Part – I : (CHEMISTRY)

SECTION – I (Total Marks : 24)

CODE - 9 **(Single Correct Answer Type)** **10/04/2011**

This section contains **8 multiple choice questions**. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

1. Oxidation states of the metal in the minerals haematite and magnetite, respectively, are

(A) II, III in haematite and III in magnetite

(B) II, III in haematite and II in magnetite

(C) II in haematite and II, III in magnetite

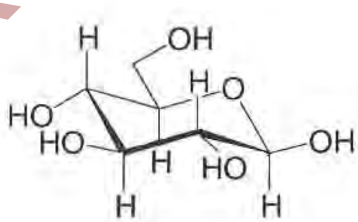
(D) III in haematite and II, III in magnetite

Ans. **[D]**

Sol. Haematite is $\overset{+3}{Fe}_2O_3$

Magnetite is Fe_3O_4 or $\overset{+2}{Fe}O.\overset{+3}{Fe}_2O_3$

2. The following carbohydrate is



(A) a ketohexose

(B) an aldohexose

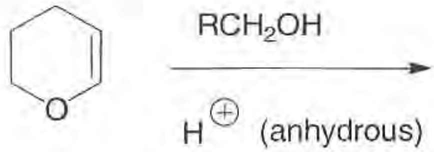
(C) an α-furanose

(D) an α-pyranose

Ans. **[B]**

Sol. Aldohexose

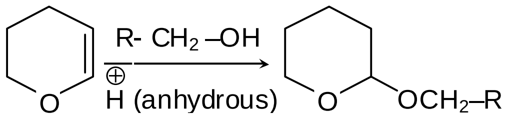
3. The major product of the following reaction is



(A) a hemiacetal
(B) an acetal
(C) an ether
(D) an ester

Ans. **[B]**

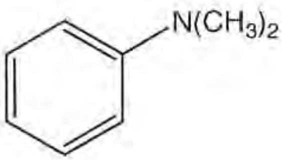
Sol.



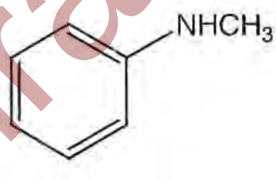
Acetal

4. Amongst the compounds given, the one that would form a brilliant colored dye on treatment with $NaNO_2$ in dil. HCl followed by addition to an alkaline solution of β-naphthol is

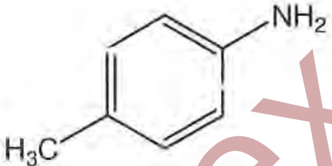
(A)



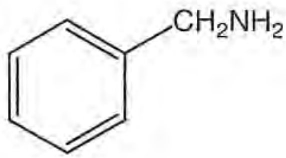
(B)



(C)

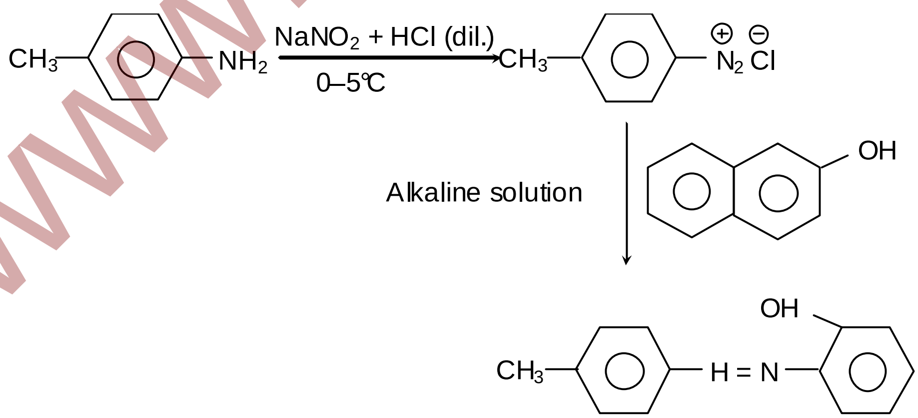


(D)



Ans. **[C]**

Sol.



Coloured dye

5. The freezing point (in $^{\circ}C$) of a solution containing 0.1 g of $K_3[Fe(CN)_6]$ (Mol. Wt. 329) in 100 g of water ($K_f = 1.86$ K kg mol^{-1}) is

(A) -2.3×10^{-2} (B) -5.7×10^{-2} (C) -5.7×10^{-3} (D) -1.2×10^{-2}

Ans. **[A]**

Sol. $\Delta T = k_f \times m \times i \times 1000$

$$= 1.86 \times \frac{0.1}{329 \times 100} \times 4 \times 1000$$

$$= 2.26 \times 10^{-2} \approx 2.3 \times 10^{-2}$$

6. Consider the following cell reaction:

$$2Fe_{(s)} + O_{2(g)} + 4H^+_{(aq)} \rightarrow 2Fe^{2+}_{(aq)} + 2H_2O(l) \qquad E^{\circ} = 1.67\ V$$

At $[Fe^{2+}] = 10^{-3}$ M, $P(O_2) = 0.1$ atm and pH = 3, the cell potential at 25 $^{\circ}C$ is

(A) 1.47 V (B) 1.77 V (C) 1.87 V (D) 1.57 V

Ans. **[D]**

$$E_{cell} = E^{\circ}_{cell} - \frac{.0591}{4} \log \frac{[Fe^{+2}]}{[P_{O_2}][H^+]^4}$$

$$= 1.67 - \frac{.591}{4} \log \frac{[10^{-3}]^2}{[.1][10^{-3}]^4}$$

$= 1.57$ V

7. Passing H_2S gas into a mixture of Mn^{2+}, Ni^{2+}, Cu^{2+} and Hg^{2+} ions in an acidified aqueous solution precipitates

(A) CuS and HgS (B) MnS and CuS

(C) MnS and NiS (D) NiS and HgS

Ans. **[A]**

Sol. **Cu^{+2}, Hg^{+2} are group II basic radicals**

8. Among the following complexes (**K**–**P**),
$K_3[Fe(CN)_6]$ (**K**), $[Co(NH_3)_6]Cl_3$ (**L**), $Na_3[Co(oxalate)_3]$ (**M**), $[Ni(H_2O)_6]Cl_2$ (**N**), $K_2[Pt(CN)_4]$ (**O**) and $[Zn(H_2O)_6](NO_3)_2$ (**P**)
the diamagnetic complexes are

(A) **K, L, M, N** (B) **K, M, O, P** (C) **L, M, O, P** (D) **L, M, N, O**

Ans. **[C]**

(L) : $[Co(NH_3)_6]Cl_3$

(M) : $Na_3[Co(Ox)_3]$

(O) : $K_2[Pt(CN)_4]$

(P) : $[Zn(H_2O)_6] (NO_3)_2$

SECTION – II (Total Marks : 16)
(Multiple Correct Answers Type)

This section contains **4 multiple choice questions**. Each questions has 4 choices (A), (B), (C) and (D), out of which **ONE OR MORE** is/are correct.

9. Reduction of the metal centre in aqueous permanganate ion involves

(A) 3 electrons in neutral medium (B) 5 electrons in neutral medium
(C) 3 electrons in alkaline medium (D) 5 electrons in acidic medium

Ans. **[A, C, D]**

Sol. → **In alkaline solution, $KMnO_4$ is first reduced to mangnate and then to insoluble MnO_2**

$$\rightarrow 2K\overset{+7}{Mn}O_4 + H_2O \xrightarrow{\text{Neutral}} 2\overset{+4}{Mn}O_2 + 2KOH + 3[O]$$

$$\rightarrow \text{I}\ MnO_4^- + 8H^+ + 5e^- \xrightarrow{\text{acidic}} Mn^{+2} + 4H_2O$$

10. For the first order reaction

$$2N_2O_5(g) \rightarrow 4NO_2(g) + O_2(g)$$

(A) the concentration of the reactant decreases exponentially with time.
(B) the half-life of the reaction decreases with increasing temperature.
(C) the half-life of the reaction depends on the initial concentration of the reactant.
(D) the reaction proceeds to 99.6 % completion in eight half-life duration.

Ans. **[A, B, D]**

Sol. $C_A = C_{A_0} e^{-kt}$ [A]

$$t_{\frac{1}{2}} = \frac{0.693}{K} = \frac{0.693}{A_0 e^{-E_a/RT}} = \frac{0.693}{A_0} e^{E_a/RT} \text{ [B]}$$

$$\frac{0.4}{100} = \left(\frac{1}{2}\right)^4 = \frac{4}{100} = n = \frac{\log\left(\frac{4}{100}\right)}{\log\left(\frac{1}{2}\right)} = 8 \text{ [D]}$$

11. The equilibrium

$$2\,Cu^{I} \rightleftharpoons Cu^{0} + Cu^{II}$$

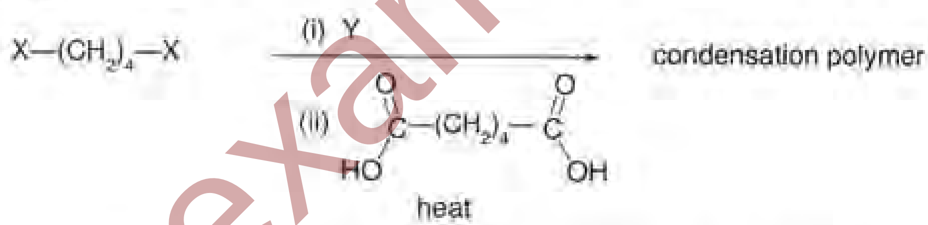
in aqueous medium at 25 °C shifts towards the left in the presence of

(A) NO_3^- (B) Cl^- (C) SCN^- (D) CN^-

Ans. **[B, C, D]**

Sol. **Cu_2Cl_2, $Cu_2(CN)_2$ and $Cu_2(SCN)_2$ are stable**

12. The correct functional group X and the reagent/reaction conditions Y in the following scheme are



(A) $X = COOCH_3$, $Y = H_2/Ni/heat$ (B) $X = CONH_2$, $Y = H_2/Ni/heat$

(C) $X = CONH_2$, $Y = Br_2/NaOH$ (D) $X = CN$, $Y = H_2/Ni/heat$

Ans. **[A, B, C, D]**

Sol. **Factual**

SECTION – III (Total Marks : 24)

(Integer Answer Type)

This section contains **6 Question.** The answer to each of the question is a **single-digit integer,** ranging from 0 to 9. The total bubble corresponding answer it to be darkened in the ORS.

