B) 0.05 mm (C) 0.1 mm (D) 0.2 mm

Ans. (D)

Sol.



20 VSD = 16 MCD

1 VSD = 0.8 MSD

Least count = MSD – VSD

= 1 mm – 0.8 mm

= 0.2 mm

44. A tiny spherical oil drop carrying a net charge q is balanced in still air with a vertical uniform electric field of strength $\frac{81\pi}{7} \times 10^5$ Vm^{-1}. When the field is switched off, the drop is observed to fall with terminal velocity 2×10^{-3} m s^{-1}. Given g = 9.8 m s^{-2}, viscosity of the air = 1.8×10^{-5} Ns m^{-2} and the density of oil = 900 kg m^{-3}, the magnitude of q is :

(A) 1.6×10^{-19} C (B) 3.2×10^{-19} C (C) 4.8×10^{-19} C (D) 8.0×10^{-19} C

Ans. **(D)**

Sol. In equilibrium,

$$mg = qE$$

In absence of electric field,

$$mg = 6\pi\eta rv$$

$\Rightarrow$ $qE = 6\pi qrv$

$$m = \frac{4}{3}\pi Rr^3 d. = \frac{qE}{g}$$

$$\frac{4}{3}\pi\left(\frac{qE}{6\pi\eta v}\right)^3 d = \frac{qE}{g}$$

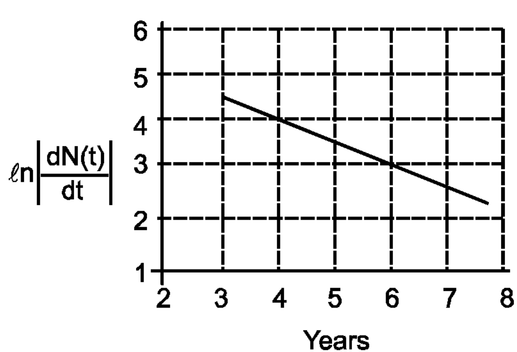
After substituting value we get,

$q = 8 \times 10^{-19}$ C **Ans.**

SECTION - II
(Integer Type)

This section contains 5 questions. The answer to each question is a single-digit integer, ranging from 0 to 9. The correct digit below the question number in the ORS is to be bubbled.

45. To determine the half life of a radioactive element, a student plots a graph of $\ell n\left|\frac{dN(t)}{dt}\right|$ versus t. Here $\frac{dN(t)}{dt}$ is the rate of radioactive decay at time t. If the number of radioactive nuclei of this element decreases by a factor of p after 4.16 years, the value of p is :



Ans. 8

Sol. $-\frac{dN}{dt} = \lambda N$

$-\frac{dN}{dt} = \lambda N_0 e^{-\lambda t}$

$\ell n\left|\frac{dN}{dt}\right| = -\lambda t + \ell n(\lambda N_0)$

y = mx + c

$m = -\lambda$

$\lambda = \frac{1}{2}$ [slope by graph = $\frac{1}{2}$]

$T = \frac{\ell n 2}{\lambda}$

$= 2 \times 0.693 = \frac{4.16}{n}$

n = 3 = no. of half life.

$p = z^3 = 8$. **Ans.**

46. Image of an object approaching a convex mirror of radius of curvature 20 m along its optical axis is observed to move from $\frac{25}{3}$ m to $\frac{50}{7}$ m in 30 seconds. What is the speed of the object in km per hour.

Ans. **3**

Sol. R = 20 m, f = 10 m

For mirror,

$$\frac{1}{V} + \frac{1}{U} = \frac{1}{f}$$

$$\frac{1}{25/3} + \frac{1}{U_1} = \frac{1}{10}$$

$$\frac{1}{U_1} = \frac{1}{10} - \frac{3}{25} = -\frac{1}{50} \Rightarrow U_1 = -50 \text{ cm}$$

& $$\frac{1}{50/7} + \frac{1}{U_2} = \frac{1}{10} \Rightarrow \frac{1}{U_2} = -\frac{1}{25} \Rightarrow U_2 = -25 \text{ cm}$$

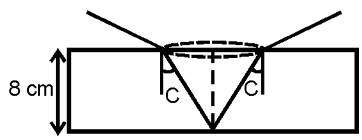
So, speed = $\left|\frac{\Delta U}{\Delta t}\right| = \frac{25}{30}$ m/sec. = $\frac{5}{6}$ m/sec.

