S.E. EXTC Sem IV / Rev

Applied Mathematics.

Con. 3803-10. (REVISED COURSE) AN-3496

(3 Hours) [Total Marks : 100

N.B. (1) Question No. **1** is **compulsory**.
(2) Attempt any **four** questions out of the remaining **six** questions.
(3) **Figures** to the **right** indicate **full** marks.

1. (a) The matrix A is given by $A = \begin{bmatrix} 1 & 2 & -3 \\ 0 & 3 & 2 \\ 0 & 0 & -2 \end{bmatrix}$ Find the eigen value of B where $B = A^2 + 2A + I - 6A^{-1}$ 5

(b) Evaluate $\int_C \frac{dz}{\sinh z}$ where C is $x^2 + 2y^2 = 16$ and define simple pole. 5

(c) Find the work done in moving a particle once round the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ in the plane Z = 0 in the force field given by $\overline{F} = (3x - 2y)\, i + (2x + 3y)\, j + y^2 k$ 5

(d) Prove that $4J_n''(x) = J_{n-2}(x) - 2J_n(x) + J_{n+2}(x)$ 5

2. (a) Prove that $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}} \sin(x)$. Hence prove that $J_{-1/2}(x) = \sqrt{\frac{2}{\pi x}} \cos(x)$ 6

(b) Show that the map of real axis of the Z-plane is a circle under the transformation $w = \frac{2}{z+i}$. Draw the figure. 7

(c) If $A = \begin{bmatrix} 3/2 & 1/2 \\ 1/2 & 3/2 \end{bmatrix}$ determine A^{10} and 4^A. 7

3. (a) Find the eigen values and eigen vectors of a matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 3 & 2 & 3 \end{bmatrix}$ 6

(b) Evaluate the line integral $\int_C \left(3x^2 y dx + 2y^3 x dy\right)$ where C is the circle $x^2 + y^2 = 1$, counter clockwise from (1, 0) to (–1, 0). 7

(c) If $\overline{F} = (2y^2 + 3z^2 - x^2)\, i + (2z^2 + 3x^2 - y^2)\, j + (2x^2 + 3y^2 - z^2)\, K$ and S is the surface enclosed by $x^2 + y^2 - 2ax + az = 0$ and $z \geq 0$, using Stoke's theorem evaluate $\iint_S (\nabla \times \overline{F}) \cdot d\overline{s}$ 7

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4. (a) Find the bilinear transformation which maps the points $z = \infty, i, 0$ onto the point $0, i, \infty$. Also find fixed points. 6

(b) Evaluate $\int_C \frac{e^{2z} \cdot dz}{(z+1)^4}$ where C is the (i) circle $|z-1| = 3$ (ii) circle $|z| = 0 \cdot 5$ 7

(c) Find the rank and signature of the real quadratic form — 7

$$2x_1^2 + x_2^2 - 3x_3^2 - 8x_2x_3 = 4x_1x_3 - 12x_1x_2$$

and reduce it to normal form through congruent transformations.

5. (a) Using Green's theorem evaluate $I = \oint_C \left[\left(xy + y^2\right)dx + x^2dy \right]$ where C is the closed curve of the region bounded by $y = x$ and $y = x^2$. 6

(b) If $f(z) = u + iv$ is analytic and $u + v = \frac{6\sin(2x)}{e^{2y} + e^{-2y} - 2\cos(2x)}$. Find f(z) 7

(c) Obtain Laurent's series for $f(z) = \frac{z^2 - 1}{z^2 + 5z + 6}$ around $z = 1$ 7

6. (a) Prove that $\frac{d}{dx}\left[x J_n J_{n+1} \right] = x\left[J_n^2 - J_{n+1}^2 \right]$ 6

(b) Show that the matrix $A = \begin{bmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{bmatrix}$ satisfies Cayley-Hamilton Theorem 7

where a, b, c are positive real nos.

(c) Evaluate $\int_0^{2\pi} \frac{\sin^2\theta \, d\theta}{5 - 4\cos\theta}$ 7

7. (a) If $f(z) = \frac{x^3(1+i) - y^3(1-i)}{x^2 + y^2}$ when $z \neq 0$ 6

$= 0$ when $z = 0$

then prove that (i) C – R equations are satisfied at origin But
(ii) $f'(0)$ does not exist.

(b) $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ find A^{50} 7

(c) If $\overline{F} = 2x^2y\, i - y^2\, j + 4xz^2\, k$ and S is the closed surface in the first octant bounded by the cylinder $y^2 + z^2 = 9$ and $x = 2$ then using Gauss divergence theorem evaluate $\iint_S \overline{N} \cdot \overline{F}\, ds$. 7