13. In 1 L saturated solution of AgCl [$K_{sp}(AgCl) = 1.6 \times 10^{-10}$], 0.1 mol of CuCl [$K_{sp}(CuCl) = 1.0 \times 10^{-6}$] is added. The resultant concentration of Ag^+ in the solution is 1.6×10^{-x}. The value of "x" is

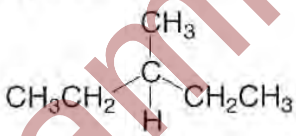
Ans. **[7]**

Sol. $[Ag^+] = \dfrac{K_1}{\sqrt{K_1 + K_2}}$ $\because K_1 << K_2$ $\therefore K_1 + K_2 \cong K_2$

$\therefore [Ag^+] = \dfrac{1.6 \times 10^{-10}}{\sqrt{1.0 \times 10^{-6}}} = 1.6 \times 10^{-7}$

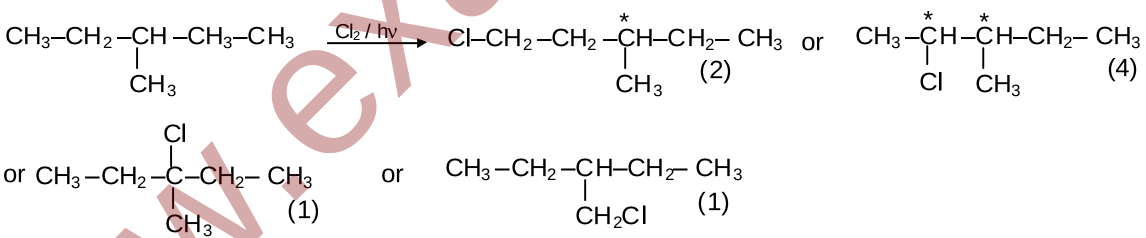
x = 7

14. The maximum number of isomers (including stereoisomers) that are possible on mono-chlorination of the following compound, is



Ans. **[8]**

Sol.



15. Among the following, the number of compounds than can react with PCl_5 to give $POCl_3$ is

O_2, CO_2, SO_2, H_2O, H_2SO_4, P_4O_{10}

Ans. **[4]**

Sol. $PCl_5 + H_2O \longrightarrow POCl_3 + 2HCl$

$PCl_5 + H_2SO \longrightarrow POCl_3 + H_2O + SO_2Cl_2$

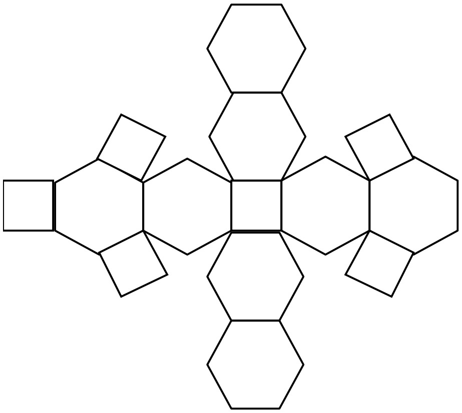
$6PCl_5 + P_4O_{10} \longrightarrow 10POCl_3$

$PCl_5 + SO_2 \longrightarrow POCl_3 + SO_2Cl_2$

16. The number of hexagonal faces that are present in a truncated octahedron is

Ans. **[8]**

Sol.



8 Hexagonal faces

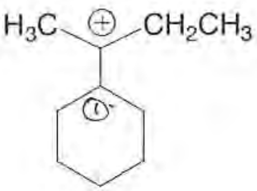
17. The volume (in mL) of 0.1 M $AgNO_3$ required for complete precipitation of chloride ions present in 30 mL of 0.01 M solution of $[Cr(H_2O)_5Cl]Cl_2$, as silver chloride is close to

Ans. **[6]**

Sol. $0.1\ V = 30 \times 0.01 \times 2$

$$V = \frac{0.3 \times 2}{0.1} = \mathbf{6\ ml}$$

18. The total number of contributing structures showing hyperconjugation (involving C-H bonds) for the following carbocation is



Ans. **[6]**

Sol. **6 (α – H $\rightarrow$ 6)**

SECTION – IV (Total Marks : 16)

(Matrix-Match Type)

This section contain **2 questions**. Each question has **four statements** (A, B, C and D) given in **column I** and five statements (p, q, r, s and t) in **column II**. Any given statement in column I can have correct matching with **ONE** or **MORE** statement (s) given in Column II. For example, if for a given question, statement B matches with the statements given in q and r, then for the particular question, against statement B, darken the bubbles corresponding to q and r in the ORS

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19. Match the transformations in **column I** with appropriate options in **column II**

Column I		**Column II**	
(A)	$CO_2(s) \rightarrow CO_2(g)$	(p)	phase transition
(B)	$CaCO_3(s) \rightarrow CaO(s) + CO_2(g)$	(q)	allotropic change
(C)	$2\,H\bullet \rightarrow H_2(g)$	(r)	ΔH is positive
(D)	$P_{(white,\ solid)} \rightarrow P_{(red,\ solid)}$	(s)	ΔS is positive
		(t)	ΔS is negative

Ans. **[A → p, r, s; B → r, s; C → t ; D → p, q, t]**

Sol. [A] $CO_2(s) \rightarrow CO_2(g)$

p, r, s

[B] $CaCO_3\,(s) \rightarrow CaO\,(s) + CO_2(g)$

r, s

[C] $2H \rightarrow H_2\,(g)$

t

[D] $P_{white} \rightarrow P_{red}$

p, q, t

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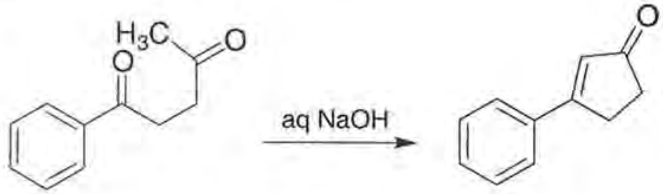
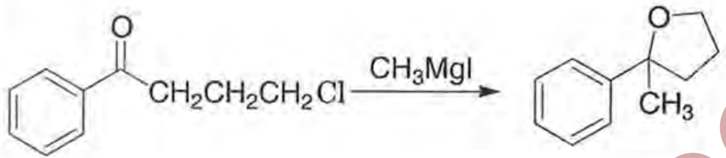
Column I

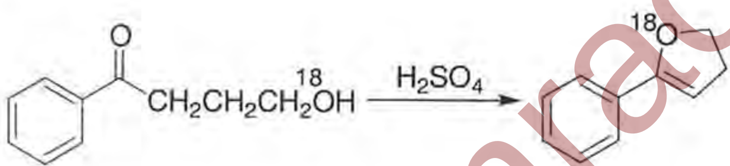
(A)



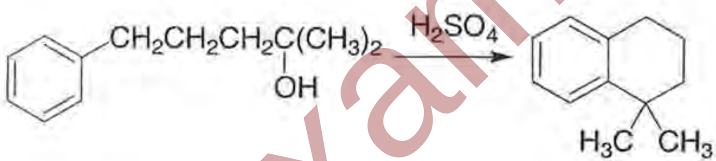
(B)



(C)



(D)



Column II

(p) Nucleophilic substitution

(q) Electrophilic substitution

(r) Dehydration

(s) Nucleophilic addition

(t) Carbanion

Ans. [A → r, t, s; B → p, s, t; C → r, s ; D → r, q]

Sol. Factual

SECTION – III (Total Marks : 24)

(Integer Answer Type)

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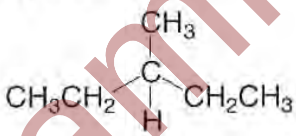

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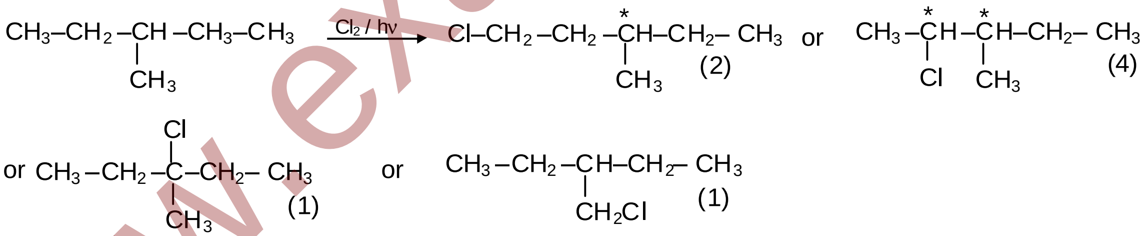
Sol.