& in km/hr = $\frac{5}{6} \times \frac{18}{5} = 3$ km/hr.

47. A large glass slab (μ = 5/3) of thickness 8 cm is placed over a point source of light on a plane surface. It is seen that light emerges out of the top surface of the slab from a circular area of radius R cm. What is the value of R?

Ans. **6**

Sol.



$\tan C = \frac{R}{8}$ ....(i)

$\frac{5}{3}\sin C = 1.\sin 90^\circ$

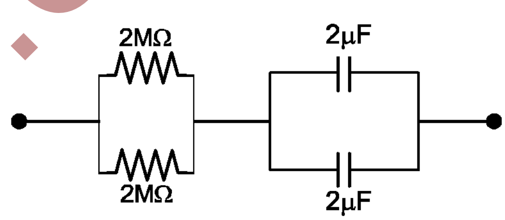
$\sin C = \frac{3}{5}$

$C = 37^\circ$

$\frac{3}{4} = \frac{R}{8}$

R = 6 cm.

48. At time t = 0, a battery of 10 V is connected across points A and B in the given circuit. If the capacitors have no charge initially, at what time (in seconds) does the voltage across them become 4 V?
[Take : $\ell n\ 5 = 1.6$, $\ell n\ 3 = 1.1$]



Ans. **t = 2 sec**

Sol. Equation of charging of capacitor,

$$V = V_0\left(1 - e^{-t/R_{eq}C_{eq}}\right)$$

$C_{eq} = 2 + 2 = 4\ \mu F$

$R_{eq} = 1\ M\Omega$

$$4 = 10\left(1 - e^{-\frac{t}{10^6 \times 4 \times 10^{-6}}}\right)$$

$e^{-t/4} = 0.6$

$\Rightarrow \quad e^{t/4} = \frac{5}{3}$

$\Rightarrow \quad \frac{t}{4} = \ell n\ 5 - \ell n\ 3$

$\Rightarrow \quad t = 0.5 \times 4$

$t = 2$ sec. **Ans.**

49. A diatomic ideal gas is compressed adiabatically to $\frac{1}{32}$ of its initial volume. If the initial temperature of the gas is T_i (in Kelvin) and the final temperature is aT_i, the value of a is :

Ans. a = 4

Sol. For adiabatic process,

$TV^{\gamma-1}$ = constant

$$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

$$T_2 = T_1(32)^{\frac{7}{5}-1}$$

$T_2 = 4T_1 \quad \Rightarrow \quad a = 4$ **Ans.**

SECTION - III
Paragraph Type

This section contains 2 Paragraphs. Based upon the first paragraph 3 multiple choice questions have to be answered. Each of these question has four choice (A), (B), (C) and (D) out of which ONLY ONE is correct.

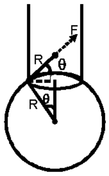
Paragraph for questions 50 to 52.

When liquid medicine of density ρ is to be put in the eye, it is done with the help of a dropper. As the bulb on the top of the dropper is pressed, a drop forms at the opening of the dropper. We wish to estimate the size of the drop. We first assume that the drop formed at the opening is spherical because that requires a minimum increase in its surface energy. To determine the size, we calculate the net vertical force due to the surface tension T when the radius of the drop is R. When this force becomes smaller than the weight of the drop, the drop gets detached from the dropper.

50. If the radius of the opening of the dropper is r; the vertical force due to the surface tension on the drop of radius R (assuming r << R) is :

(A) $2\pi rT$ (B) $2\pi RT$ (C) $\frac{2\pi r^2 T}{R}$ (D) $\frac{2\pi R^2 T}{r}$

Ans. (C)

Sol. 

Due to surface tension, vertical force on drop = $F_v = T2\pi r \sin\theta = T2\pi r\frac{r}{R} = \frac{T2\pi r^2}{R}$

51. If $r = 5 \times 10^{-4}$ m, $\rho = 10^3$ kgm^{-3}, g = 10 ms^{-2}, T = 0.11 Nm^{-1}, the radius of the drop when it detaches from the dropper is approximately :

(A) 1.4×10^{-3} m (B) 3.3×10^{-3} m (C) 2.0×10^{-3} m (D) 4.1×10^{-3} m

Ans. **(A)**

Sol. Equating forces on the drop :

$$\frac{T2\pi r^2}{R} = \rho\frac{4}{3}\pi R^3 g$$ (Assume drop as a complete sphere)

$$R = \left(\frac{3Tr^2}{2\rho g}\right)^{1/4}$$

$$= \left(\frac{3 \times 0.11 \times 25 \times 10^{-8}}{2 \times 10^3 \times 10}\right)^{1/4}$$

