MASTER OF ARTS (ECONOMICS)

Term-End Examination

June, 2006

MEC-003 : QUANTITATIVE METHODS

Time : 3 hours *Maximum Marks : 100*

Note : *Answer **two** questions from Section A, **four** from Section B and **two** from Section C.*

SECTION A

*Answer any **two** questions from this section.* 2×20

1. (i) Explain the meaning of necessary and sufficient conditions encountered in optimisation problems.

(ii) A monopoly firm produces two products and faces the demand functions,

$$X_1 = 40 - 2P_1 + P_2$$
$$X_2 = 15 + P_1 + P_2.$$

If its total cost function is given as

$$C = X_1^2 + X_1 X_2 + X_2^2,$$

find the profit maximising necessary and sufficient conditions and get the equilibrium output, profit and prices.

2. (i) Write a differential equation of first order and list the steps you will pass through for obtaining its solution.

(ii) If the production function is given as $q = A L^{\alpha} K^{1-\alpha}$, where A is a positive constant and $0 < \alpha < 1$, follow the logic of Solow's growth model to solve for K using differential equation. You know that a constant fraction s of output is saved with $0 < s < 1$. Moreover, you also know that the saving is used to augment the capital stock.

3. (i) What is an input coefficient matrix ?

(ii) You are given the following technology matrix. Find the equilibrium prices if the wage rate is Rs. 10 per day.

	Steel	Coal	Final Demand
Steel	0·4	0·1	50
Coal	0·7	0·6	100
Labour	5	2	

4. (i) Discuss the assumptions made in formulating a linear programming problem.

(ii) Using Simplex method, solve the problem :

Maximise $\pi = 6x_1 + 2x_2 + 5x_3$

$$\text{subject to} \begin{bmatrix} 2 & 3 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \leq \begin{bmatrix} 10 \\ 8 \\ 19 \end{bmatrix}$$

SECTION B

*Answer any **four** questions from this section.* *4×10*

5. You are given a Cobb-Douglas production function, $q = A L^{\alpha} K^{1-\alpha}$ with A, $\alpha > 0$ and q, L and K are level of output, labour and capital respectively. Prove that it is homogeneous of degree 1 and explain its meaning.

6. Find the extreme value(s) of the equation $q = P^2 - 6$ and determine whether they are maxima or minima.

7. Let the consumer's demand function be $p = 20 - q$. Derive the consumer's surplus for $p = 8$.

8. Find the time path represented by the equation

$$y_t = 2\left(-\frac{4}{5}\right)^t + 9.$$

9. Use Cramer's rule to solve the following national income model :

$$Y = C + I + G \quad \text{and} \quad C = a + bY$$

10. The correlation coefficient between height and weight for a group of 20 Indian adult males was found to be 0·203. Test if there is any correlation between these traits in the population. You are given the table values of $t_{0.025,\,18} = 2{\cdot}101$ and $t_{0.005,\,18} = 2{\cdot}878$.

SECTION C

*Answer any **two** questions from this section.* *2×10*

11. Define the following terms :

(i) Type II error

(ii) Monotone function

(iii) Inverse function

(iv) L'Hospital's rule

(v) Homogeneous function

12. Answer the following as directed :

(i) Use Lagrange multiplier method to find the stationary value of z, if $z = x - 3y - xy$ subject to $x + y = 6$.

(ii) Find the limit of the function

$$y = \frac{3x+5}{x+2} \text{ when } x \to 0.$$

13. (i) From the function $\phi(x) = \frac{1-x}{1+x}$, find the Taylor's series with $n = 4$ and $x_0 = -2$.

(ii) Find the inner product of the following pair of vectors :

(2, 3, 4) and (4, 5, 5).