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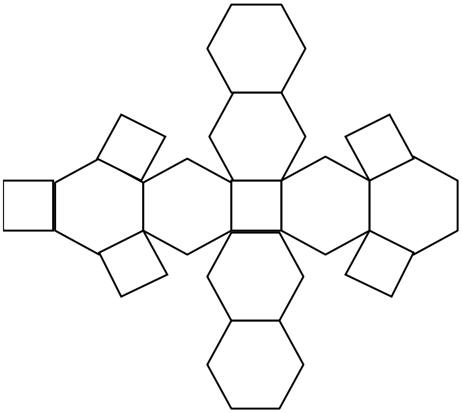

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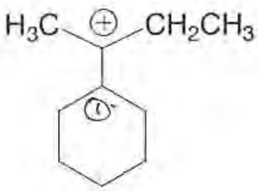

Ans. **[6]**

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(Matrix-Match Type)

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		(t)	ΔS is negative

Ans. **[A → p, r, s; B → r, s; C → t ; D → p, q, t]**

Sol. [A] $CO_2(s) \rightarrow CO_2(g)$

p, r, s

[B] $CaCO_3\,(s) \rightarrow CaO\,(s) + CO_2(g)$

r, s

[C] $2H \rightarrow H_2\,(g)$

t

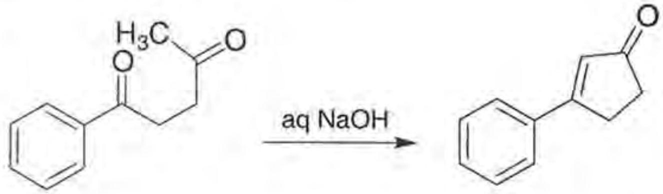
[D] $P_{white} \rightarrow P_{red}$

p, q, t

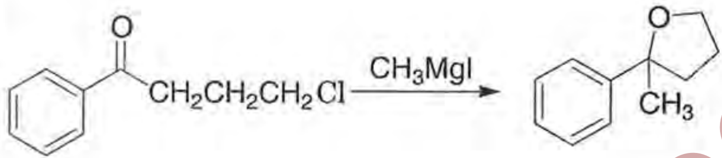
20. Match the reactions in **column I** with appropriate types of steps/reactive intermediate involved in these reactions as given in **column II**

Column I

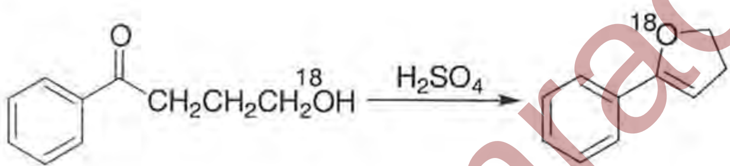
(A)



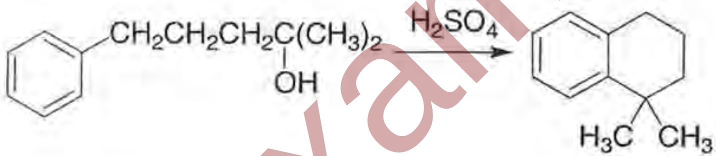
(B)



(C)



(D)



Column II

(p) Nucleophilic substitution

(q) Electrophilic substitution

(r) Dehydration

(s) Nucleophilic addition

(t) Carbanion

Ans. [A → r, t, s; B → p, s, t; C → r, s ; D → r, q]

Sol. Factual

PART II : PHYSICS

SECTION – I (Total Marks : 24)

Code : 9 **(Single Correct Answer Type)** **10/04/2011**

This section contains **8 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

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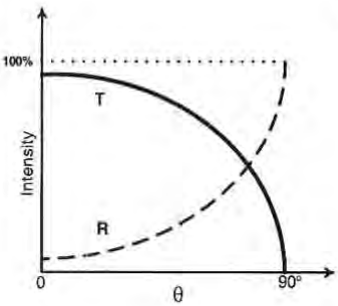
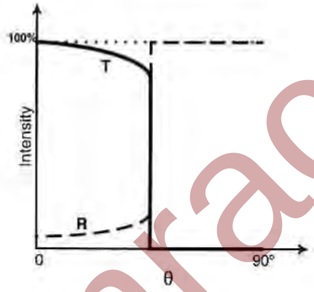
21. A light ray traveling in glass medium is incident on glass-air interface at an angle of incidence θ. The reflected (R) and transmitted (T) intensities, both as function of θ, are plotted. The correct sketch is

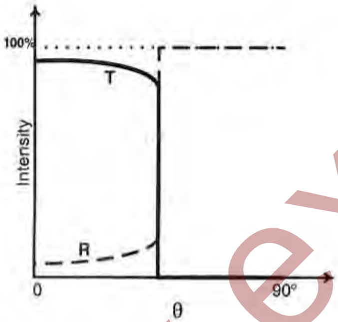
(A)



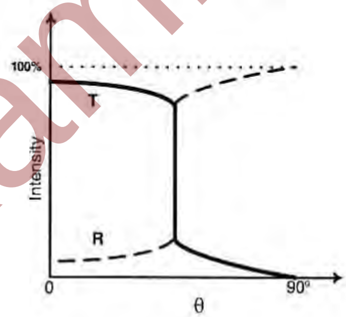
(B)



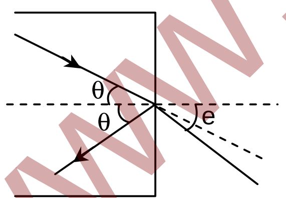
(C)



(D)

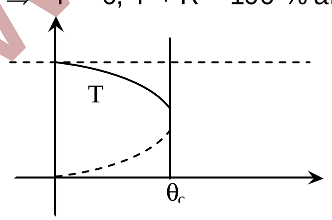


Ans.[C]

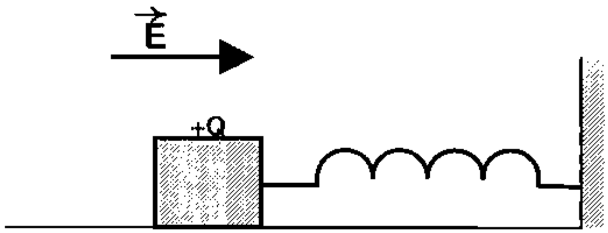


When $\theta > \theta_C$, no ray will transmit

$\Rightarrow$ T = 0, T + R = 100 % and R > 0



22. A wooden block performs SHM on a frictionless surface with frequency, ν_0. The block carries a charge +Q on its surface. If now a uniform electric field $\vec{E}$ is switched-on as shown, then the SHM of the block will be



(A) of the same frequency and with shifted mean position.

(B) of the same frequency and with the same mean position.

(C) of changed frequency and with shifted mean position.

(D) of changed frequency and with the same mean position.

Ans.[A] In order to have net force zero, the mean position will be shifted towards right but the time period will remain unaffected.

23. The density of a solid ball is to be determined in an experiment. The diameter of the ball is measured with a screw gauge, whose pitch is 0.5 mm and there are 50 divisions on the circular scale. The reading on the main scale is 2.5 mm and that on the circular scale is 20 divisions. If the measured mass of the ball has a relative error of 2%, the relative percentage error in the density is

(A) 0.9% (B) 2.4% (C) 3.1% (D) 4.2%

Ans.[C] Pitch = 0.5 mm

divisions on the = 50

circular scale

$\Rightarrow$ least count of screw gauge = $\frac{0.5}{50} = 0.01$

main scale, reading = 2.5 mm

circular scale reading = 20

$\Rightarrow$ reading = 2.5 mm + (20 × 0.01) mm

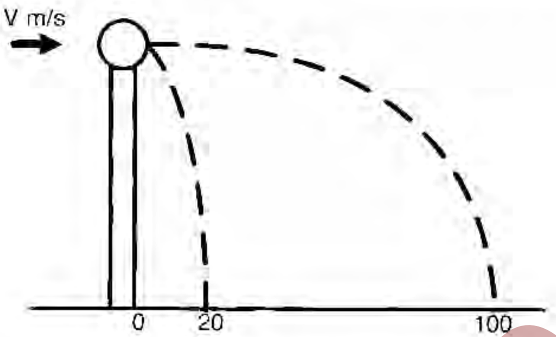
= 2.5 mm + 0.2 mm = 2.7 mm

$$\rho = \frac{m}{\frac{4\pi}{3}\left[\frac{D}{2}\right]^3}$$

$$\frac{\Delta\rho}{\rho} = \frac{\Delta m}{m} + 3\,\frac{\Delta D}{D}$$

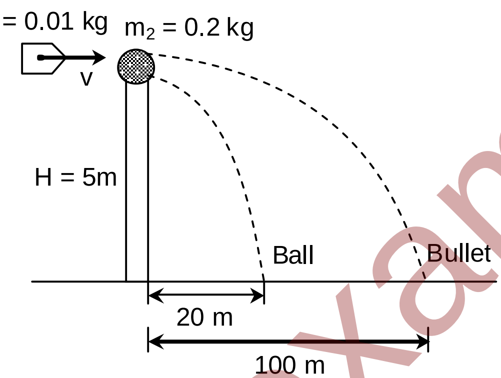
$$\%\text{error} = \frac{\Delta\rho}{\rho} \times 100 = 2\,\% + 3\left(\frac{0.01}{2.7}\right) \times 100 = 3.1.$$

24. A ball of mass 0.2 kg rests on a vertical post of height 5 m. A bullet of mass 0.01 kg, traveling with a velocity V m/s in a horizontal direction, hits the centre of the ball. After the collision, the ball and bullet travel independently. The ball hits the ground at a distance of 20 m and the bullet at a distance of 100 m from the foot of the post. The initial velocity V of the bullet is



(A) 250 m/s (B) $250\sqrt{2}$ m/s (C) 400 m/s (D) 500 m/s

Ans.[D]



$$T = \sqrt{\frac{2H}{g}} = 1 \text{ sec}$$

Let v_1 & v_2 be velocity of bullet & ball respectively just after collision.