$= 14.25 \times 10^{-4}$ m $= 1.425 \times 10^{-3}$ m

52. After the drop detaches, its surface energy is :

(A) 1.4×10^{-6} J (B) 2.7×10^{-6} J (C) 5.4×10^{-6} J (D) 8.1×10^{-6} J

Ans. **(B)**

Sol. Surface energy of the drop

U = TA

= $0.11 \times 4\pi (1.4 \times 10^{-3})^2$

= 2.7×10^{-6} J

Paragraph for questions 53 to 55

The key feature of Bohr's theory of spectrum of hydrogen atom is the quantization of angular momentum when an electron is revolving around a proton. We will extent this to a general rotational motion to find quantized rotational energy of a diatomic molecule assuming it to be rigid. The rule to be applied is Bohr's quantization condition.

53. A diatomic molecule has moment of inertia I. By Bohr's quantization condition its rotational energy in the n^{th} level (n = 0 is not allowed) is :

(A) $\frac{1}{n^2}\left(\frac{h^2}{8\pi^2 I}\right)$ (B) $\frac{1}{n}\left(\frac{h^2}{8\pi^2 I}\right)$ (C) $n\left(\frac{h^2}{8\pi^2 I}\right)$ (D) $n^2\left(\frac{h^2}{8\pi^2 I}\right)$

Ans. (D)

Sol. $I\omega = \frac{nh}{2\pi}$

Rotational kinetic energy = $\frac{1}{2}I\omega^2 = \frac{1}{2}\frac{n^2h^2}{4\pi^2 I} = \frac{n^2h^2}{8\pi^2 I}$

Ans. (D)

54. It is found that the excitation frequency from ground to the first excited state of rotation for the CO molecule is close to $\frac{4}{\pi}\times 10^{11}$ Hz. Then the moment of inertia of CO molecule about its centre of mass is close to (Take $h = 2\pi \times 10^{-34}$ J s)

(A) 2.76×10^{-46} kg m^2 (B) 1.87×10^{-46} kg m^2 (C) 4.67×10^{-47} kg m^2 (D) 1.17×10^{-47} kg m^2

Ans. (B)

Sol. hf = change in rotational kinetic energy (f = frequency)

$hf = \frac{3h^2}{8\pi^2 I}$

$I = \frac{3h}{8\pi^2 f} = \frac{3\times 2\pi \times 10^{-34}}{8\pi^2 \times \frac{4}{\pi}\times 10^{11}} = 0.1875 \times 10^{-45}$

$I = 1.875 \times 10^{-46}$ kg m^2 .

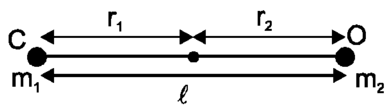
55. In a CO molecule, the distance between C (mass = 12 a.m.u.) and O (mass = 16 a.m.u.), where 1 a.m.u. = $\frac{5}{3}\times 10^{-27}$ kg, is close to :

(1 a.m.u. = $\frac{5}{3}\times 10^{-27}$ kg) :

(A) 2.4×10^{-10} m (B) 1.9×10^{-10} m (C) 1.3×10^{-10} m (D) 4.4×10^{-11} m

Ans. (C)

Sol.



$m_1r_1 = m_2r_2$

$12r_1 = 16r_2$

$$\frac{r_1}{r_2} = \frac{4}{3} \quad \Rightarrow \quad \frac{r_1}{\ell} = \frac{4}{7}$$

$$r_1 = \frac{4}{7}\ell$$

Now, $I = m_1r_1^2 + m_2r_2^2$

$= m_1r_1(\ell)$

$$= m_1\left(\frac{4}{7}\ell\right)\ell$$

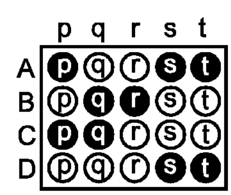
$$I = \left(\frac{4m_1}{7}\right)\ell^2 \quad \Rightarrow \quad \ell = \sqrt{\frac{7I}{4m_1}}$$

$$\ell = \sqrt{\frac{7 \times 1.87 \times 10^{-46}}{4 \times 12 \times \frac{5}{3} \times 10^{-27}}}$$

$= 0.128 \times 10^{-9}$ m $= 1.28 \times 10^{-10}$ m

SECTION - IV (Matrix - Type)

This section contains 2 questions. Each question has four statements (A, B, C and D) given in **Column-I** and five statements (p,q,r,s and t) in **Column-II**. Any given statement in **Column-I** can have correct matching with **ONE OR MORE** statement(s) in **Column-II**. For example, if for a given question, statement B matches with the statements given in q and r, then for that particular question against statement B, darken the bubbles corresponding to q and r in the ORS.



