S.E. Sem 4. Applied Maths 4 06/01/09
Etrx & EXTC.

Con. 5926-08. RC-6065

(3 Hours) [Total Marks : 100

N.B. (1) Questions No. **1** is **compulsory**.
(2) Attempt any **four** questions from the remaining **six** questions.
(3) **Figures** to the **right** indicate **full** marks.

1. (a) Find the characteristic equation of the matrix $A = \begin{bmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$ and show that the equation is also satisfied by A. **5**

(b) A vector field is given by $F = (\sin y)\, i + x(1 + \cos y)\, j$ then evaluate $\int_C F \cdot dr$ over a circular path given by $x^2 + y^2 = a^2,\ z = 0$. **5**

(c) If $F = 3xyi + 4yz^2j - 5xzk$ and $\phi = y^2 - zx$ then find : Div (ϕF) and curl (ϕF). **5**

(d) State and prove Cauchys Integral formula. **5**

2. (a) Find the directional derivative of the function $f = x^2 - y^2 + 2z^2$ at the point P(1, 2, 3) in the direction of the line PQ where Q is the point (5, 0, 4). **6**

(b) Find all the eigenvalues and eigenvectors of the matrix : **6**

$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

(c) Find all possible Laurent's expansion of the function $f(z) = \dfrac{z^3 - 6z - 1}{(z-1)(z-3)(z+2)}$ about $z = 3$ indicating the region of convergence. **8**

3. (a) Prove that $F = (y^2 \cos x + z^3)\, i + (2y \sin x - 4) j + (3xz^2 + 2)\, k$ is a conservative field and find scalar potential ϕ such that $F = \nabla \phi$. **6**

(b) Define : **6**
(1) Singular point
(2) Isolated singular point
(3) Essential singular point, with one example.

(c) Reduce the quadratic form **8**
$2x^2 + 9y^2 + 6z^2 + 8xy + 8yz + 6xz$ to sum of squares by linear transformation. Also find the rank, index and signature.

4. (a) State Residue theorem and evaluate the integral $\oint_C \frac{z-3}{z^2+2z+5}\,dz$ where C is the circle 6

(1) $|z+1-i| = 2$ (2) $|z+1+i| = 2$.

(b) Test whether the matrix $A = \begin{bmatrix} 7 & 4 & -1 \\ 4 & 7 & -1 \\ -4 & -4 & 4 \end{bmatrix}$ is derogatory ? 6

(c) Verify Green's theorem in the plane for 8

$$\oint_C (3x^2 - 8y^2)\,dx + (4y - 6xy)\,dy$$

where C is the boundry of the region defined by $y = \sqrt{x}$, $y = x^2$.

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5. (a) If $A = \begin{bmatrix} 2 & 3 \\ -3 & -4 \end{bmatrix}$ then prove that $A^{50} = \begin{bmatrix} -149 & -150 \\ 150 & 151 \end{bmatrix}$. 6

(b) Evaluate $\iint_S F \cdot ds$ for $F = 18zi - 12xj + 3yk$ where S is area of the plane $2x + 3y + 6z = 12$ lying in the first octant. 6

(c) Evaluate (1) $\int_0^{2\pi} \frac{1}{5 + 4\cos\theta} d\theta$ 8

(2) $\int_0^{\infty} \frac{1}{(x^2 + a^2)} dx$.

6. (a) Verify Cauchys Integral theorem for $f(z) = z^2$ along a circle C, $|z| = 1$. 6

(b) Apply Stokes theorem to evaluate— 6

$$\int_C [(x + 2y)dx + (x - z)dy + (y - z)dz]$$

where C is the boundry of the triangle with vertices (2, 0, 0), (0, 3, 0) and (0, 0, 6) oriented in the counter clockwise direction.

(c) Test whether the matrix $A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$ is diagonalisable ? If yes, find the transforming matrix P and the diagonal matrix D. 8

7. (a) (i) If λ_1, λ_2 and λ_3 are the eigenvalues of the matrix 3

$\begin{bmatrix} -2 & -9 & 5 \\ -5 & -10 & 7 \\ -9 & -21 & 14 \end{bmatrix}$ then find the values of (1) $\lambda_1 + \lambda_2 + \lambda_3$ (2) $\lambda_1 \lambda_2 \lambda_3$

(ii) Prove that the characteristic roots of a Hermitian matrix are real. 3

(b) If $f(a) = \int_C \frac{4z^2 + z + 4}{z - a} dz$ and C is $4x^2 + 9y^2 = 36$ then find : (1) $f(i)$ (2) $f'(1)$ (3) $f''(i)$. 6

(c) Verify divergence theorem for $F = 2x^2yi + y^2j + 4xz^2k$ taken over the region in the first octant bounded by $y^2 + z^2 = 9$ and $x = 2$. 8