$v_2 \times 1 = 20 \Rightarrow v_2 = 20$

& $v_1 = 100$

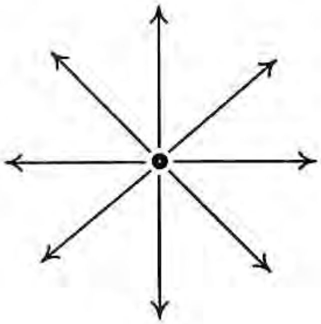
From conservation of momentum

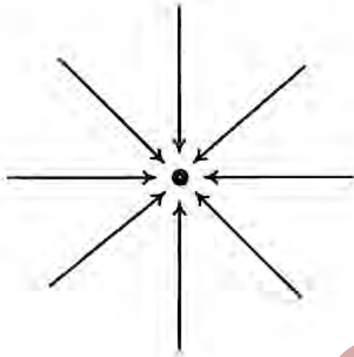
$0.01 \times v = (0.01 \times 100) + (0.2 \times 20)$

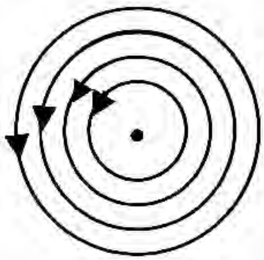
$0.01\, v = 1 + 4 = 5$

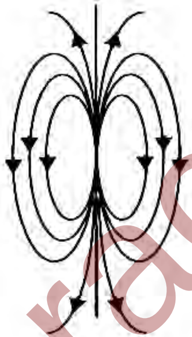
$$v = \frac{5}{10^{-2}} = 500 \text{ m/sec.}$$

25. Which of the field patterns given below is valid for electric field as well as for magnetic field?

(A) 

(B) 

(C) 

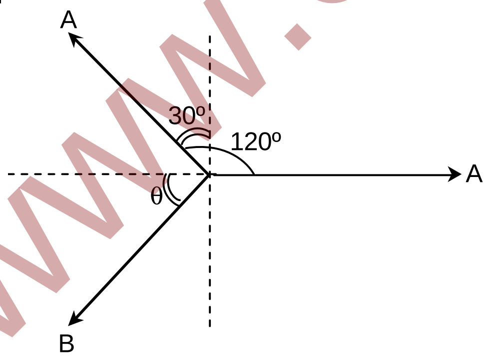
(D) 

Ans.[C] Electric lines of force for induced electric field is closed loop.

26. A point mass is subjected to two simultaneous sinusoidal displacements in x-direction, $x_1(t) = A\sin\omega t$ and $x_2(t) = A\sin\left(\omega t + \frac{2\pi}{3}\right)$. Adding a third sinusoidal displacement $x_3(t) = B\sin(\omega t + \phi)$ brings the mass to a complete rest. The values of B and ϕ are

(A) $\sqrt{2}A, \frac{3\pi}{4}$ (B) $A, \frac{4\pi}{3}$ (C) $\sqrt{3}A, \frac{5\pi}{6}$ (D) $A, \frac{\pi}{3}$

Ans.[B]

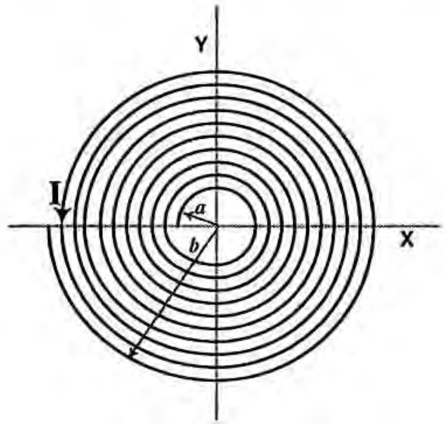


Here $\phi = \pi + \theta$

$A\cos 30^\circ = B\sin\theta \Rightarrow B\sin\theta = \frac{\sqrt{3}A}{2}$ and $A\sin 30^\circ + B\cos\theta = A \Rightarrow B\cos\theta = \frac{A}{2}$

Solving above, $B = A$ and $\theta = 60^\circ = \frac{\pi}{3}$. Hence $\phi = 240^\circ = \frac{4\pi}{3}$

27. A long insulated copper wire is closely wound as a spiral of 'N' turns. The spiral has inner radius 'a' and outer radius 'b'. The spiral lies in the X-Y plane and a steady current 'I' flows through the wire. The Z-component of the magnetic field at the center of the spiral is



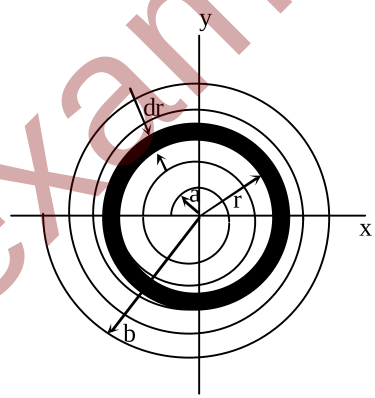
(A) $\frac{\mu_0 N I}{2(b-a)} \ln\left(\frac{b}{a}\right)$

(B) $\frac{\mu_0 N I}{2(b-a)} \ln\left(\frac{b+a}{b-a}\right)$

(C) $\frac{\mu_0 N I}{2b} \ln\left(\frac{b}{a}\right)$

(D) $\frac{\mu_0 N I}{2b} \ln\left(\frac{b+a}{b-a}\right)$

Ans.[A]



No. of turns per unit thickness = $\frac{N}{b-a}$

magnetic field at centre due to element = $\frac{\mu_0 (dN) i}{2r}$

$$dB = \frac{\mu_0 i}{2r}\left(\frac{N}{b-a}\right) dr$$

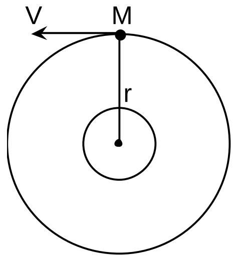
$$B = \frac{\mu_0 i N}{2(b-a)} \int_a^b \frac{dr}{r}$$

$$= \frac{\mu_0 i N}{2(b-a)} \ln\left(\frac{b}{a}\right)$$

28. A satellite is moving with a constant speed 'V' in a circular orbit about the earth. An object of mass 'm' is ejected from the satellite such that it just escapes from the gravitational pull of the earth. At the time of its ejection, the kinetic energy of the object is

(A) $\frac{1}{2}mV^2$ (B) mV^2 (C) $\frac{3}{2}mV^2$ (D) $2mV^2$

Ans.[B]



$\Rightarrow \quad \frac{mv^2}{r} = \frac{GmM_e}{r^2} \quad \Rightarrow \quad r = \frac{GM_e}{V^2}$ ....(1)

If K.E. of mass m = was k then from

$$E = K - \frac{GmM_e}{r} = 0 \Rightarrow K = m\left(\frac{GM_e}{r}\right) = mv^2$$

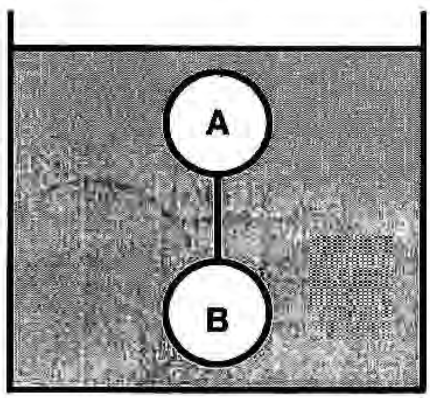
SECTION – II (Total Marks : 16)

(Multiple Correct Answer(s) Type)

This section contains **4 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONE or MORE** may be correct.

---

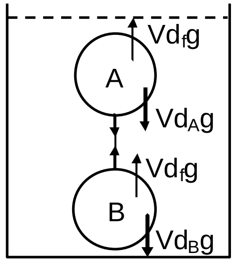
29. Two solid spheres A and B of equal volumes but of different densities d_A and d_B are connected by a string. They are fully immersed in a fluid of density d_F. They get arranged into an equilibrium state as shown in the figure with a tension in the string. The arrangement is possible only if



(A) $d_A < d_F$ (B) $d_B > d_F$

(C) $d_A > d_F$ (D) $d_A + d_B = 2\,d_F$

Ans.[A,B,D]



system will be in equilibrium with tension in string only if $d_f > d_A$ and $d_B > d_f$. If both A & B are considered as a system then

$$2Vd_fg = V(d_A + d_b)g \quad \Rightarrow \quad d_A + d_B = 2d_f$$

30. Which of the following statement(s) is/are correct?

(A) If the electric field due to a point charge varies as $r^{-2.5}$ instead of r^{-2}, then the Gauss law will still be valid.

(B) The Gauss law can be used to calculate the field distribution around an electric dipole.

(C) If the electric field between two point charges is zero somewhere, then the sign of the two charges is the same.

(D) The work done by the external force in moving a unit positive charge from point A at potential V_A to point B at potential V_B is $(V_B - V_A)$.

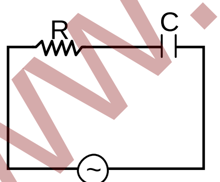
Ans.[C,D]

31. A series R-C circuit is connected to AC voltage source. Consider two cases; (A) when C is without a dielectric medium and (B) when C is filled with dielectric of constant 4. The current I_R through the resistor and voltage V_C across the capacitor are compared in the two cases. Which of the following is/are true?