56. Two transparent media of refractive indices μ_1 and μ_3 have a solid lens shaped transparent material of refractive index μ_2 between them as shown in figures in **column II.** A ray traversing these media is also shown in the figures. In **Column I** different relationships between μ_1, μ_2 and μ_3 are given. Match them to the ray diagrams shown in **Column II.**

Column I

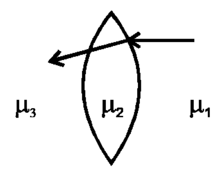
(A) $\mu_1 < \mu_2$

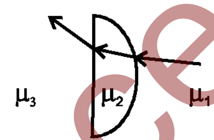
(B) $\mu_1 > \mu_2$

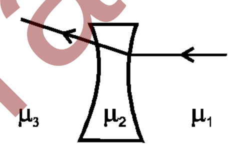
(C) $\mu_2 = \mu_3$

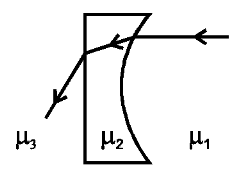
(D) $\mu_2 > \mu_3$

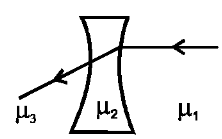
Column II

(p) 

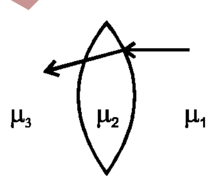
(q) 

(r) 

(s) 

(t) 

Ans. **(A) – p,r ; (B) – q,s,t ; (C) – p,r,t ; (D) – q, s**

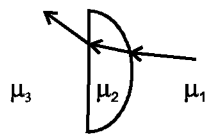
Sol. (A) 

$\mu_2 = \mu_3$

As there is no deviation. As the light bends towards normal in denser medium $\mu_2 > \mu_1$

p – A & C

(B)



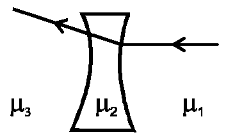
As light bends away from normal

$\mu_2 < \mu_1$

& $\mu_3 < \mu_2$

q – B & D

(C)

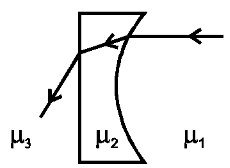


$\mu_2 = \mu_3$ (As no deviation)

$\mu_2 > \mu_1$ (As light bends + towards normal)

r – C & A

(D)



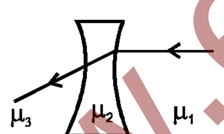
$\mu_2 < \mu_1$

$\mu_3 < \mu_2$

As light bends away from normal

s – B, D

(E)



$\mu_2 = \mu_3$ As no deviation of light

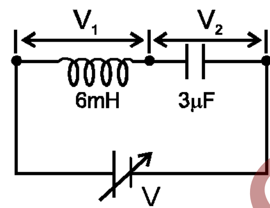
$\mu_2 < \mu_1$ As light bend away from normal

t – C & B

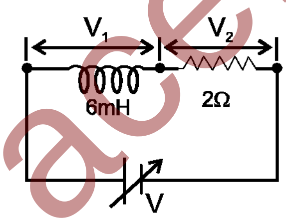
57. You are given many resistances, capacitors and inductors. These are connected to a variable DC voltage source (the first two circuits) or an AC voltage source of 50 Hz frequency (the next three circuits) in different ways as shown in **Column II.** When a current I (steady state for DC or rms for AC) flows through the circuit, the corresponding voltage V_1 and V_2. (indicated in circuits) are related as shown in **Column I.** Match the two

Column I | **Column II**

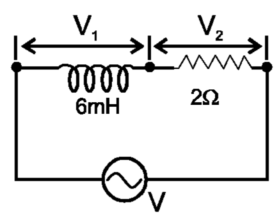
(A) $I \neq 0, V_1$ is proportional to I

(p) 