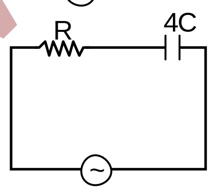
(A) $I_R^A > I_R^B$ (B) $I_R^A < I_R^B$

(C) $V_C^A > V_C^B$ (D) $V_C^A < V_C^B$

Ans.[B,C]



$$Z_1 = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$$

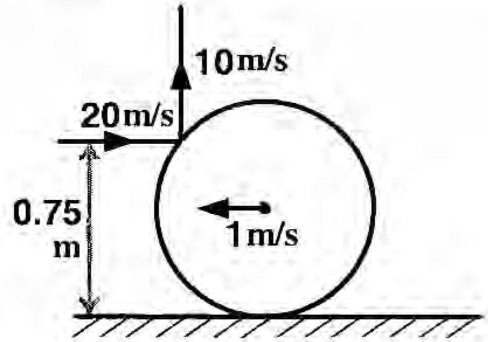


$$Z_2 = \sqrt{R^2 + \left(\frac{1}{4\omega C}\right)^2}$$

$z_1 > z_2 \quad \therefore \quad I_R^A < I_R^B$

$V_C^A = \frac{I_K^A}{\omega C}$; $V_C^B = \frac{I_K^B}{4\omega C}$; $V_C^B < V_C^A$

32. A thin ring of mass 2 kg and radius 0.5 m is rolling without slipping on a horizontal plane with velocity 1 m/s. A small ball of mass 0.1 kg, moving with velocity 20 m/s in the opposite direction, hits the ring at a height of 0.75 m and goes vertically up with velocity 10 m/s. Immediately after the collision



(A) the ring has pure rotation about its stationary CM.

(B) the ring comes to a complete stop.

(C) friction between the ring and the ground is to the left.

(D) there is no friction between the ring and the ground.

Ans.[A,C]

SECTION – III (Total Marks : 24)

(Integer Answer Type)

This section contains **6 questions**. The answer to each of the questions is a **single-digit integer**, ranging from 0 to 9. The bubble corresponding to the correct answer is to be darkened in the ORS.

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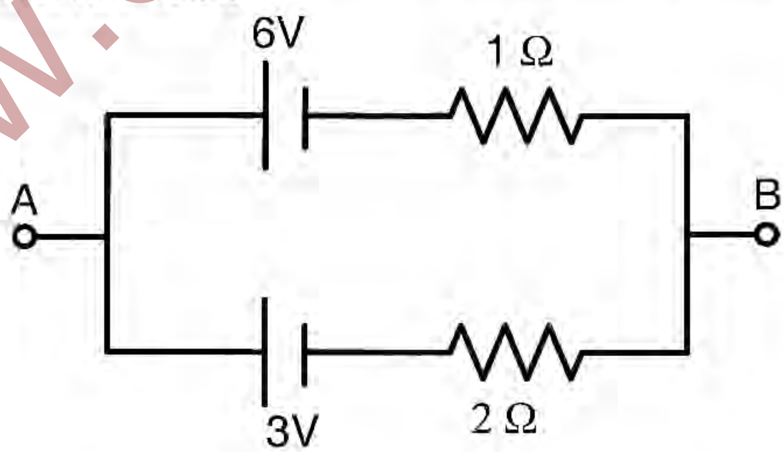
33. Two batteries of different emfs and different internal resistances are connected as shown. The voltage across AB in volts is



Ans.[5]

$$V_A - V_B = \frac{\frac{6}{1}+\frac{3}{2}}{\frac{1}{1}+\frac{1}{2}} = \frac{6+1.5}{1.5} = \frac{7.5}{1.5} = 5V$$

34. A series R-C combination is connected to an AC voltage of angular frequency $\omega = 500$ radian/s. If the impedance of the R-C circuit is $R\sqrt{1.25}$, the time constant (in millisecond) of the circuit is

Ans.[4] $Z = R\sqrt{1.25}$

$\tau = RC$

$$R^2 + \left(\frac{1}{500C}\right)^2 = Z^2$$

$$R^2 + \left(\frac{1}{500C}\right)^2 = R^2 \times 1.25$$

$$\left(\frac{1}{500C}\right)^2 = 0.25\ R^2 \qquad \Rightarrow \frac{1}{500C} = 0.5R$$

$$\frac{1}{2500} = RC$$

$$\frac{1}{250} = RC$$

0.004 sec = RC

RC = 4 mill second.

35. A train is moving along a straight line with a constant acceleration 'a'. A boy standing in the train throws a ball forward with a speed of 10 m/s, at an angle of 60° to the horizontal. The boy has to move forward by 1.15 m inside the train to catch the ball back at the initial height. The acceleration of the train, in m/s^2, is

Ans.[5] $T = \frac{2\times10\times\sqrt{3}}{2\times10} = \sqrt{3}$ sec

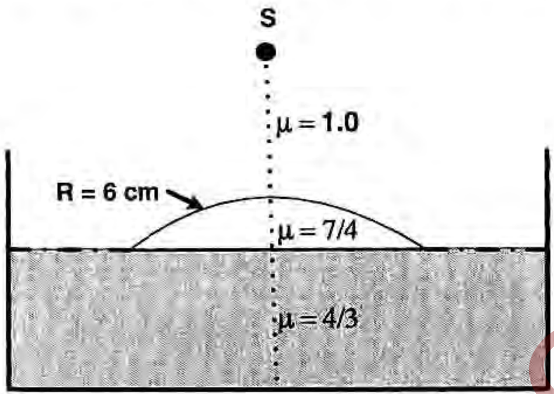
$x = 10\cos 60°(T) = 5\sqrt{3}$ m

In frame of train,

$5\sqrt{3} = \frac{1}{2}\times a \times (\sqrt{3})^2 + 1.15$ (a : acceleration of train)

$a = 5\ m/sec^2$

36. Water (with refractive index = $\frac{4}{3}$) in a tank is 18 cm deep. Oil of refractive index $\frac{7}{4}$ lies on water making a convex surface of radius of curvature 'R = 6 cm' as shown. Consider oil to act as a thin lens. An object 'S' is placed 24 cm above water surface. The location of its image is at 'x' cm above the bottom of the tank. Then 'x' is



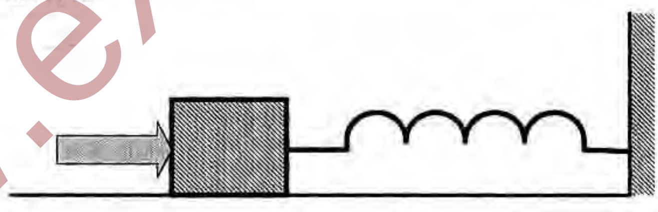
Ans.[2]

$$\frac{\mu_3}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R_1} + \frac{\mu_3 - \mu_2}{R_2}$$

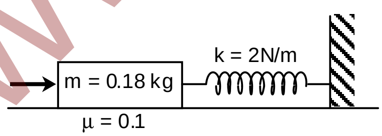
$$\frac{4}{3V} - \frac{1}{-24} = \frac{\frac{7}{4} - 1}{6} + \frac{\frac{4}{3} - \frac{7}{4}}{\infty} \Rightarrow \frac{4}{3V} + \frac{1}{24} = \frac{1}{8} \Rightarrow \frac{4}{3V} = \frac{1}{12} \Rightarrow V = 16 \text{ cm}$$

$\therefore$ Ans. = (18 – 16) cm = 2 cm

37. A block of mass 0.18 kg is attached to a spring of force-constant 2 N/m. The coefficient of friction between the block and the floor is 0.1. Initially the block is at rest and the spring is un-stretched. An impulse is given to the block as shown in the figure. The block slides a distance of 0.06 m and comes to rest for the first time. The initial velocity of the block in m/s is V = N/10. Then N is



Ans.[4]



Using W – E theorem

$$\frac{1}{2} \times m(u)^2 = \frac{1}{2}K(x)^2 + \mu mg(x)$$

$$\frac{1}{2} \times (0.18)u^2 = \frac{1}{2} \times 2 \times 36 \times 10^{-4} + 0.1 \times 0.18 \times 10 \times 0.06$$

$\Rightarrow$ u = 0.4 m/sec.

$\Rightarrow \frac{4}{10}$ m/sec.

38. A silver sphere of radius 1 cm and work function 4.7 eV is suspended from an insulating thread in free-space. It is under continuous illumination of 200 nm wavelength light. As photoelectrons are emitted, the sphere gets charged and acquires a potential. The maximum number of photoelectrons emitted from the sphere is $A \times 10^{Z}$ (where $1 < A < 10$). The value of 'Z' is

Ans.[7] Energy of photon $\approx \frac{1240}{200}$ ev = 6.2 eV

Maximum KE of a electron = 6.2 eV – 4.7 eV

When potential on surface of sphere becomes equal to 1.5V

$$\frac{q}{4\pi\epsilon_0 r} = 1.5\text{ V} \Rightarrow q = 1.5 \times (4\pi\varepsilon_0) \times r$$

No. of photoelectron emitted $n = \frac{1.5 \times (4\pi\varepsilon_0)r}{1.6 \times 10^{-19}} = 1.04 \times 10^{7}$

SECTION – IV (Total Marks : 16)

(Matrix-Match Type)

This section contains **2 questions**. Each question has **four statements** (A, B, C and D) given in **Column I** and **five statements** (p, q, r, s and t) in **Column II**. Any given statement in Column I can have correct matching with **ONE** or **MORE** statement(s) given in Column II. For example, if for a given question, statement B matches with the statements given in q and r, then for the particular question, against statement B, darken the bubbles corresponding to q and r in the ORS.

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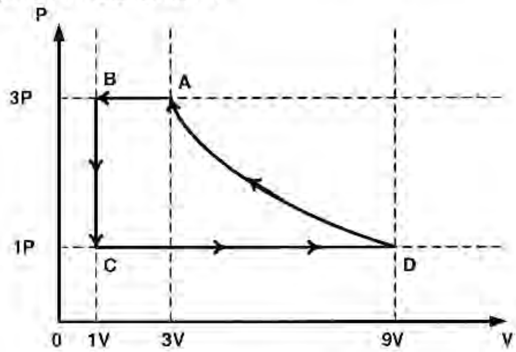
39. One mole of a monatomic ideal gas is taken through a cycle ABCDA as shown in the P-V diagram. **Column II** gives the characteristics involved in the cycle. Match them with each of the processes given in **Column I**.



Column I	Column II
(A) Process $A \to B$	(p) Internal energy decreases.
(B) Process $B \to C$	(q) Internal energy increases.
(C) Process $C \to D$	(r) Heat is lost.
(D) Process $D \to A$	(s) Heat is gained.
	(t) Work is done on the gas.

Ans. (A) → p,r,t; (B) → p,r; (C) → q,s; (D) → r,t

Process AB : (Pressure is constant)

If $T_A = T \Rightarrow T_B = \frac{T}{3}$

So ΔU = Negative [$\because$ $\Delta U = nC_v\Delta T$]

$\Delta W = nR\Delta T$ = Negative

$\Delta Q = \Delta U + \Delta W$ = Negative

Process BC : (Volume is constant)

If $T_B = \frac{T}{3}$ then $T_C = \frac{T}{9}$

$\Delta U = nC_v\Delta T$ = Negative

ΔW = Zero

ΔQ = Negative

Process $C \to D$: (Pressure is constant)

If $T_C = \frac{T}{9}$ then $T_D = T$

$\Delta U = nC_v\Delta T$ = positive

ΔW = positive

ΔQ = positive

Process $D \to A$:

$T_D = T$ and $T_A = T$

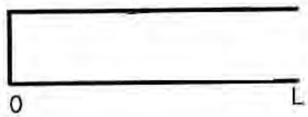
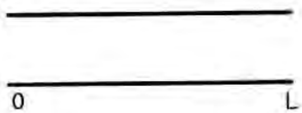
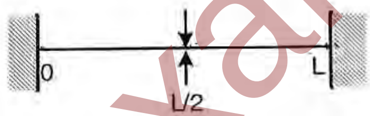
Hence process is isothermal

$\Delta U = 0$

ΔW = negative

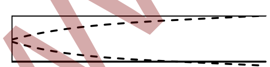
ΔQ = negative

40. **Column I** shows four systems, each of the same length L, for producing standing waves. The lowest possible natural frequency of a system is called its fundamental frequency, whose wavelength is denoted as λ_f. Match each system with statements given in **Column II** describing the nature and wavelength of the standing waves.

Column I	Column II
(A) Pipe closed at one end 	(p) Longitudinal waves
(B) Pipe open at both ends 	(q) Transverse waves
(C) Stretched wire clamped at both ends 	(r) $\lambda_f = L$
(D) Stretched wire clamped at both ends and at mid-point 	(s) $\lambda_f = 2L$
	(t) $\lambda_f = 4L$

Ans. (A) → p,t; (B) → p,s; (C) → q,s; (D) → q,r

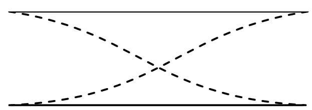
(A)



$$\frac{\lambda_f}{4} = L$$

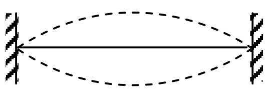
$$\Rightarrow \lambda_f = 4L$$

(B) Longitudinal waves



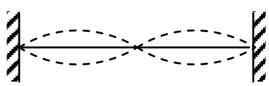
$$\frac{\lambda_f}{2} = L$$

(C) Stretched wire clamped at both ends



$$\frac{\lambda_f}{2} = L \Rightarrow \lambda_f = 2L$$

(D)



$$\frac{\lambda_f}{2} + \frac{\lambda_f}{2} = L$$

$$\Rightarrow \lambda_f = L$$

Part – III : (MATHEMATICS)

SECTION – I (Total Marks : 24)

Code-9 (Single Correct Answer Type) **10/04/2011**

This section contains **8 multiple choice questions**. Each question has four choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

41. Let $P(6,3)$ be a point on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$. If the normal at the point P intersects the x-axis at $(9,0)$, then the eccentricity of the hyperbola is

(A) $\sqrt{\frac{5}{2}}$ (B) $\sqrt{\frac{3}{2}}$ (C) $\sqrt{2}$ (D) $\sqrt{3}$

Ans. **[B]**

Sol. Equation of the normal at (6, 3) is

$$\frac{a^2x}{6}+\frac{b^2y}{3}=a^2+b^2$$

it passes through (9, 0)

so $\frac{9a^2}{6}=a^2+b^2$

$\Rightarrow b^2=\frac{a^2}{2}$

Now $b^2=a^2(e^2-1)$

$\therefore e^2-1=\frac{1}{2}$

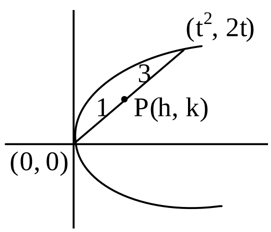
$e^2=\frac{3}{2}$ $\Rightarrow e=\sqrt{\frac{3}{2}}$

42. Let (x, y) be any point on the parabola $y^2=4x$. Let P be the point that divides the line segment from $(0,0)$ to (x,y) in the ratio $1:3$. Then the locus of P is

(A) $x^2=y$ (B) $y^2=2x$ (C) $y^2=x$ (D) $x^2=2y$

Ans. **[C]**

Sol. $h=\frac{t^2}{4}$, $k=\frac{2t}{4}$



$t^2=4h$, $t=2k$

so $4k^2=4h$

$\therefore k^2 = h$

hence required locus is $y^2 = x$

43. Let $f(x) = x^2$ and $g(x) = \sin x$ for all $x \in \mathbb{R}$. Then the set of all x satisfying $(f \circ g \circ g \circ f)(x) = (g \circ g \circ f)(x)$, where $(f \circ g)(x) = f(g(x))$, is

(A) $\pm\sqrt{n\pi}$, $n \in \{0, 1, 2, ...\}$ (B) $\pm\sqrt{n\pi}$, $n \in \{1, 2, ...\}$

(C) $\frac{\pi}{2} + 2n\pi$, $n \in \{..., -2, -1, 0, 1, 2, ...\}$ (D) $2n\pi$, $n \in \{..., -2, -1, 0, 1, 2, ...\}$

Ans. [A]

Sol. $gof(x) = gf(x) = g(x^2) = \sin x^2$

$go(gof(x)) = g(\sin x^2) = \sin(\sin x^2)$

$fo(gogof(x)) = f(\sin(\sin x^2)) = (\sin(\sin x^2))^2$

$\therefore (\sin(\sin x^2))^2 = \sin(\sin x^2)$

$\sin(\sin x^2)(\sin(\sin x^2) - 1) = 0$

$\sin(\sin x^2) = 0$ or $\sin(\sin x^2) = 1$

$\sin x^2 = n\pi$ | $\sin x^2 = 2n\pi + \frac{\pi}{2}$

At n = 0 | At n = 0

$\sin x^2 = 0$ | $\sin x^2 = \frac{\pi}{2}$

$x^2 = n\pi$ | Not possible

$x = \pm\sqrt{n\pi}$; $n \in \{0, 1, 2, ...\}$

44. Let $f : [-1, 2] \to [0, \infty)$ be a continuous function such that $f(x) = f(1-x)$ for all $x \in [-1, 2]$. Let $R_1 = \int_{-1}^{2} x f(x)\,dx$, and R_2 be the area of the region bounded by $y = f(x)$, $x = -1$, $x = 2$, and the x-axis. Then

(A) $R_1 = 2R_2$ (B) $R_1 = 3R_2$ (C) $2R_1 = R_2$ (D) $3R_1 = R_2$

Ans. [C]

Sol. $R_1 = \int_{-1}^{2} x f(x)\,dx$ ... (i)

$R_1 = \int_{-1}^{2} (1-x) f(1-x)\,dx$

$= \int_{-1}^{2} (1-x) f(x)\,dx$ ... (ii)

(i) + (ii)

$2R_1 = \int_{-1}^{2} f(x)\,dx = R_2$

$\therefore 2R_1 = R_2$

45. If

$$\lim_{x \to 0} \left[1 + x \ln(1+b^2)\right]^{\frac{1}{x}} = 2b\sin^2\theta,\ b>0 \text{ and } \theta \in (-\pi, \pi],$$

then the value of θ is

(A) $\pm\frac{\pi}{4}$ (B) $\pm\frac{\pi}{3}$ (C) $\pm\frac{\pi}{6}$ (D) $\pm\frac{\pi}{2}$

Ans. [D]

Sol. $\lim_{x\to 0} (1 + x\,\ell n(1 + b^2)]^{1/x} = 2b \sin^2\theta$ $b > 0;\ \theta \in (-\pi, \pi)$

$$\lim_{x\to 0}\left([1+x\ell n(1+b^2)]^{\frac{1}{x\,\ell n(1+b^2)}}\right)^{\ell n(1+b^2)} = 2b\sin^2\theta$$

$e^{\ell n(1+b^2)} = 2b \sin^2\theta$

$1 + b^2 = 2b \sin^2\theta$

$2\sin^2\theta = b + \frac{1}{b}$

$\text{RHS} = b + \frac{1}{b} \geq 2$ as $b > 0$

But $\text{LHS} = 2\sin^2\theta \leq 2$

Only possibility

$2\sin^2\theta = 2$

$\sin^2\theta = 1$

$\theta = \pm\frac{\pi}{2}$

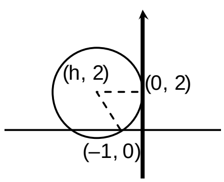
46. The circle passing through the point $(-1, 0)$ and touching the y-axis at $(0, 2)$ also passes through the point

(A) $\left(-\frac{3}{2}, 0\right)$ (B) $\left(-\frac{5}{2}, 2\right)$ (C) $\left(-\frac{3}{2}, \frac{5}{2}\right)$ (D) $(-4, 0)$

Ans. [D]

Sol. $\therefore (h - 0)^2 + (2 - 2)^2 = (h + 1)^2 + (2 - 0)^2$

$h^2 = h^2 + 1 + 2h + 4$



$h = -\frac{5}{2}$

Equation of circle is

$$\left(x+\frac{5}{2}\right)^2 + (y - 2)^2 = \left(-\frac{5}{2} - 0\right)^2$$

$$x^2 + \frac{25}{4} + 5x + y^2 + 4 - 4y = \frac{25}{4}$$

$x^2 + y^2 + 5x - 4y + 4 = 0$

from given points only point (–4, 0) satisfies this equation.