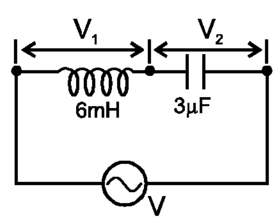
(B) $I \neq 0, V_2 > V_1$

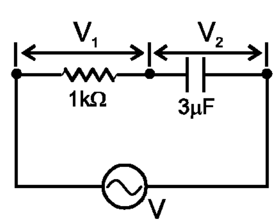
(q) 

(C) $V_1 = 0, V_2 = V$

(r) 

(D) $I \neq 0, V_2$ is proportional to I

(s) 

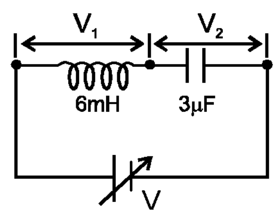
(t) 

Ans. (A) – r,s,t ; (B) – q,r,s,t ; (C) – p,q ; (D) – q,r,s,t

As per given conditions, there will be no steady state in circuit 'p', so it should not be considered in options of 'c'.

Ans. (A) – r,s,t ; (B) – q,r,s,t ; (C) – q ; (D) – q,r,s,t

Sol. (p)



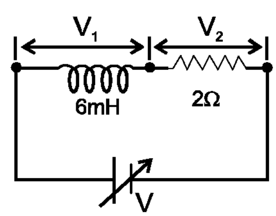
As I is steady state current

$V_1 = 0$; $I = 0$

Hence, $V_2 = V$

So , answer of P $\Rightarrow$ C

(q)



In the steady state ;

$V_1 = 0$ as $\frac{dI}{dt} = 0$

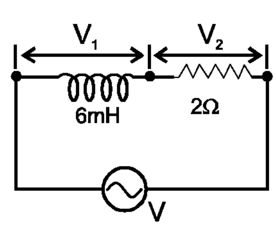
$\therefore$ $V_2 = V = IR$

or $V_2 \propto I$

and $V_2 > V_1$

So , answer of q $\Rightarrow$ B, C, D

(r)



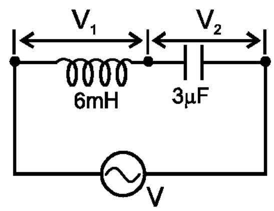
Inductive reactance $X_L = \omega L$

$X_L = 6\pi \times 10^{-1}\ \Omega$

and resistance $= R = 2\Omega$

So, $V_1 = IX_L$
and $V_2 = IR$
Hence, $V_2 > V_1$
So, Answer of $r \Rightarrow$ A,B,D

(s)

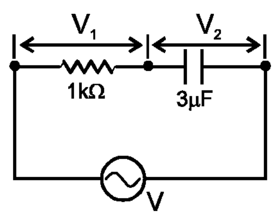


Here, $V_1 = IX_L$, where, $X_L = 6\pi \times 10^{-1}\ \Omega$

Also, $V_2 = IX_C$, where, $X_C = \frac{10^4}{3\pi}$

So, $V_2 > V_1$
$V_1 \propto I$
$V_2 \propto I$
So, answer of $s \Rightarrow$ A,B,D

(t)



Here, $V_1 = IR$, where, $R = 1000\ \Omega$, $X_C = \frac{10^4}{3\pi}\Omega$

$V_2 = IX_C$, where, $X_C = \frac{10^4}{3\pi}\Omega$

So, $V_2 > V_1$
and $V_1 \propto I$
$V_2 \propto I$
So, answer of $t \Rightarrow$ A,B,D

Ans. **(A) – r,s,t ; (B) – q,r,s,t ; (C) – p,q ; (D) – q,r,s,t**

Note : For circuit 'p' :

$$V - \frac{Ldi}{dt} - \frac{q}{C} = 0 \quad \text{or} \quad CV = CL\frac{di}{dt} + q \quad \text{or} \quad 0 = LC\frac{d^2i}{dt^2} + \frac{dq}{dt} \quad \text{or} \quad \frac{d^2i}{dt^2} = -\frac{1}{LC}\frac{dq}{dt}$$

So, $$i = i_0 \sin\left(\frac{1}{\sqrt{LC}}t + \phi_0\right)$$

As per given conditions, there will be no steady state in circuit 'p'. So it should not be considered in options of 'c'.

Ans. **(A) – r,s,t ; (B) – q,r,s,t ; (C) – q ; (D) – q,r,s,t**