47. Let $\omega \neq 1$ be a cube root of unity and S be the set of all non-singular matrices of the form

$$\begin{bmatrix} 1 & a & b \\ \omega & 1 & c \\ \omega^2 & \omega & 1 \end{bmatrix},$$

where each of a, b, and c is either ω or ω^2. Then the number of distinct matrices in the set S is

(A) 2 (B) 6 (C) 4 (D) 8

Ans. [A]

Sol.
$$\begin{vmatrix} 1 & a & b \\ \omega & 1 & c \\ \omega^2 & \omega & 1 \end{vmatrix} \neq 0$$

$(1 - \omega c) - a(\omega - \omega^2 c) + b(\omega^2 - \omega^2) \neq 0$

$1 - \omega c - a\omega + ac\omega^2 \neq 0$

$(1 - \omega c) - a\omega(1 - \omega c) \neq 0$

$(1 - \omega c)(1 - a\omega) \neq 0$

$c \neq \omega^2$ & $a \neq \omega^2$ & $b = \omega$ or ω^2

$(a, b, c) \equiv (\omega, \omega, \omega)$ or $(\omega, \omega^2, \omega)$

48. A value of b for which the equations

$$x^2 + bx - 1 = 0$$
$$x^2 + x + b = 0,$$

have one root in common is

(A) $-\sqrt{2}$ (B) $-i\sqrt{3}$ (C) $i\sqrt{5}$ (D) $\sqrt{2}$

Ans. [B]

Sol. $x^2 + bx - 1 = 0$ ... (i)

$x^2 + x + b = 0$ ... (ii)

(i) – (ii) we get $x = \frac{b+1}{b-1}$

Put this value in (i)

$$\left(\frac{b+1}{b-1}\right)^2 + b\left(\frac{b+1}{b-1}\right) - 1 = 0$$

$\Rightarrow b^3 + 3b = 0$

$\Rightarrow b(b^2 + 3) = 0$

$\Rightarrow b = 0$ or $b = \pm i\sqrt{3}$

SECTION – II (Total Marks : 16)
(Multiple Correct Answers Type)

This section contains **4 multiple choice questions**. Each question has four choices (A), (B), (C) and (D), out of which **ONE OR MORE** may be correct.

49. If

$$f(x)=\begin{cases} -x-\frac{\pi}{2}, & x\le -\frac{\pi}{2} \\ -\cos x, & -\frac{\pi}{2}<x\le 0 \\ x-1, & 0<x\le 1 \\ \ln x, & x>1, \end{cases}$$

then

(A) $f(x)$ is continuous at $x=-\frac{\pi}{2}$

(B) $f(x)$ is not differentiable at $x=0$

(C) $f(x)$ is differentiable at $x=1$

(D) $f(x)$ is differentiable at $x=-\frac{3}{2}$

Ans. **[A, B, C, D]**

Sol. At $x = -\frac{\pi}{2}$

LHL = 0, RHL = 0, $f\left(-\frac{\pi}{2}\right) = 0$

So f(x) is continuous at $x = -\frac{\pi}{2}$

At x = 0

LHD = 0; RHD = 1

So f(x) is not differentiable at x = 0

At x = 1

LHD = 1, RHD = 1

So f(x) is differentiable at x = 1

in $\left(-\frac{\pi}{2}, 0\right]$; f(x) = – cos x

so f(x) is differentiable at $x = -\frac{3}{2}$

50. Let L be a normal to the parabola $y^2 = 4x$. If L passes through the point $(9, 6)$, then L is given by

(A) $y - x + 3 = 0$
(B) $y + 3x - 33 = 0$
(C) $y + x - 15 = 0$
(D) $y - 2x + 12 = 0$

Ans. **[A, B, D]**

Sol. $y = mx - 2m - m^3$

It passes through (9, 6)

$6 = 9m - 2m - m^3$

$m^3 - 7m + 6 = 0$

$(m - 1)(m - 2)(m + 3) = 0$

$\therefore m = -3, 1, 2$

Hence equations will be

$y = x - 3$, $y = 2x - 12$ and $y = -3x + 33$

51. Let E and F be two independent events. The probability that exactly one of them occurs is $\frac{11}{25}$ and the probability of none of them occurring is $\frac{2}{25}$. If $P(T)$ denotes the probability of occurrence of the event T, then

(A) $P(E) = \frac{4}{5}$, $P(F) = \frac{3}{5}$
(B) $P(E) = \frac{1}{5}$, $P(F) = \frac{2}{5}$
(C) $P(E) = \frac{2}{5}$, $P(F) = \frac{1}{5}$
(D) $P(E) = \frac{3}{5}$, $P(F) = \frac{4}{5}$

Ans. **[A, D]**

Sol. $P(E)(1 - P(F)) + (1 - P(E))P(F) = \frac{11}{25}$

$P(E) + P(F) - 2P(E)P(F) = \frac{11}{25}$ … (1)

$(1 - P(E))(1 - P(F)) = \frac{2}{25}$

$1 - P(E) - P(F) + P(E)P(F) = \frac{2}{25}$

$P(E) + P(F) - P(E)P(F) = \frac{23}{25}$ … (2)

From (1) & (2)

$P(E)P(F) = \frac{12}{25}$

and $P(E) + P(F) = \frac{7}{5}$

so either

$P(E) = \frac{4}{5}$, $P(F) = \frac{3}{5}$ and $P(E) = \frac{3}{5}$, $P(F) = \frac{4}{5}$

52. Let $f:(0,1)\to\mathbb{R}$ be defined by

$$f(x)=\frac{b-x}{1-bx},$$

where b is a constant such that $0<b<1$. Then

(A) f is not invertible on $(0,1)$

(B) $f\neq f^{-1}$ on $(0,1)$ and $f'(b)=\frac{1}{f'(0)}$

(C) $f=f^{-1}$ on $(0,1)$ and $f'(b)=\frac{1}{f'(0)}$

(D) f^{-1} is differentiable on $(0,1)$

Ans. **[A, B]**

Sol. f : (0, 1) $\to$ R

$f(x) = \frac{b-x}{1-bx} \quad \forall\ b \in (0, 1)$

$f'(x) = \frac{b^2-1}{(1-bx)^2} = (-)$ ve

So f(x) is monotonically decreasing for $x \in (0, 1)$

so for $x \in (0, 1)$

$f(x) \in (f(1), f(0))$

$f(x) \in (-1, b)$

so f(x) is not onto.

so f(x) is not invertible function.

SECTION – III (Total Marks : 15)
(Integer Answer Type)

This section contains **6 questions**. The answer to each of the questions is a **single-digit integer**, ranging from 0 to 9. The bubble corresponding to the correct answer is to be darkened in the ORS

53. Let $y'(x)+y(x)g'(x)=g(x)g'(x)$, $y(0)=0$, $x\in\mathbb{R}$, where $f'(x)$ denotes $\frac{d\,f(x)}{dx}$ and $g(x)$ is a given non-constant differentiable function on $\mathbb{R}$ with $g(0)=g(2)=0$. Then the value of $y(2)$ is

Ans. **[0]**

Sol. $\frac{dy}{dg} + y = g$

I. F. = $\int 1.dg = g$

$y.e^g = \int ge^g.dg = ge^g - \int e^g.dg$

$ye^g = ge^g - e^g + c$

$y = g - 1 + ce^{-g}$

$\because$ y(0) = 0 & g(0) = 0

at x = 0

$0 = 0 - 1 + Ce^{-0}$

C = 1

$y = g - 1 + e^{-g}$

at x = 2

$y(2) = 0 - 1 + e^{-0} = 0$

54. Let $\vec{a} = -\hat{i} - \hat{k}$, $\vec{b} = -\hat{i} + \hat{j}$ and $\vec{c} = \hat{i} + 2\hat{j} + 3\hat{k}$ be three given vectors. If $\vec{r}$ is a vector such that $\vec{r} \times \vec{b} = \vec{c} \times \vec{b}$ and $\vec{r}.\vec{a} = 0$, then the value of $\vec{r}.\vec{b}$ is

Ans. **[9]**

Sol. $\vec{a} = -\hat{i} - \hat{k}$, $\vec{b} = -\hat{i} + \hat{j}$, $\vec{c} = \hat{i} + 2\hat{j} + 3\hat{k}$

$(\vec{r} - \vec{c}) \times \vec{b} = 0 \Rightarrow \vec{r} - \vec{c} = \lambda\vec{b} \Rightarrow \vec{r} = \vec{c} + \lambda\vec{b}$

$\because \quad \vec{r} \cdot \vec{a} = 0$

$\Rightarrow \quad \vec{a}.\vec{c} + \lambda\vec{b}.\vec{a} = 0$

$\Rightarrow \quad \lambda = -\dfrac{\vec{a}.\vec{c}}{\vec{b}.\vec{a}} = 4$

$\Rightarrow \quad \vec{r}.\vec{b} = \vec{c}.\vec{b} + \lambda|\vec{b}|^2 = 9$

55. Let $\omega = e^{i\pi/3}$, and a, b, c, x, y, z be non-zero complex numbers such that

$$a + b + c = x$$
$$a + b\omega + c\omega^2 = y$$
$$a + b\omega^2 + c\omega = z.$$

Then the value of $\dfrac{|x|^2 + |y|^2 + |z|^2}{|a|^2 + |b|^2 + |c|^2}$ is

Ans. **[*]**

Sol. wrong question if $\omega = e^{i2\pi/3}$ then ans is 3. If $\omega = e^{i\pi/3}$ then no integral solution is possible.

56. Let M be a 3×3 matrix satisfying

$$M\begin{bmatrix}0\\1\\0\end{bmatrix} = \begin{bmatrix}-1\\2\\3\end{bmatrix}, \quad M\begin{bmatrix}1\\-1\\0\end{bmatrix} = \begin{bmatrix}1\\1\\-1\end{bmatrix}, \text{ and } M\begin{bmatrix}1\\1\\1\end{bmatrix} = \begin{bmatrix}0\\0\\12\end{bmatrix}.$$

Then the sum of the diagonal entries of M is

Ans. [9]

Sol. Let $M = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$

$\because\ M \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix} \Rightarrow b = -1,\ e = 2,\ h = 3$

$\therefore\ M \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} \Rightarrow a = 0,\ d = 3,\ g = 2$

$M \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 12 \end{bmatrix} \Rightarrow c = 1,\ f = -5,\ i = 7$

So $a + e + i = 0 + 2 + 7 = 9$

57. The number of distinct real roots of $x^4 - 4x^3 + 12x^2 + x - 1 = 0$ is

Ans. [2]

Sol. Let $f(x) = x^4 - 4x^3 + 12x^2 + x - 1$

Let $\alpha, \beta, \gamma, \delta$ are the root of equation.

$\therefore$ $\alpha\beta\gamma\delta = -1$ so the equation has at least two real roots. .....(i)

$f'(x) = 4x^3 - 12x^2 + 24x + 1$

$f''(x) = 12x^2 - 24x + 24 = 12((x + 1)^2 + 1)$

so $f''(x) > 0$ so $f'(x) = 0$ has only one real roots so $f(x) = 0$ has at most two real roots. ....(ii)

from (i) & (ii)

$f(x) = 0$ has exactly two real roots.

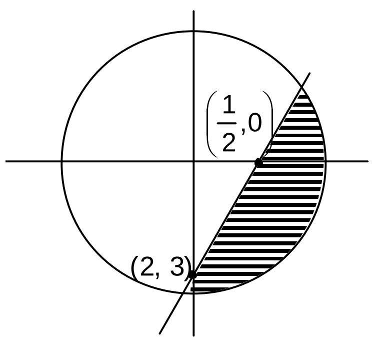
58. The straight line $2x - 3y = 1$ divides the circular region $x^2 + y^2 \le 6$ into two parts. If

$$S = \left\{\left(2, \frac{3}{4}\right), \left(\frac{5}{2}, \frac{3}{4}\right), \left(\frac{1}{4}, -\frac{1}{4}\right), \left(\frac{1}{8}, \frac{1}{4}\right)\right\},$$

then the number of point(s) in S lying inside the smaller part is

Ans. [2]

Sol.



Pont (x_1, y_1) lies inside the region if $x_1^2 + y_1^2 - 6 \le 0$ & $2x_1 - 3y_1 - 1 \le 0$.

$P_1 \equiv \left(2, \frac{3}{4}\right)$ $\quad 4 + \frac{9}{16} - 6 \le 0$ True

$\quad 4 - \frac{9}{4} - 1 > 0$ True

$P_2 \equiv \left(\frac{5}{2}, \frac{3}{4}\right)$ $\quad \frac{25}{4} + \frac{9}{16} - 6 \le 0$ False

$P_3 \equiv \left(\frac{1}{4}, \frac{-1}{4}\right)$ $\quad \frac{1}{16} + \frac{1}{16} - 6 \le 0$ True

$\quad \frac{2}{4} + \frac{3}{4} - 1 > 0$ True

$P_4 \equiv \left(\frac{1}{8}, \frac{1}{4}\right)$ $\quad \frac{1}{64} + \frac{1}{16} - 6 \le 0$ True

$\quad \frac{2}{8} - \frac{3}{4} - 1 > 0$ False

So P_1 & P_3 lies in the interval

SECTION – IV(Total marks : 28)
(Integer Answer Type)

This section contains **2 questions**. Each question has **four statements** (A, B, C and D) given in **Column I** and **five statements** (p, q, r, s and t) in **Column II**. Any given statement Column I can have correct matching with **ONE** or **MORE** statement(s) given in Column II. For example, if for a given question, statement B matches with the statements given in q and r, then for the particular question, against statement B, darken the bubbles corresponds to q and r in the ORS.

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59. Match the statements given in **Column I** with the values given in **Column II**

Column I

(A) If $\vec{a} = \hat{j} + \sqrt{3}\hat{k}$, $\vec{b} = -\hat{j} + \sqrt{3}\hat{k}$ and $\vec{c} = 2\sqrt{3}\hat{k}$ form a triangle, then the internal angle of the triangle between $\vec{a}$ and $\vec{b}$ is

(B) If $\int_a^b (f(x) - 3x)\,dx = a^2 - b^2$, then the value of $f\left(\frac{\pi}{6}\right)$ is

(C) The value of $\frac{\pi^2}{\ln 3} \int_{7/6}^{5/6} \sec(\pi x)\,dx$ is

(D) The maximum value of $\left|Arg\left(\frac{1}{1-z}\right)\right|$ for $|z| = 1$, $z \ne 1$ is given by

Column II

(p) $\frac{\pi}{6}$

(q) $\frac{2\pi}{3}$

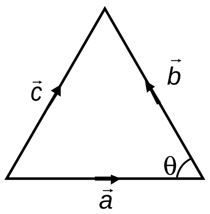
(r) $\frac{\pi}{3}$

(s) π

(t) $\frac{\pi}{2}$

Ans. **[A → q ; B → p ; C → s ; D → t]**

Sol. **(A)**



$$\cos\theta = \frac{-\vec{a}\cdot\vec{b}}{|-\vec{a}||\vec{b}|} = -\frac{1}{2} \Rightarrow \theta = \frac{2\pi}{3}$$

(B) $\int_a^b (f(x) - 3(x))dx = a^2 - b^2$

differentiating w.r.t (b).

f(b) – 3b = – 2b

$$\boxed{f(b) = b}$$

So $f\left(\frac{\pi}{6}\right) = \frac{\pi}{6}$

(C) $I = \frac{\pi^2}{\ell n3}\int_{7/6}^{5/6} \sec \pi x \, dx$

$$I = \frac{\pi^2}{\pi \ell n3}\left|\ell n|\sec \pi x + \tan \pi x|\right|_{7/6}^{5/6}$$

$$I = \frac{\pi}{\ell n3}\cdot \ell n3 = \pi$$

(D) ∴ |z| = 1

z = cos θ + *i* sin θ. ∀ θ∈ (– π. π] and θ ≠ 0.

$$\left|Arg\frac{1}{(1-z)}\right| = \left|Arg\left(\frac{1}{1-\cos\theta - i\sin\theta}\right)\right| = \left|Arg\left(\frac{1}{2} + \frac{i\cot\frac{\theta}{2}}{2}\right)\right| = \left|\frac{\theta}{2}\right|$$ so maximum values is π/2.

60. Match the statements given in **Column I** with the intervals/union of intervals given in **Column II**

Column I	Column II
(A) The set $\left\{ \mathrm{Re}\left(\frac{2iz}{1-z^2}\right) : z \text{ is a complex number}, \lvert z \rvert = 1, z \neq \pm 1 \right\}$ is	(p) $(-\infty, -1) \cup (1, \infty)$
	(q) $(-\infty, 0) \cup (0, \infty)$
(B) The domain of the function $f(x) = \sin^{-1}\left(\frac{8(3)^{x-2}}{1-3^{2(x-1)}}\right)$ is	(r) $[2, \infty)$
(C) If $f(\theta) = \begin{vmatrix} 1 & \tan\theta & 1 \\ -\tan\theta & 1 & \tan\theta \\ -1 & -\tan\theta & 1 \end{vmatrix}$, then the set $\left\{ f(\theta) : 0 \le \theta < \frac{\pi}{2} \right\}$ is	(s) $(-\infty, -1] \cup [1, \infty)$
(D) If $f(x) = x^{3/2}(3x - 10)$, $x \ge 0$, then $f(x)$ is increasing in	(t) $(-\infty, 0] \cup [2, \infty)$

Ans. **[A → p, r, s ; B → r, t ; C → r ; D → r]**

Sol. **(A)** Let z = cos θ + *i* sin θ

so $\frac{2iz}{1-z^2} = \frac{2i(\cos + i\sin\theta)}{1-\cos 2\theta - i\sin 2\theta} = -\text{cosec}\theta \quad \forall\, \theta \neq (2n+1)\frac{\pi}{2}$

so $\mathrm{Re}\left(\frac{2iz}{1-z^2}\right) = -\text{cosec}\,\theta \in (-\infty, -1] \cup [1, \infty)$

(B) $\frac{8 \times 3^{x-2}}{1-3^{2x-2}} = \frac{8 \times 3^x}{9 - 3^{2x}}$

Let $3^x = t$

So $f(x) = \sin^{-1}\left(\frac{8 \times 3^x}{9 - 3^{2x}}\right) = \sin^{-1}\left(\frac{8t}{9-t^2}\right)$

$-1 \le \frac{8t}{9-t^2} \le 1$ on solving

$x \in (-\infty, 0] \cup [2, \infty) \cup \{1\}$

(C) $f(\theta) = 2\sec^2\theta$

so $f(\theta) \in [2, \infty)$

(D) $f(x) = 3x^{5/2} - 10x^{3/2}$

$f'(x) = \frac{15\sqrt{x}}{2}(x-2)$

So f(x) is increasing for $f'(x) \ge 0$

$x \in [2, \infty)$